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BS (Software engineering. 2nd Semester.

Qs No: 1

Compute adjoint of:

$$A = \begin{bmatrix} 1 & 2 & \text{2nd-I.D} \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

Solution

$$A = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

First of all we find cofector of A.

$$A_{ij} = (-1)^{i+j} \cdot M_{ij}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 1(6-1) = \boxed{5}$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = -1(4-3) = \boxed{-1}$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = 1(2-9) = \boxed{-7}$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 6 \\ 1 & 2 \end{vmatrix} = -1(4-6) = \boxed{2}$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 6 \\ 3 & 2 \end{vmatrix} = 1(2-18) = \boxed{-16}$$

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$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -1(1-6) = \boxed{5}$$

$$A_{31} = (-1)^{3+1} = \begin{vmatrix} 2 & 6 \\ 3 & 1 \end{vmatrix} = +1(2-18) = \boxed{-16}$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 6 \\ 2 & 1 \end{vmatrix} = -1(1-12) = \boxed{11}$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1(3-4) = \boxed{-1}$$

Now find adjoint of A.

$$A = \begin{bmatrix} 5 & -1 & -7 \\ 2 & -16 & 5 \\ -16 & 11 & -1 \end{bmatrix}$$

Taking transpose of A.

$$A^t = \begin{bmatrix} 5 & 2 & -16 \\ -1 & -16 & 11 \\ -7 & 5 & -1 \end{bmatrix}$$

So this is a required answer.

P-T-O

$$B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$$

Solution:

first of all we find cofector of B.

$B_{ij} = (-1)^{i+j} \cdot M_{ij}$

$B_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -1 & 8 \\ -2 & 8 \end{vmatrix} = 1(-8+16) = \boxed{8}$

$B_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & 8 \\ 5 & 8 \end{vmatrix} = -1(16-40) = \boxed{24}$

$B_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = 1(4-5) = \boxed{-1}$

$B_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 4 & 5 \\ -2 & 8 \end{vmatrix} = -1(32+10) = \boxed{-42}$

$B_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix} = 1(24-25) = \boxed{-1}$

$B_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = -1(-6-20) = \boxed{26}$

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$$B_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix} = 1(40+5) = \boxed{45}$$

$$B_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix} = -1(24-10) = \boxed{-14}$$

$$B_{33} = (-1)^{3+3} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = 1(-3-8) = \boxed{-11}$$

Now we find adjoint ~~of~~

B.

$$B = \begin{bmatrix} 8 & 24 & 1 \\ -42 & -24 & 26 \\ 45 & -14 & -11 \end{bmatrix}$$

Now taking transpose on B.

$$B^t = \begin{bmatrix} 8 & -42 & 45 \\ 24 & -24 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

So it is required Answer.

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Ques No: 02

Cofector of A_{21}, A_{31}, A_{33} if

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix}$$

$$= -1(-4+9) = \boxed{-5}$$

Cofectors of A_{21} is -5

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix}$$

$$= +1(-2-9) = \boxed{-11}$$

Cofectors of A_{31} is -7

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$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix}$$

$$= -1(3-4) = \boxed{+1}$$

Cofecter of A_{33} is $+1$

Cofecters of

$A_{21} = \boxed{-5}$

$A_{31} = \boxed{-11}$

$A_{33} = \boxed{1}$

Ans

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Question No : 03

Find λ given value and given vector, if

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \quad \text{and} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution :

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$$

Det$(A-\lambda I) = 0 \rightarrow$ values.

$$\left| \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right|$$

$$\left| \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right|$$

$$= \left| \begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{bmatrix} \right|$$

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$(2-\lambda)\left[(3-\lambda)(2-\lambda)-2\right]-1(2-\lambda+2)+1(1+3-\lambda)=$

$=(2-\lambda)\left[6-3\lambda-2\lambda+\lambda^2-2\right]-\left[2-\lambda+2\right)+1(4-\lambda)=0$

$=(2-\lambda)(4-5\lambda+\lambda^2)-(4-\lambda)+(4-\lambda)=0$

$=8-10\lambda+2\lambda^2-4\lambda+5\lambda^2-\lambda^3-\cancel{4}+\cancel{\lambda}+\cancel{4}-\cancel{\lambda}=0$

$=-\lambda^3+7\lambda^3-14\lambda+8=0$

+2	-1	7	-14	8
	↓	-2	+10	-8
	-1	5	-4	0

$-\lambda^2+5\lambda-4=0$

$\lambda^2-5\lambda+4=0$

$\lambda^2-4\lambda-\lambda+4=0$

$\lambda(\lambda-4)-1(\lambda-4)=0$

$(\lambda-1)(\lambda-4)=0$

P.T.O

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$\lambda - 1 = 0$, $\lambda - 4 = 0$

$\boxed{\lambda = 1}$, $\boxed{\lambda = 4}$

$\lambda = 2$, $\lambda = 1$, $\lambda = 4$

Eigen values $\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$

For Eigen vector

Det $(A - \lambda I)x = 0$

$$\left| \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{vmatrix} 2-2 & 1 & 1 \\ 1 & 3-2 & 2 \\ -1 & 1 & 2-2 \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & 0 \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

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$$0x + y + z = 0 \quad \text{——— (1)}$$
$$x + y + 2z = 0 \quad \text{——— (2)}$$
$$-x + y + 0z = 0 \quad \text{——— (3)}$$

Sub (1) → (3)

$$y + z = 0$$
$$\mp x \pm y \pm 0 = 0$$
$$x + z = 0$$

$$x = -z \quad \text{— (4)} \rightarrow \text{put in eq (2)}$$

$$-z + y + 2z = 0$$

$$y - z = 0 \quad \text{— (5)}$$

Add (5) + (1)

$$y - z = 0$$
$$y + z = 0$$

$$2y = 0$$

$$\boxed{y = 0} \rightarrow \text{put in eq (1)}$$

$$0 + z = 0 \Rightarrow \boxed{z = 0}$$

$$x + y + 2z = 0$$
$$0 + 0 + 0 = 0 \Rightarrow \text{proves}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \text{Eigen vector.}$$