

Q1

Solution:

Let a be the first term and d be the common difference of the arithmetic sequence.

Then

$$a_n = a + (n-1)d \quad n \geq 1$$

$$\Rightarrow a_3 = a + (3-1)d$$

and $a_8 = a + (8-1)d$

Given that $a_3 = 7$ and $a_8 = 17$. Therefore

$$7 = a + 2d \dots\dots\dots (1)$$

and $17 = a + 7d \dots\dots\dots (2)$

Subtracting (1) from (2), we get,

$$10 = 5d$$

$$\Rightarrow d = 2$$

Substituting $d = 2$ in (1) we have

$$7 = a + 2(2)$$

which gives $a = 3$

Thus, $a_n = a + (n-1)d$

$$a_n = 3 + (n-1)2 \text{ (using values of } a \text{ and } d)$$

Hence the value of 36th term is

$$a_{36} = 3 + (36-1)2$$

$$= 3 + 70$$

$$= 73$$

Ans

Q2 Given $f(x) = 2x + 3$ and $g(x)$

$= -x^2 + 5$ find $(f \circ f)(x)$.

Sol: $(f \circ f)(x) = f(f(x))$

$$= f(2x + 3)$$

$= 2(\quad) + 3$... setting up to insert

the input

$$= 2(2x+3) + 3$$

$$= 4x + 6 + 3$$

$$= 4x + 9$$

Given $f(x) = 2x + 3$ and $g(x) = -x^2 + 5$
Find $(g \circ g)(x)$.

$$(g \circ g)(x) = g(g(x))$$

$$= -(\quad)^2 + 5 \quad \text{Setting up to insert the input}$$

$$= -(-x^2 + 5)^2 + 5$$

$$= -(x^4 - 10x^2 + 25) + 5$$

$$= -x^4 + 10x^2 - 25 + 5$$

$$= -x^4 + 10x^2 - 20$$

Ans

Q3

Sol: Basis Step

$P(1)$ is ~~the~~ true

for $n=1$

$$\text{L.H.S of } P(1) = 1^2 = 1$$

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$$\begin{aligned} \text{R.H.S of } P(1) &= \frac{1(1+1)(2(1)+1)}{6} \\ &= \frac{(1)(2)(3)}{6} = \frac{6}{6} = 1 \end{aligned}$$

So L.H.S = R.H.S of $P(1)$. Hence

$P(1)$ is true

inductive step

Suppose $P(k)$ is true for some integer $k \geq 1$:

$$1^2 + 2^2 + 3^2 + \dots + k^2 = k(k+1)(2k+1)$$

To prove $P(k+1)$ is true; i.e;

$$1^2 + 2^2 + 3^2 + \dots + (k+1)^2 = (k+1)(k+1+1)(2(k+1)+1)$$

Consider LHS of above equation

Consider LHS of above equation (2)

$$1^2 + 2^2 + 3^2 + \dots + (k+1)^2$$

$$= 1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= (k+1) \left[\frac{k(2k+1)}{6} + (k+1) \right]$$

$$= (k+1) \left[\frac{k(2k+1) + 6(k+1)}{6} \right]$$

$$= (k+1) \left[\frac{2k^2 + k + 6k + 6}{6} \right]$$

$$= (k+1)(2k^2 + 7k + 6)$$

$$= (k+1)(k+2)(2k+3)$$

$$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

This $P(k+1)$ is ~~also~~ true whenever $P(k)$ is true.

Hence from the Principle of Induction the statement $P(n)$ is true for all natural number n .

Q4

Ans: Relations and its types concepts are one of the important topics of Set theory. Sets, relations and functions all three are interlinked topics. Sets denote the collection of ordered elements whereas relations and functions define the operations performed on sets.

The relations define the connection between the two given sets.

Also, there are types of relations stating the connection between the sets, hence, here we will learn about

Relations and their types
in detail.

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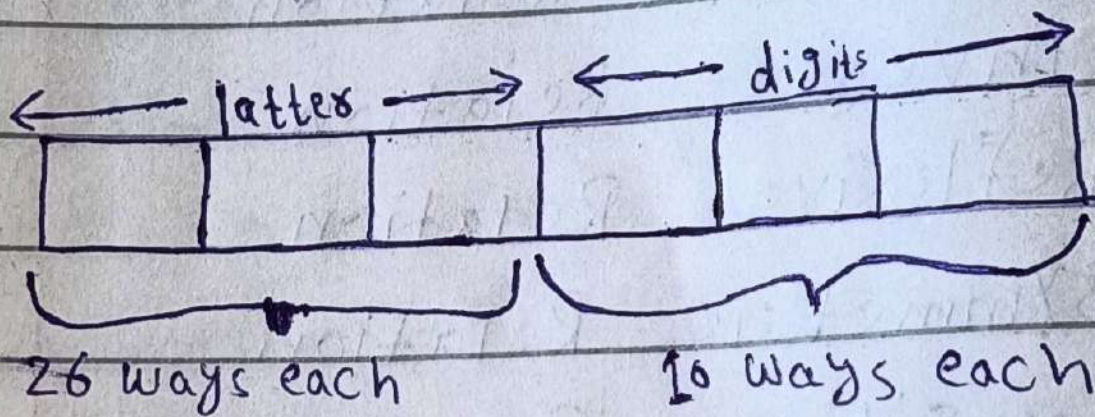
- Definition
- Types
 - Empty Relation
 - Universal Relation
 - Identity Relation
 - Inverse Relation
 - Reflexive Relation
 - Symmetric Relation
 - Transitive Relation
 - Equivalence Relation

Representation

Q5

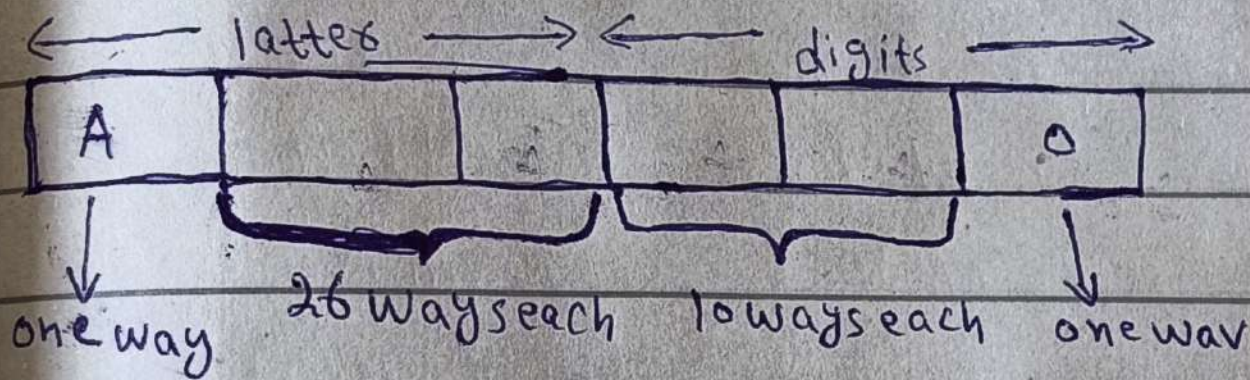
Solution:

1) Each of the three letters can be written in 26 different ways, and each of the three digits can be written in 10 different ways.



Hence, by the Product Rule, there is a total of $26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17,576,000$ different license plates possible.

2) The first and last Place can be filled in one way only, while each of second and third Place can be filled in 26 ways and each of fourth and fifth Place can be filled in 10 ways.



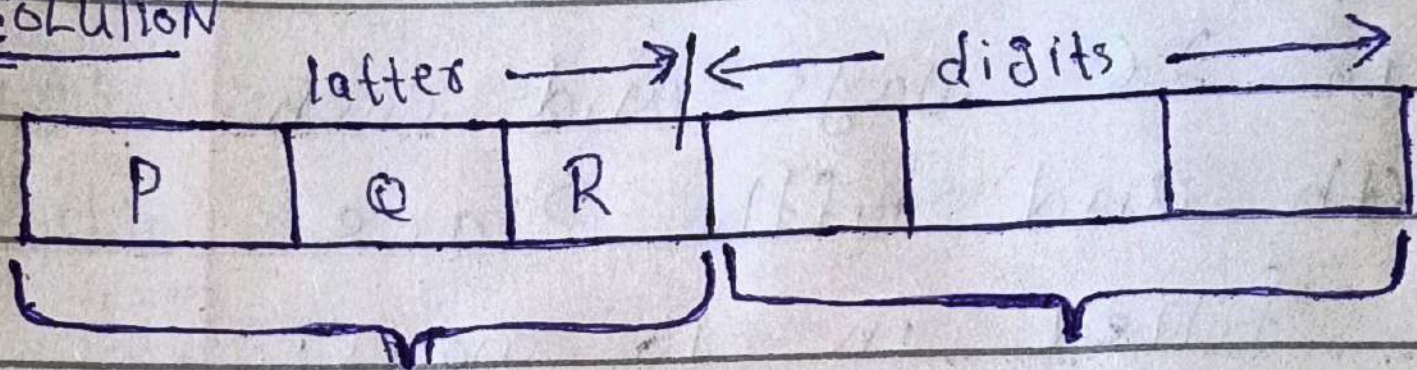
Number of license Plates that begin with A and end in O are $1 \times 26 \times 26 \times 10 \times 10 \times 1 = 67600$

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3) Number of license plates that begin with POR are

$$1 \times 1 \times 1 \times 10 \times 10 \times 10 = 1000$$

SOLUTION



one way each

10 ways each