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Degree * B.S (software Engr)

Assignment * Differential
Equation

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QUESTION - 1

Use any of the method for solving the Ordinary Equations.

Q 12:-

$$x^2 y'' - 4xy' + 6y = 0, \quad y(1) = 0.4, \quad y'(1) = 0$$

Sol:-

put $y = x^m$

$$y' = m x^{m-1}$$

$$y'' = m(m-1) x^{m-2}$$

put y'' in given D.E

$$x^2 m(m-1) x^{m-2} - 4x m x^{m-1} + 6x^m = 0$$

$$x^2 m(m-1) x^m x^{-2} - 4x m x^m x^{-1} + 6x^m = 0$$

Dropping Common factor (x^m)

$$m(m-1) - 4m + 6 = 0$$

$$m^2 - 5m + 6 = 0$$

Now find roots

$$m^2 - 5m + 6 = 0$$

$$m_{1/2} = \frac{5 \pm \sqrt{(-5)^2 - 4 \cdot 6}}{2}$$

$$m_{1/2} = \frac{5 \pm 1}{2}$$

$$m = 3, \quad m_2 = 2$$

This provide two real solution

$$y_1 = x^m = x^3 \quad \& \quad y_2 = x^{m_2} = x^2$$

General solution is

$$y = C_1 y_1 + C_2 y_2$$

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$$\Rightarrow C_1 y_1 + C_2 y_2$$

$$y' = 3C_1 x^2 + 2C_2 x$$

Now to determine C_1 & C_2

$$\begin{cases} 0.4 = y(1) = C_1 \cdot 1^3 + C_2 \cdot 1^2 \\ 0 = y'(1) = 3C_1 \cdot 1^2 + 2C_2 \cdot 1^2 \end{cases}$$

$$\begin{cases} 0.4 = C_1 + C_2 \\ 0 = 3C_1 + 2C_2 \end{cases}$$

$$\begin{cases} 0.4 - C_2 = C_1 \\ 0 = 3(0.4 - C_2) + 2C_2 \end{cases}$$

$$\begin{cases} 0.4 - C_2 = C_1 \\ 0.2 = C_2 \end{cases}$$

$$\begin{cases} 0.4 - 0.2 = C_1 \\ 0.2 = C_2 \end{cases}$$

$$\begin{cases} 0.2 = C_1 \\ 0.2 = C_2 \end{cases}$$

The particular solution is

$$y = (-0.8x^3) + 1.2x^2 \text{ Ans}$$

Q1 \rightarrow 13

$$x^2 y'' + 3xy' + 0.75y = 0$$

$$y(1) = 1$$

$$y'(1) = 1.5$$

Sol:-

$$\text{put } y = x^m$$

$$y' = m x^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

putting in the given

$$x^2 m(m-1)x^{m-2} + 3xm x^{m-1} + 0.75x^m = 0$$

$$x^2 m(m-1)x^{m-2} + 3xm x^{m-1} + 0.75x^m = 0$$

Dropping Common factor x^m

$$m(m-1) + 3m + 0.75 = 0$$

$$m^2 + 2m + 0.75 = 0$$

finding root

$$m^2 + 2m + 0.75 = 0$$

$$m = \frac{-2 \pm \sqrt{2^2 - 4(1)(0.75)}}{2}$$

$$m = \frac{-2 \pm 1}{2}$$

$$m_1 = -1/2 \quad \& \quad m_2 = -3/2$$

we have two real solution

$$y_1 = x^{m_1} = x^{-1/2} = x^{-0.5} \quad \& \quad y_2 = x^{m_2} = x^{-3/2}$$

The general solution is

$$y' = -0.5C_1 x^{-1.5} - 1.5C_2 x^{-2.5}$$

To determine C_1 and C_2

$$\begin{cases} 1 = y(1) = C_1 1^{-0.5} + C_2 1^{-1.5} \\ 1.5 = y'(1) = -0.5C_1 1^{-1.5} - 1.5C_2 1^{-2.5} \end{cases}$$

$$\begin{cases} 1 = C_1 + C_2 \\ 1.5 = -0.5C_1 - 1.5C_2 \end{cases}$$

$$\begin{cases} 1 = C_1 + C_2 \\ 3 = C_1 + 3C_2 \end{cases}$$

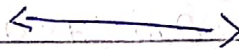
$$\begin{cases} 1 - C_2 = C_1 \\ 2 = 2C_2 / (2) \end{cases}$$

$$\begin{cases} 1 - C_2 = C_1 \\ 1 = C_2 \end{cases}$$

$$\begin{cases} 0 = C_1 \\ 1 = C_2 \end{cases}$$

particular solution is

$$y = x^{-1.5} \text{ Ans.}$$



Q1 → 14

$$x^2 y'' + xy' + 9y = 0$$

$$y(1) = 0$$

$$y'(1) = 2.5$$

Sol:-

put $y = x^m$ and $y'' = m(m-1)x^{m-2}$

$$x^2 m(m-1)x^{m-2} + mx^m + 9x^m = 0$$

$$x^2 m(m-1)x^{m-2} + mx^m + 9x^m = 0$$

Dropping common factor x^m

$$m(m-1) + m + 9 = 0$$

$$m^2 - m + m + 9 = 0$$

$$m^2 + 9 = 0$$

finding root

$$m^2 + 9 = 0 \Rightarrow m^2 - (3i)^2 = 0 \Rightarrow (m - 3i)(m + 3i) = 0$$

$$m_1 = 3i \quad \& \quad m_2 = -3i$$

$$x^{m_1} = x^{3i} = (e^{mx})^{3i} = e^{3i \ln x}$$

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$$x^{m^2} = x^{-3i} = e^{\ln x} = e^{-3i \ln x}$$

$$e^a = e^{a+ib} = e^a (\cos b + i \sin b)$$

$$\Rightarrow e^{3i \ln x} = e^0 (\cos(3 \ln x) - i \sin(3 \ln x)) = \cos(3 \ln x) - i \sin(3 \ln x)$$

$$\Rightarrow x^{m^2} = \cos(3 \ln x) + i \sin(3 \ln x)$$

$$x^{m^2} = \cos(3 \ln x) - i \sin(3 \ln x)$$

$$\Rightarrow x^{m^1} + x^{m^2} = \cos(3 \ln x) + i \sin(3 \ln x) + \cos(3 \ln x) - i \sin(3 \ln x) = 2 \cos(3 \ln x)$$

$$\Rightarrow \frac{x^{m^1} + x^{m^2}}{2} = \cos(3 \ln x)$$

Subtracting 2nd equation from 1st and dividing by $2i$

$$\frac{x^{m^1} - x^{m^2}}{2i}$$

$$\Rightarrow \frac{x^{m^1} - x^{m^2}}{2i} = \frac{\cos(3 \ln x) + i \sin(3 \ln x) - \cos(3 \ln x) + i \sin(3 \ln x)}{2i} = \sin(3 \ln x)$$

$$\Rightarrow \frac{x^{m^1} - x^{m^2}}{2} = \sin(3 \ln x)$$

$$y_1 = \cos(3 \ln x) \text{ \& } y_2 = \sin(3 \ln x)$$

$$y = C_1 y_1 + C_2 y_2$$

$$C_1 \cos(3 \ln x) + C_2 \sin(3 \ln x)$$

$$y' = C_1 \sin(3 \ln x) \cdot (3 \ln x)' + C_2 \cos(3 \ln x) \cdot (3 \ln x)'$$

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$$= -\frac{3c_1}{2} \sin(3 \ln x) + \frac{3c_2}{2} \cos(3 \ln x)$$

Now to find c_1 and c_2

$$\begin{cases} 0 = y(1) = C_1 \cos(3 \ln x) + C_2 (\sin)(3 \ln x) \\ 2.5 = y'(1) = -3C_1 \sin(3 \ln x) + 3C_2 \cos(3 \ln x) \end{cases}$$

$$\begin{cases} 0 = C_1 \cos(0) + C_2 (\sin)(0) \\ 2.5 = -3C_1 \sin(0) + 3C_2 \cos(0) \end{cases}$$

$$\begin{cases} 0 = C_1 \\ 5/2 = 3C_2 / 3 \end{cases}$$

$$\begin{cases} 0 = C_1 \\ 5/6 = C_2 \end{cases}$$

The particular solution is

$$y = 5/6 \sin(3 \ln x)$$

←→ Ans.

Q1 → 15

$$x^2 y'' + 3xy' + y = 0$$

$$y(1) = 3.6$$

$$y'(1) = 0.4$$

Sol:

$$\text{Put } y = x^m, y'' = m(m-1)x^{m-2}$$

$$x^m(m-1)x^{m-2} + 3mx^m + x^m = 0$$

$$x^2 m(m-1)x^m \cdot x^{-2} + 3mx^m + x^m = 0$$

Dropping common factor

$$m(m-1) + 3m + 1 = 0$$

$$m^2 - m + 3m + 1 = 0$$

$$m^2 + 2m + 1 = 0, (m+1)^2 = 0$$

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$$m = 1$$

$$y_1 = x^m = x^{-1} = 1/x$$

$$y'' + 3/x \cdot y' + \frac{1}{x^2} \cdot y = 0$$

$$p(x) = \frac{3 \cdot 1}{x} \Rightarrow \int p dx = 3 \ln(x)$$

$$y_2 = u y_1$$

$$u = \int v du \quad \& \quad u = \frac{1}{y_1} e^{-\int p dx}$$

To find u

$$e^{-\int p dx} = e^{-3 \ln(x)} = (e^{\ln(x)})^{-3} = x^{-3}$$

$$u = x^{-3} \cdot \frac{1}{x^2} = x^{-3+2} = x^{-1} = \frac{1}{x}$$

$$x = \int \frac{dx}{x} = \ln(x)$$

$$y_2 = u y_1 = y_1 \ln x = \frac{1}{x} \ln x$$

$$y = C_1 y_1 + C_2 y_2$$

$$C_1 \cdot \frac{1}{x} + C_2 \cdot \frac{1}{x} \ln x$$

$$\frac{1}{x} C_1 + C_2 \ln x$$

$$y' = (x^{-1})' (C_1 + C_2 \ln x) + x^{-1} (C_2 \ln x)'$$

$$= -x^{-2} (C_1 + C_2 \ln x) + \frac{1}{x} \ln \frac{1}{x}$$

$$\frac{1}{x^2} (-C_1 - C_2 \ln x + C_2)$$

Now finding C_1 and C_2

$$\begin{cases} 3.6 = y(1) = \frac{1}{1} (C_1 + C_2 \ln 1) \\ 0.4 = y'(1) = \frac{1}{1} (-C_1 - C_2 \ln 1 + C_2) \end{cases}$$

$$\begin{cases} 3.6 = C_1 \\ 0.4 = -C_1 + C_2 \end{cases}$$

$$\begin{cases} 3.6 = C_1 \\ 0.4 = -3.6 + C_2 \end{cases}$$

$$\begin{cases} 3.6 = C_1 \\ 4.0 = C_2 \end{cases}$$

$$y = (3.6 + 4.0 \cdot 0 \ln x)^{1/2}$$

←→ Ans.

Q 1 → 16

$$x^2 D^2 - 3x D + 4I y = 0$$

$$y(1) = -1, y'(1) = 2\pi$$

Sol:-

$$\begin{aligned} x^2 D^2 y - 3x Dy + 4Iy &= x^2 D(Dy) - 3x Dy + 4y \\ &= x^2 y'' - 3xy' + 4y \end{aligned}$$

$$\Rightarrow x^2 y'' - 3xy' + 4y = 0$$

part $y = x^m$ and $y'' = m(m-1)x^{m-2}$, $y' = mx^{m-1}$

$$x^2 m(m-1)x^{m-2} - 3x m x^{m-1} + 4x^m = 0$$

$$x^2 m(m-1)x^{m-2} - 3x m x^{m-1} + 4x^m = 0$$

Dropping x^m

$$m^2 - 4m + 4 = 0$$

$$m(m-4) - 3m + 4 = 0, m^2 - 4m + 4 = 0$$

hence $y = x^m$ is a solution

$$m^2 - 4m + 4 = 0, (m-1)^2 = 0$$

$$(1) y_1 = x^1 = x$$

$$y'' = -\frac{3}{x} \cdot y' + \frac{4}{x^2} \cdot y = 0$$

$$P(x) = -3 \cdot \frac{1}{x} \Rightarrow \int P(x) dx = -3 \ln(x)$$

$$y_2 = u y_1$$

→

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$$x = \int u du \quad \& \quad u = \frac{1}{y^2} \int P du$$

To find u

$$e^{P du} = e^{3 \ln(x)} = (e^{\ln(x)})^3 = x^3$$

$$u = x^3 \cdot \frac{1}{(x^2)^2} = x^{3-4} = x^{-1} = \frac{1}{x}$$

$$u = \int \frac{du}{x} = \ln(x)$$

$$y_2 = u y_1 = y_1 \ln(x) = x \ln(x)$$

$$y_1 = y_2 \in R$$

General Solution is

$$y = C_1 y_1 + C_2 y_2$$

$$C_1 x^2 + x^2 \ln x$$

$$x^2 (C_1 + C_2 \ln x)$$

$$y' = (x^2)' (C_1 + C_2 \ln x) + x^2 (C_1 + C_2 \ln x)'$$

$$= 2x (C_1 + C_2 \ln x) + C_2 x^2 = \frac{1}{x}$$

$$= 2C_1 x + 2C_2 x \ln x + C_2 x$$

$$2C_1 x + C_2 x (2 \ln x + 1)$$

$$\begin{cases} -x = y(1) = 1^2 (C_1 + C_2 \ln 1) \\ 2x = y'(1) = 2C_1 + C_2 (2 \ln 1 + 1) \end{cases}$$

$$\begin{cases} -x = C_2 \\ 2x = 2C_1 + C_2 \end{cases}$$

$$\begin{cases} -x = C_2 \\ 4x = C_2 \end{cases}$$

Particular solution is

$$y = x^2 (-x + 4x \ln x)$$

Ans.

Q1 \rightarrow 18

$$(9x^2 D^2 + 3x D + I)y = 0$$

$$y(1) = 1$$

$$y'(1) = 0$$

Sol:- Apply given operation to the equation

$$= 9x^2 D^2 y + 3x Dy + Iy = 9x^2 D(Dy) + 3x Dy$$

$$= 9x^2 y'' + 3x y' + y$$

$$9x^2 y'' + 3x y' + y = 0$$

$$\text{let } y = x^m, y' = m x^{m-1}, y'' = m(m-1) x^{m-2}$$

$$9x^2 m(m-1) x^{m-2} + 3x m x^{m-1} + x^m = 0$$

$$9x^2 m(m-1) x^{m-2} + 3x m x^{m-1} + x^m = 0$$

$$9m(m-1) + 3m + 1 = 0, 9m^2 - 9m + 3m + 1 = 0$$

$$9m^2 - 6m + 1 = 0$$

finding the root of equation

$$m^2 - 4m + 4 = 0, (m-2)^2 = 0$$

$$9m^2 - 6m + 1 = 0, m/2 = \frac{6 \pm \sqrt{6^2 - 4 \cdot 9}}{18}$$

18

$$m/2 = 6/18$$

$$m/2 = 1/3$$

$$m = 1/3$$

$$y_1 = x^m = x^{1/3}$$

$$y'' + \frac{1}{3x} y' + \frac{1}{9x^2} y = 0$$

$$P(x) = \frac{1}{3} \cdot \frac{1}{x} \Rightarrow \int P dx = \frac{1}{3} \ln(x)$$

$$y^2 = u y_1$$

$$u = \int u \, du \quad \& \quad u = \frac{1}{y}, \quad e^{\int p \, dx}$$

finding u

$$u e^{-\int p \, dx} = e^{-\frac{1}{2} \ln(x)} = (e)^{\ln(x) \cdot -\frac{1}{2}} = x^{-\frac{1}{2}}$$

$$u = x^{-\frac{1}{2}} \cdot \frac{1}{(x^{\frac{1}{2}})^2} = x^{-\frac{1}{2} - 2 \cdot \frac{1}{2}} = x^{-1} = \frac{1}{x}$$

$$u = \int \frac{dx}{x} = \ln(x)$$

$$y_1 = u y_1 = y_1 \ln(x) = x^{\frac{1}{2}} \ln x$$

hence

$$y = C_1 y_1 + C_2 y_2$$

$$C_1 x^{\frac{1}{2}} + x^{\frac{1}{2}} \ln x$$

$$x^{\frac{1}{2}} (C_1 + C_2 \ln x)$$

$$\begin{aligned} y' &= (x^{\frac{1}{2}}) (C_1 + C_2 \ln x) + x^{\frac{1}{2}} (C_1 + C_2 \ln x) \\ &= \frac{1}{2} \cdot x^{-\frac{1}{2}} (C_1 + C_2 \ln x) + x^{\frac{1}{2}} C_2 \end{aligned}$$

$$\begin{cases} 1 \cdot y(1) = 1^{\frac{1}{2}} (C_1 + C_2 \ln 1) \\ 0 = y'(1) = \frac{1}{2} \cdot 1^{-\frac{1}{2}} (C_1 + C_2 \ln(1)) + 1^{\frac{1}{2}} C_2 \end{cases}$$

$$\begin{cases} 1 = C_1 \\ -\frac{1}{2} = C_2 \end{cases}$$

$$y = x^{\frac{1}{2}} (1 - \frac{1}{2} \ln x)$$

Ans.

Q 19 :-

$$(x^2 D^2 - xD - 15I)y = 0$$

$$y(1) = 1$$

$$y'(1) = 4.5$$

Sol:- Applying given operation on the equation.

$$x^2 D^2 y - x D y - 15 I y = x^2 D(Dy) - x D y - 15 y$$

$$\Rightarrow x^2 y'' - x y' - 15 y$$

let

$$y = x^m, y' = m x^{m-1}, y'' = m(m-1) x^{m-2}$$

$$x^2 m(m-1) x^{m-2} - x m x^{m-1} - 15 x^m = 0$$

$$x^2 m(m-1) x^{m-2} - x m x^{m-1} - 15 x^m = 0$$

Dropping x^m

$$\Rightarrow m(m-1) - m - 15 = 0$$

$$\Rightarrow m^2 - 2m - 15 = 0$$

finding roots

$$m^2 - 2m - 15 = 0$$

$$m_{1/2} = \frac{2 \pm \sqrt{2^2 + 4 \cdot 15}}{2}$$

$$m_{1/2} = \frac{2 \pm 8}{2}$$

$$m_1 = 5 \quad \& \quad m_2 = -3$$

The two real solution are

$$y_1 = x^{m_1} = x^5 \quad \& \quad y_2 = x^{m_2} = x^{-3}$$

Now the general solution is

$$y = C_1 y_1 + C_2 y_2$$

$$= C_1 x^5 + C_2 x^{-3}$$

$$y' = 5C_1 x^4 - 3C_2 x^{-4}$$

Now to determine the C_1 and C_2

$$\begin{cases} 0.1 = y(1) = C_1 \cdot 1^5 + C_2 \cdot 1^{-3} \\ -4.5 = y'(1) = 5C_1 \cdot 1^4 - 3C_2 \cdot 1^{-4} \end{cases}$$

$$\begin{cases} 0.1 = C_1 + C_2 \\ 4.5 = 5C_1 + (-3C_2) \end{cases}$$

$$\begin{cases} 0.1 - C_2 = C_1 \\ 4.5 = 5C_1 + (-3C_2) \end{cases}$$

$$\begin{cases} 0.1 - C_2 = C_1 \\ -4.5 = 5(0.1 - C_2) - 3C_2 \end{cases}$$

$$\begin{cases} 0.1 - C_2 = C_1 \\ 5 = 8C_2 / 8 \end{cases}$$

$$\begin{cases} 0.1 - 0.625 = C_1 \\ 0.625 = C_2 \end{cases}$$

$$\begin{cases} -0.525 = C_1 \\ 0.625 = C_2 \end{cases}$$

Particular solution

$$y = -0.528x^5 + 0.628x^{-3}$$

Ans.

QUESTION - No - 2

Q.1:

Use the method of separation of variables to find the general solution to the following.

(A) :-

$$x' = \sqrt{x}$$

Sol:-

$$\frac{dx}{dt} = \sqrt{x}$$

$$\frac{dx}{\sqrt{x}} = 1 dt$$

$$\frac{1}{\sqrt{x}} dx = dt$$

$$\int \frac{1}{\sqrt{x}} dx = \int dt$$

$$2\sqrt{x} + C_1 = t + C_2$$

$$2\sqrt{x} = t + C$$

$$4(x) = (t + C)^2$$

$$\Rightarrow x = \frac{(t + C)^2}{4}$$

Ans



(B) :- $x' = e^{-2x}$

Sol:- $\frac{dx}{dt} = e^{-2x} \Rightarrow \frac{dx}{e^{-2x}}$

$= \int \frac{dx}{e^{-2x}} = \int dt$

$= \frac{e^{2x}}{2} = t + c$

$e^{2x} = 2(t + c)$

$2x = \ln 2(t + c)$

$\Rightarrow x = \frac{\ln 2(t + c)}{2}$

Ans

(C)

$y' = 1 + y^2$

Sol:-

$\frac{dy}{dt} = 1 + y^2 \Rightarrow \frac{dy}{1 + y^2} = dt$

$\int dy \frac{1}{1 + y^2} = \int dt$

$y'(1) = \tan(t + c)$ Ans

$$(D) :- u' = \frac{1}{5-2u}$$

$$\underline{\text{Sol:-}} \quad \frac{dy}{dt} = \frac{1}{5-2u} \Rightarrow \frac{dy}{5-2u} = dt \quad (1)$$

$$= \int \frac{dy}{5-2u} = \int dt$$

$$= -(u-5)u + C_1 = t + C_2$$

$$\Rightarrow -(u-5)u = t + C$$

$$\Rightarrow \frac{t+C}{-(u-5)} \text{ Ans}$$



$$(F) :- Q' = \frac{Q}{4+Q^2}$$

$$\underline{\text{Sol:-}} \quad \frac{dQ}{dt} = \frac{Q}{4+Q^2} \Rightarrow \frac{Q^2 dQ}{Q} = dt$$

$$= \int \frac{Q^2 dQ}{Q} = \int dt$$

$$3 \ln |Q| + \frac{Q^2}{2} + C$$

Ans



(G)

$$x' = e \cdot x^2$$

$$\frac{dx}{dt} = e x^2$$

$$\frac{dx}{e x^2} = dt$$

$$\int \frac{1}{e^{2x}} dx = \int db$$

$$\frac{\sqrt{x} e^f(x)}{2} = b + e$$

$$x = \frac{2b + e}{\sqrt{x} e^f}$$

$$x(1) = \frac{2b + e}{\sqrt{\pi}} \quad \text{Ans}$$

$$(h) \quad y' = r(a - y)$$

$$\text{sol} \quad \frac{dy}{dt} = r(a - y)$$

$$\frac{dy}{r(a - y)} = dt$$

$$\int \frac{1}{r(a - y)} dy = \int dt$$

$$\frac{(a - y)x}{r} = t + c$$

$$x(t) = \frac{r(ct + e)}{a - y} \quad \text{Ans}$$

Q2-2

Solve $y' = r(a-y)$, where r and a are constants

Sol $\frac{dy}{dt} = r(a-y)$

$$\frac{dy}{r(a-y)} = dt$$

$$\int \frac{1}{r(a-y)} dy = \int dt$$

$$a-y = \underbrace{K (r)^{1/2}}$$

Q2-3, (a)

(a) $x(0) = 1$

$$x(t) = \frac{(t+e)^2}{4}$$

$x(0) = 1, \quad y(t) = x(0) = 1$

$$1 = \frac{(0+e)^2}{4}$$

$4 = e^2, \quad (e=2) \text{ Ans}$



$$1b, \quad u \cos = 1$$

$$u(t) = \frac{\ln 2(t) + e}{2}$$

$$x(0) = 1$$

$$1 = \frac{\ln 2(0) + e}{2}$$

$$x(0) = 1$$

$$1 = \frac{\ln 2(0) + e}{2}$$

$$1 = \ln \frac{e}{2} + e$$

$$\frac{2 - \ln x e^2}{2 - 4} \quad \text{Ans}$$

Q (a)

$$x' = \frac{2x}{t+1}$$

$$\frac{dx}{dt} = \frac{2x}{t+1}$$

$$\frac{dx}{dt} = \frac{2x}{t+1}$$

$$\frac{dx}{2x} = \frac{1}{t+1} dt$$

$$\int \frac{dx}{2x} = \int \frac{1}{t+1} dt$$

$$\frac{x^2}{4} = \ln(t+1) + C$$

$$\frac{x^2}{4 \ln(t+1)} = C \Rightarrow C = \frac{x^2}{4 \ln(t+1)} \quad \text{Ans}$$

24 (b)

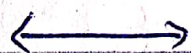
$$Q' = t \sqrt{t^2 + 1} \sec \theta$$

$$\frac{dQ}{dt} = t \sqrt{t^2 + 1} \sec \theta$$

$$\frac{dQ}{\sec \theta} = t \sqrt{t^2 + 1} dt$$

$$\int \frac{dQ}{\sec \theta} = \int t \sqrt{t^2 + 1} dt$$

$$\sin \theta = \frac{(t^2 + 1)^{3/2}}{3} + C \quad C = 3 \sin \theta \quad \text{Ans}$$



Qu: (c)

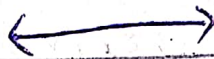
$$(2041)0 - (5+1) = 0$$

sol $(2u+1) \frac{du}{dt} = t+1$

$(2u+1) du = (t+1) dt$
 $\int (2u+1) du = \int (t+1) dt$

$u^2 + u = \frac{t^2}{2} + t + e$
 $2(u^2 + u) - t^2 = e$

$e = 2 \frac{(u^2 + u)}{t^2} - t$ Ans



Q 2-4 (D)

$R' = (t-1)(R^2+1)$

$\frac{dR}{dt} = (t+1)(R^2+1)$

$\frac{dR}{R^2+1} = (t+1) dt$

$\int \frac{dR}{R^2+1} = \int (t+1) dt$

$\cot(R) = \frac{t^2}{2} + t + e$

$e = 2 \cot(R) - t^2 - 2t$ Ans

