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ID # 14989

Dept # BS (CS) 4th Semester

Submitted to Daud Khan Khalil

Subject :

QNO 1:-

Answer: 1:-

$\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8)$
 $(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (2,7), (2,8)$
 $(3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (3,7), (3,8)$
 $(4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (4,7), (4,8)$
 $(5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (5,7), (5,8)$
 $(6,1), (6,2), (6,3), (6,4), (6,5), (6,6), (6,7), (6,8)$
 $(7,1), (7,2), (7,3), (7,4), (7,5), (7,6), (7,7), (7,8)$
 $(8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (8,8)\}$

Set

$A = \{ \text{The sum is } 7 \}$

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$B = \{ \text{The sum is even} \}$

$C = \{ \text{The sum is greater than 8} \}$

$D = \{ \text{The two die had the same outcomes} \}$

Now

$A = \{ (1,6), (2,5), (3,4), (5,2), (6,1), (4,3) \}$

$B = \{ (1,1), (1,3), (1,5), (1,7), (2,2), (2,4), (2,6), (2,8)$

$(3,1), (3,3), (3,5), (3,7), (4,2), (4,4), (4,6)$

$(4,8), (5,1), (5,3), (5,5), (5,7), (6,2)$

$(6,4), (6,6), (6,8), (7,1), (7,3), (7,5)$

$(7,7), (8,2), (8,4), (8,6), (8,8) \}$

$C = \{ (1,8), (2,7), (2,8), (3,6), (3,7), (3,8)$

$(4,5), (4,6), (4,7), (4,8), (5,4), (5,5)$

$(5,6), (5,7), (5,8), (6,3), (6,4), (6,5)$

$(6,6), (6,7), (6,8), (7,2), (7,3), (7,4)$

$(7,5), (7,6), (7,7), (7,8), (8,1), (8,2)$

$(8,3), (8,4), (8,5), (8,6), (8,7)$

$(8,8) \}$

$D = \{ (1,1), (2,2), (3,3), (4,4), (5,5),$

$(6,6), (7,7), (8,8) \}$

$$A \cap B = \{ \} \text{ OR } \emptyset$$

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$$A \cap C = \{ \}$$

$$A \cap D = \{ \}$$

$$P(A) = 6/64, P(B) = 32/64$$

$$P(C) = 36/64, P(D) = 8/64$$

$$P(A \cap B) = 0, P(A \cap C) = 0, P(A \cap D) = 0$$

Hence,
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 0 + 32/64$$

$$P(A|B) = 0$$

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = 0 \times 36/64$$

$$P(A|C) = 0$$

$$P(A|D) = \frac{P(A \cap D)}{P(D)} = 0 \times 8/64$$

$$P(A|D) = 0$$

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Question No 38

Answers

- 1:- Exactly 4 games.
- 2:- At least 4 games
- 3:- From 3 to 6 games.

Sol:-

Given that $p = \frac{2}{3}$ $n = 8$

$$q = 1 - p$$
$$= 1 - \frac{2}{3}$$

$$q = \frac{1}{3}$$

Let "x" denotes the number of games
win by A, Then

(i) $P(x=4)$

$$= \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4$$

$$= \frac{1120}{6561}$$

$$= 0.1707$$

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$$(ii) P(x \geq 4)$$

$$= 1 - \sum_{x=0}^3 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$$

$$= 1 - \left[\binom{8}{0} \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^8 + \binom{8}{1} \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^7 + \binom{8}{2} \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^6 + \binom{8}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 \right]$$

$$= 1 - \frac{1}{6561} [1 + 16 + 112 + 448]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561}$$

$$= 0.9121$$

$$(iii) P(3 \leq x \leq 6)$$

$$\sum_{x=3}^6 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$$

$$\binom{8}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 + \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 + \binom{8}{5} \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^3 + \binom{8}{6} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^2$$

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$$= \frac{8 [56 + 140 + 224 + 224]}{(3)^8}$$

$$= \frac{8 \times 644}{6561}$$

$$= \frac{5152}{6561}$$

$$= 0.7852$$

$$= 0.7852$$

Q No 48

Answers

Proofs

Since the E_i 's form a partition of the sample space, we can apply the law of total probability for $P(A \cap B)$

$$P(A \cap B) = \sum_{i=1}^m P(A \cap B | E_i) P(E_i)$$

\therefore (A and B are conditionally independent)

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$$P(A \cap B) = \sum_{i=1}^M P(A|C_i) P(B) P(C_i)$$

\therefore (B is independent of all C_i 's)

$$P(A \cap B) = P(B) \sum_{i=1}^M P(A|C_i) P(C_i)$$

$$P(A \cap B) = P(B) P(A)$$

\therefore (law of total probability)

Hence A and B are independent.

Q NOS 8-

Ans:-

Mean and Variance of Binomial Random Variables

The probability function for a binomial random variable is

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

This is the probability of having x successes in a series of n independent trials when the probability of success in any one of the trials is p . If X is a random variable with the probability distribution-

$$E(X) = \sum_{x=0}^n x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^n x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^n \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

Since $x=0$ term vanishes let $y = x-1$

and $m = n-1$, substiting $x = y+1$ and

$n = m+1$ into the last sum

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$$E(x) = \sum_{y=0}^m \frac{(m+1)!}{y!(m-y)!} p^{y+1} (1-p)^{m-y}$$

$$= (m+1)p \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= np \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y b^{m-y} (1-p)^{m-y}$$

By Binomial theorem

$$(a+b)^m = \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

Set $a = p$ and $b = 1-p$

$$\sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

$$= (a+b)^m$$

$$= (p + 1-p)^m$$

$$= 1$$

So that

$E(x) = np$

Similarly but this time using

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$$y = x-2 \text{ and } m = n-2.$$

$$E(x(x-1)) = \sum_{x=0}^n x(x-1) \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^n x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=2}^n \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^n \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{y=0}^m \frac{m!}{y!(m-y)!} (p^y (1-p)^{m-y})$$

$$= n(n-1)p^2 (p + (1-p))^m$$

$$= n(n-1)p^2$$

So the variance of x is

$$E(x^2) - E(x)^2 = E(x(x-1)) + E(x) - (np)^2$$

$$E(x^2) = n(n-1)p^2 + np - (np)^2$$

$$= np(1-p)$$

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Q No 8

Ans: Binomial Distributions.

A binomial distribution can be thought of as simply the probability of a success or failure outcome in an experiment or survey that is repeated multiple times.

$$P(X=x) = f(x) = \binom{n}{x} p^x q^{n-x}$$

Bi-nomial frequency Distribution:-

if the binomial probability distribution is multiplied by N , the number of experiments or sets, the resulting distribution is known as the Bi-nomial frequency distribution.

$$N \binom{n}{x} p^x q^{n-x}$$

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Q No 2:-

Answer: When we are rolling two dice, there are 36 different combinations. Counting those up, there are 15 possibilities less than 7: (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1).

The probability of getting less than a 7 is

$$\frac{15}{36} = \frac{5}{12}$$

There are 6 possible combinations of getting a 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1), which gives a probability of $\frac{6}{36} = \frac{1}{6}$

This means that 21 possibilities account for getting less than or equal to 7, so there are 15 remaining possibilities of getting less than 7. This is the same

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As the probability of getting less than 7. So the probability must be $5/12$ as well. In calculating this, we must assume that each combination is equally likely to roll as any other and therefore the dice are fair or else the calculating don't work.

Q NO 78

Answer:-

Coefficient of variation:-

For Data Set A:-

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$CV = \frac{3}{45} \times 100$$

$$CV = 6.7$$

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For Data Set B:-

$$EV = 11/60 \times 100$$

$$EV = 18.3$$

For Data Set C &

$$EV = 5/50 \times 100$$

$$EV = 10$$

For Data Set D &

$$EV = 15/25 \times 100$$

$$EV = 60$$