NAME # KIRAMAT ULLAH

ID NO # 13290

SUBJECT # DIFFERENTIAL EQUATION

SEMESTER # 8TH

DEPARTMENT # BEE

DATE # 26-09-2020

TEACHER NAME #

SIR HIMAYAT ULLAH

QUESTION

NO 1

Estimate general solution of
$4y'' - 20y' + 25y = 0$

SOLUTION:-

This is second order homogeneus differential equation with constant coefficient

$$ay'' + by' + cy = 0$$

and the solution for this is
$y = e^{\lambda}$ — (i)

General solution
$$y = C_1 e^{\lambda} + C_2 + e^{\lambda}$$

Now

$4 \frac{d^2}{dx^2}(y) - 20 \frac{d}{dx}(y) + 25(y) = 0$ eq (A)

put eq (i) in eq (A)

$$\Rightarrow 4\frac{d^2}{dx^2}\left(e^{\lambda x}\right) - 20\frac{d}{dx}\left(e^{\lambda x}\right) + 25e^{\lambda x} = 0$$

$$\Rightarrow \frac{d^2}{dx^2}e^{\lambda x} = \lambda^2 e^{\lambda x} \quad — \quad (B)$$

put eq (B) and eq (1) in eq (A)

$$\Rightarrow 4\lambda^2 e^{\lambda x} - 20e^{\lambda x} + 25e^{\lambda x} = 0$$

$$\Rightarrow e^{\lambda x}\left(4\lambda^2 - 20\lambda + 25\right) = 0$$

$$\Rightarrow e^{\lambda x} \neq 0$$

$$\Rightarrow 4x^2 - 20\lambda + 25 = 0$$

$$\Rightarrow (2\lambda - 5)^2 = 0$$

$$\Rightarrow \lambda\ 5/2 \quad \text{or} \quad \lambda\ 5/2$$

$$\Rightarrow y(x) = y_1(x) + y_2(x)$$

$$\Rightarrow \boxed{y(x) = C_1 e^{5/2\, x} + C_2 x e^{5/2\, x}}$$

Answer

QUESTION NO 2

PART (A)

Calculate the initial value problem $y'' + 2y' + y = 0$
$y(0) = 4.\ y'(0) = -6$

SOLUTION:

$y + 2y' + y = 0$

$\lambda^2 + 2\lambda + 1 = 0$

$\lambda^2 + \lambda + \lambda + 1 = 0$

$\lambda(\lambda + 1) + 1(\lambda + 1) = 0$

$\lambda = -1,\ \lambda = 1$

Root and Real and Equation

$x = -1$

$y(x) = y_1(x) + y_2(x) = C_1 e^{-x} + C_2 e^{-x} x$

$y' = C_1 e^{-x} + C_2 e^{-x}$

$y' = C_1 e^{-x} + C_2 e^{-x} - x e^{-x}$

Solve for the unknown Constant using

The initial conditions.

$\frac{dy(x}{dx}$

$\frac{dy(x)}{dx} = \frac{d}{dx}\left(c_1 e^{-x} + C_2 e^{-x} x\right)$

$= c_1 e^{-x} + C_2 e^{-x} - C_2 e^{-x} x$

Now

$y = 4 \;, x = 0$

Subsitute $y = 4$ into $y(x) = e^{x} c_1 + e^{-x} x C_2$

$4 = c_1$ — ①

$C_1 = 4$

Now

$x = 0 \;, y = 6$

$y'(0) = -6$

$-6 c_1 e^{-x} + C_2 e^{-x} - C_2 e^{-x} x$

$= -6 = C_1 + C_2 \Rightarrow$ — ②

Add ① and ②

$$4 = \cancel{C_1}$$

$$\underline{-6 = -\cancel{C_1} + C_2}$$

$$2 = C_2$$

Now

$$C_1 = 4$$

$$C_2 = 2$$

Substitute $C_1 = 4$ and $C_2 - 2$

into

$$y' = C_1 e^{-x} + C_2 e^{-x} x$$

$$y' = -2e^{-x}(x-2) \text{ Ans}$$

QUESTION NO 2

PART (B)

Analyze the general solution of $x^2y'' + 3xy' + y = 0$.

SOLUTION:-

$a = 3 \quad , b = 1$

$m^2 + (a-1)m + b = 0$

$m^2 + (3-1)m + 1 = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = -1 \quad , m = -1$

Roots are real and equal

So, $\boxed{y = (C_1 + C_2 \ln x)x^m}$

Answer

Question No 3

Examine the method of Undetermined coefficient method for
$y'' + y' - 6y = 6x^3 - 3x^2 + 12x.$

Solution:-

$y'' + y' - 6y = 6x^3 - 3x^2 + 12x$ — ①

for Homogenous equation

$a = 1 \quad , \quad b = -6$

$\lambda^2 + \lambda - 6$

$\lambda^2 + 3\lambda - 2\lambda - 6$

$\lambda(\lambda + 3) - 2(\lambda + 3)$

$\lambda - 2 = 0 \quad , \quad \lambda + 3 = 0$

$\lambda = 2 \quad , \quad \lambda = 3$

So,

$y = C_1 e^{2x} + C_2 e^{-3x}$

$y_p = K_3 x^3 + K_2 x^2 + K_1 x^1 + K_0$

$y_0 = 3K_3 x^3 + 2K_2 x^2 + K_1$

$y_p = 6K_3 x + 2x_2$

put in eq (1)

$\Rightarrow \underbrace{\cancel{6} x_3 x + 2x}_{y''} + 3K_3 x^2 + 2K_2 x + x_1$

$-6K_{\cancel{3}} x^3 - 6x_2 x^2 - 6x_1 x - 6x_0$

$= 6x_3 \cancel{x^3} - 3K_2 x^2 + 12x_1$

$-6x_3 x^2 + (3K_3 - 6x_2)x^2 + (6K_3 + 2K_2) - 6K_1 x$

$+2K_2 + K_1 - 6K_0 = 6x^3 - 3x^2 + 12x$

$-6K_3 = 6$

$\Rightarrow \boxed{K_3 = -1}$

$3K_3 - 6x_2 = -3$

$3(-1) - 6x_2 = -3$

$-K_2 = 0 \Rightarrow \boxed{K_2 = 0}$

$6K_3 - 2K_2 - 6K_1 = 12$

$6(-1) + 1(0) - 6k_1 = 12$

$-6 + 0 - 6k_1 = 12$

$-6k_1$

$k_1 = -18/6$

$\boxed{k_1 = -3}$

$-2k_1 + k_1 + k_0 = 0$

$-2\cancel{k}(-3) - 2 + k_2 = 0$

$\boxed{k_0 = -1/2}$

So, $y_p = -x^3 + 0x^2 - 3x - 1/2 = -x^3 - 3x - 1/2$

Answer

QUESTION No 4

Examine the method of variation of Parameters for
$y'' - 4y' + 4y = x^2 e^{2x}$.

SOLUTION:-

$y'' - 4y + 4y' = x^2 e^{2x}$

For equation

$\Rightarrow y'' - 4y' + 4y = 0$

$\Rightarrow \lambda^2 - 4\lambda + 4 = 0$

$\Rightarrow \lambda^2 - 2\lambda - 2\lambda + 4 = 0$

$\Rightarrow \lambda(\lambda - 2) - 2(\lambda - 2) = 0$

$\Rightarrow \lambda = 2 \quad , \quad \lambda = 2$

Roots are Real and equal

$\Rightarrow y = (c_1 + c_2 x)e^{2x}$

$\Rightarrow y_1 = C_1 e^{1x} + C_2 x e^{2x}$

$\Rightarrow y_1 = e^{2x}, \; y_2 = x e^{2x}$

$\Rightarrow y_1' = 2e^{2x}, \; y_2' = e^{2x} + 2ex^{2x}$

$$\Rightarrow W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$\Rightarrow W = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & e^{2x} + 2e^{x2x} \end{vmatrix}$$

$\Rightarrow W = e^{4x} + 2\cancel{x}e^{4} - 2\cancel{x}e^{4x}$

$$\boxed{W = e^{4x}}$$

$$yp = -y_1 \int \frac{y_2 \, (x)}{w} + y_2 \int \frac{y_1 \, (x)}{w}$$

$$yp = -e^{2x} \int \frac{xe^{2x} \cdot x^2 e^{2x}}{e^{4x}} dx$$

$$+ xe^{2x} \int \frac{e^{2x} \, x^2 e^{2x}}{e^{4x}} dx$$

$$yp = -e^{2x} \int x^3 \frac{\cancel{e^{4x}}}{\cancel{e^{4x}}} + xe^{2x} \int x^2 \frac{\cancel{e^{4x}}}{\cancel{e^{4x}}} dx$$

$$y_p = -e^{2x}\int x^2 dx + xe^{2x}\int x^2 dx$$

$$y_p = -e^{2x}\cdot\frac{x^4}{4} + xe^{2x}\frac{x^3}{3}$$

so

$$y = y_x + y_p$$

$$\boxed{y = C_1 e^{2x} + C_2 x^{2x} - e^{2x}\frac{x^4}{4} + x_3^{2x}\frac{x^3}{3}}$$

Answer

QUESTION

No 5

Identify an ODE $y'' + ay' + by = 0$ for the basis $1, e^{-3x}$..

SOLUTION :-

e^{-3x} will e^{3x}

$\Rightarrow y_1 = e^{0x}$, $y_2 e^{3x}$

$\Rightarrow y = C_1 e^{0x} + C_2 e^{3x}$

So, roots are real and distinct

$$y = (c_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x})$$

So,

$$\lambda_1 = 0 , \lambda_2 = 3$$

$$\lambda_1 = 0 , \lambda_2 - 3 = 0$$

$$(\lambda)(\lambda - 3) = 0$$

$$\lambda^2 - 3\lambda = 0$$

So,

$$\lambda^2 - a\lambda + b = 0$$

As $a = -3$, $b = 0$

So,

$$y'' + ay' + by = 0$$

$$\boxed{y'' - 3y' = 0}$$

Answer