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SECTION : A

SUBJECT : Differential equation

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Assignment # 2nd

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Q. Cauchy Euler Method.

①

Question #1

The Cauchy Euler Method

① $x^3y''' + 2x^2y' + 2y - 10x + \frac{10}{x}$

Solution:

$$x^3\frac{d^3y}{dx^3} + 2x^2\,dy/dx + 2y = 10x + 10x^{-1}$$

$$x^3D^3y + 2x^2D^2 + 2y = 10x + 10x^{-1}$$

$$x^3D^3 + 2x^2D^2 + 2y = 10x + 10x^{-1}$$

$$(x^3D^3 + 2x^2D + 2)y = 10x + 10x^{-1} \rightarrow (i)$$

let $x = e^t = t = \ln x$.

$$xD = D$$

$$x^2D^2 = D(\Delta - 1) = \Delta^2 - \Delta$$

$$x^3D^3 = \Delta(\Delta-1)(\Delta-2)$$

Subtituting into eq (i)

$$(\Delta - 3D^2 + 2\Delta + 2(\Delta^2 - \Delta) + 2)y = 10^x + 10x^{-1}$$

$$(\Delta^3 - \Delta^2 + 2)y = 10x + 10x^{-1}$$

$$(m^3 - m^2 + 2)y = 10e^t + \frac{10}{e^t}$$

Using Synthetic division

$$\begin{array}{r|rrrr} & 1 & -1 & 0 & 2 \\ -1 & & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & \boxed{0} \end{array}$$

$$\Delta^2 - 2\Delta + 2 = 0$$

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NOW!

Using Quadratic Formula

$a=1,\ b=-2,\ c=2$

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = \frac{-(-2) \pm \sqrt{-2^2 - 4(1)(2)}}{2a}$$

$$\Delta = \frac{2 \pm \sqrt{4-8}}{2}$$

$$\Delta = \frac{2 \pm \sqrt{-4}}{2}$$

$$\Delta = \frac{2 \pm \sqrt{-4}}{2}$$

$$\Delta = \frac{2 \pm \sqrt{-1} \times \sqrt{4}}{2}$$

$$\Delta = \frac{2 + 2i}{2}$$

$$\Delta = \frac{\cancel{2}(1 \pm i)}{\cancel{2}}$$

$\Delta = 1 \pm i$

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Since roots are Complex

$y_c = e^{-x}(c_1 \cos t + c_2 \sin t)$

NOW!

Particular integration

$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^3 - D^2 + 2} \cdot 10/e^t$

$= \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^{-t}}{(1)^3 - (1)^2 + 2}$

$= \frac{\cancel{10}^{5} e^t}{\cancel{2}} + \frac{\cancel{10} e^{-t}}{\cancel{(1)^3 - (1)^2 + 2}\ \cancel{2}}$

$5e^t + 5e^{-t}$

$y_p = 5e^t + 5e^{-t}$

General Solution:

$y = y_c + y_p$

$y = e^{-x}(c_1 \cos t + c_2 \sin t) + 5e^t + 5e^{-t}$

put $e^t = x$ and $t = \ln x$

$y = e^{-x}(c_1 \ln x + c_2 \sin \ln x) + 5e^x + 5e^{-x}$

Question NO 2:

a) $x^3 \frac{d^3y}{dx} + 4x^2 \frac{d^2y}{dx} - 5x\frac{dy}{dx} - 15y = x^4$

Sol:

let $\frac{d}{dx} = D$

$x^3D^3y + 4x^2D^2y - 5xDy - 15y - 15y = x^4$

$(x^3D^3 + 4x^2D^2 - \cancel{0}5xD - 15)\,y = x^4$

let $x = e^t = t = \ln x$

$xD = D$

$x^2D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta.$

$\Delta^3D^3 = \Delta(\Delta-1)(\Delta-2) = \Delta^3 - 3\Delta^2 + 2\Delta$

NOW!

Subtituting:

$(x^3D^3 + 4x^2D^2 - 5xD - 15)\,y = x^4$

$(\Delta^3 - 3\Delta^2 + 2\Delta + 4(\Delta^2 - D) - 5(\Delta) - 15)y = e^{4t}$

$(D^3 + \Delta^2 - 7D - 15)y - e^{4t}$

Synthetic division:

$$\begin{array}{c|cccc} & 1 & +1 & -7 & -15 \\ 5 & & 3 & 12 & 15 \\ \hline & 1 & 4 & 5 & \boxed{0} \end{array}$$

$\Delta^2 + 4\Delta + 5 = 0$

Quadratic formula.

$$D = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{4^2-4(1)(5)}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$$D = \frac{\cancel{2}(-2 \pm i)}{\cancel{2}}$$

$$y_c = e^{5x}(c_1 \cos t + c_2 \sin t)$$

For $y_p = ?$

$$y_p = \frac{1}{D^3 - D^2 - 7D - 15} e^{4t}$$

$$= \frac{1}{(4^3) + (4^2) - 7(4) - 15} e^{4t}$$

$$y_p = \frac{1}{37} e^{4t}$$

Hence:

$$y = y_c + y_p$$

$$y = (c_1 \cos t + c_2 \sin t) + \frac{1}{37} e^{4t}$$

Again put $t = \ln x$ and $x = \ln x$

$$y = e^{34}(c_1 \cos \ln x + c_2 \sin \log x) + \frac{1}{37} e^{4x}$$

$\boxed{\text{Ans}}$

Question NO 3:

$$x^2 y'' + 2xy' - 6y = 10x^2$$

Sol

$y(1) = 1$ and $y'(1) = -6$

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 6y = 10x^2$$

put

$xD = \Delta \Rightarrow x^2 D^2 = \Delta(\Delta - 1) = \Delta^2 - D$

$x = e^t$ and $\log x = t$

$\Rightarrow (\Delta^2 - \Delta + 2\Delta - 6)y = 10e^{2t}$

$(\Delta^2 + \Delta - 6)y = 10e^{2t}$

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$y_c = ?$

$y_c = c_1 e^{-3t} + c_2 e^{2t}$

$\Rightarrow$ For $y_p = ?$

$$y_p = \frac{1}{D^2 - D - 6} 10 e^{2t}$$

$$= \frac{10}{D^2 - D - 6} e^{2t}$$

$$= 10 \frac{1}{0} e^{2t} \text{ fails}$$

Now!

$$10 \frac{1}{\frac{d}{dD}(D^2 + D - 6)} e^{2t}$$

$$= 10 \frac{t}{2D+1} e^{2t}$$

$$= 10 \frac{1 \cdot t}{4+1} e^{2t}$$

$y_p = 2te^{2t}$

General Solution

$y = y_c + y_p$

$c_1 e^{-3t} + c_2 x^2 + 2(\log x) x^2$ —— (B)

put $(1) = 1$ i-e $x = 1, y = 1$ in (B)

$1 = c_1(1)^{-3} + c_2(1)^2 + 2\log(1)$

$1 = c_1 + c_2$ —— (C)

Now differentiate eq. B w.r.t x

$y' = -3c_1 x^{-4} + 2c_2 x + \frac{2}{x}(x^2) + 4x\log x$

$-6 = -3c_1 + 2c_2 + 2 + 0$

$-6 - 2 = -3c_1 + 2c_2 + 2$

$-8 = -3c_1 + 2c_2$ —— (D)

Xing eq C with 2 and -tract from D

$2c_1 + \cancel{2c_2} = 2$

$\mp 3c_1 \pm \cancel{2c_2} = \mp 8$

$5c_1 = 10$

$c_1 = \frac{10}{5}$ $c_1 = 2$

$-8 = -3(2) + 2c_2$

$-8 = -6 + 2c_2$

$2c_2 = -8 + 6$

$2c_2 = -2$

$c_2 = \frac{-2}{2}$

$\boxed{c_2 = -1}$

Now! put the value of c_1 and c_2 in eq (B)

$y = 2x^{-3} - x^2 + 2\ln x \, x(x^2)$

$y = 2/x^3 - x^2 + 2x^2\log x$. Answer

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Question NO 5:

$(x+1)^2 y'' = -3(x+1)' y' + 4y = x^2$

Solution

$(x+1)^2 \frac{d^2}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2$

$((x+1)^2 \frac{d^2}{dx^2} - 3(x+)\frac{dy}{dx} + 4)y = x^2$

$[(x+1)^2 D^2 - 3(x+1)D + 4]y = x^2$ (A)

putt $(x+1)D = \Delta \Rightarrow (x+1)^2 D^2 = \Delta(\Delta - 1)$

$x = e^t$ in eq A. $= \Delta^2 - \Delta$

$= (\Delta^2 - \Delta - 3\Delta + 4)y = e^{2t}$

$(\Delta^2 - 4\Delta + 4)y = e^{2t}$

for y_c we findout the roots.

$\Delta^2 - 4\Delta + 4 = 0$

$\Delta^2 - 2\Delta - 2\Delta + 4 = 0$

$(\Delta - 2) - 2(\Delta - 2) = 0$

$\Delta - 2 = 0$, $\Delta = 2$

$\Delta - 2 = 0$, $\Delta = 2$

So!

: General Solution as

$y = (c_1 + c_2 x)^{2x}$

$y = (c_1 + c_2 x)^{2x}$

for $y_p = ?$

$y_p = \frac{1}{\Delta^2 - 4\Delta + 4}$

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$yp = \frac{2}{2D-4} e^{2t}$

If we put "2"

$2D - 4 = 2(2) - 4 = 0$

we take again derivation

$y = (c_1 + c_2 x)^{2t} + e^{2t}$ → General sol

(Answer)