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I.D : 6997

Assignment : Sessional Assignment.

Date: 29/6/2020.

Subject : DSP

Submitted To: Sir Mehr Ali shah.

Q No: 1 (a)

Determine the response $y(n)$, $n \geq 0$ . . . .

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

To The Input $x(n) = 4^n u(n)$.

Solution: $y_h(n) = c_1(-1)^n + c_2(4)^n$

$$y_p(n) = k\,(4)^n u[n]$$

Upon substitution we obtain.

$$kn(4)^n u(n) - 3k(n-1)(4)^{n-1} u(n-1) - 4k(n-2)(4)^{n-2} u(n-2)$$

$$= (4)^n u(n) + 2(4)^{n-1} u(n-1)$$

To Determine k, we avaluate this equation for any $n \geq 2$. we select $n = 2$. from we obtain

$k = \frac{6}{5}$. Therefore.

$$y_p(n) = \frac{6}{3} n (4)^n u(n).$$

The Total Solution To the diffrence equation.

$$y(n) = c_1(-1)^n + c_2(4)^n + \frac{6}{3} n (4)^n \quad n \geq 0$$

Where The constants c_1 and c_2 are determined Such that the Initial Conditions are Satisfied.

$$y(0) = 3y(-1) + 4y(-2) + 1$$

$$y(1) = 3y(0) + 4y(-1) + 6$$

$$= 13\,y(-1) + 12y(-2) + 9$$

We evaluate ~~at~~ at $n = 0$. and $n = 1$.

$$y(0) = c_1 + c_2.$$

$$y(1) = -c_1 + 4c_2 + \frac{24}{5}$$

We can equate these two sets of Relations to obtain C_1 and C_2 -

We can Simplyfy the computations above by setting $y(-1) = y(-2) = 0$

Then we have.

$$c_1 + c_2 = 1$$

$$-c_1 + 4c_2 + \frac{24}{5} = 9$$

Hence $c_1 = -\frac{1}{25}$ and $c_2 = \frac{26}{25}$

finally we have ther zero none state response To the forcing f/n $x(n) = (4)^n u(n)$

$$y_{zs}(n) = -\frac{1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6}{5}\,n(4)^n \quad n \geq 0.$$

X————————X

Q No: 1 (B)

Sol: $y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

$$y(n) - 0.6y(n-1) + 0.08y(n-2) = x(n)$$

To obtain the homogeneus equation set input $x(n) = 0$

$$y(n) - 0.6y(n-1) + 0.08y(n-2) = 0$$

$$y_h(n) = \lambda^n$$

Subtitute the solution to the homogeneus equation.

$$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$$

$$\lambda^{n-2}[\lambda^2 - 0.6\lambda + 0.08) = 0$$

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$$(\lambda - 0.2)(\lambda - 0.4) = 0$$

$$\lambda_1 = 0.2 \quad \lambda_2 = 0.4$$

$$y_h(n) = c_1(\lambda_1)^n + C_2(\lambda_2)^n$$

$$y(n) = c_1 (0.2)^n + c_2 (0.4)^n \quad \text{--- (1)}$$

The particular solution is,

$$y_p(n) = k(-1)^n u[n]$$

$$k(-1)^n u[n] - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) =$$

$$= (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$, $k(1+4+4) = 2$

$$k = 2/9$$

The Total solution is.

$$y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n \right] u(n)$$

from the Initial condition we obtain.

$$y(-1) = y(-2) = 0$$

$$c_1 + 2/9 = 0$$

$$\boxed{c_1 = -2/9}$$

$$2c_1 + 2c_2 - 2/9 = 0$$

$$\boxed{c_2 = 1/3}$$

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Q No: 2 (a)

Determine the Casual signal $x(n)$

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Sol:

$$\frac{X(z)}{z} = \frac{z^2}{(2z-1)(z-1)^2}$$

$$\frac{X(z)}{z} = \frac{A_1}{2z-1} + \frac{B_1}{z-1} + \frac{C}{(z-1)^2}$$

Find A, B, and C.

$A = 4$

$B = -3$

$c = -1$

Hence $x(n) = [4(2)^n - 3 - n]\,u(n)$

Continue -

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(6)

$$X(z) = \frac{1}{4}\frac{1}{(1-z^{-1})} + \frac{3}{4}\frac{1}{1-z^{-1}} + \frac{1}{2}\frac{z^{-1}}{(1-z^{-1})^2}$$

By Applying the Inverse z-Transform.

$$x(n) = \frac{1}{4}(-1)^n u(n) - \frac{3}{4}u(n) - \frac{1}{2}nu(n) =$$

$$\left[\frac{1}{4}(-1)^n - \frac{3}{4} + \frac{n}{2}\right]u(n)$$

x ——————— x

QNo: 2 (b)

Determine The Partial fraction expansion of The Propar function.

$$X(z) = \frac{1}{1 - 1.5 z^{-1} + 0.5 z^{-2}}$$

Solution: First we eliminate the negative Powers by Multiplying both nuerator and denominator by z^{+2}. Thus.

$$X(z) = \frac{z^2}{z^2 - 1.5 z + 0.5}$$

The Poles are $P_1 = 1$ and $P_2 = 0.5$.

$$\frac{X(z)}{(z)} = \frac{z}{(z-1)(z-0.5)} = \frac{A_1}{z-1} + \frac{A_2}{z-0.5}$$

A very simple method to determine A_1, and A_2 is to simply Multiply the equation by denominator Term $(z-1)(z-0.5)$

Thus

$$z = (z - 0.5)A_1 + (z-1)A_2 \quad -(1)$$

Set $z = P_1 = 1$ in eq (1).

$$1 = (1 - 0.5) A_1$$

$A_1 = 2$

Set $z = P_2 = 0.5$ in eq ①

~~$0.5 = (0.5 - 1) A_1$~~

$0.5 = (0.5 - 1) A_2$

$A_2 = -1$

$$\frac{X(z)}{z} = \frac{2}{z-1} - \frac{1}{z-0.5}$$

we can determine the coefficients $A_1, A_2, \ldots A_N$ by Multiplying both sides ① by each terms $(z - P_k)$ $k = 1, 2, 3 \ldots N$

$$\frac{(z - P_k)\,X(z)}{z} = \frac{(z - P_k)A_1}{z - P_1} + \cdots \frac{(z - P_k)A_N}{z - P_N}$$

$z = P_k$

$$A_k = \left.\frac{(z - P_k)\,X(z)}{z}\right|_{z = P_k} \quad k = 1, 2, \ldots N$$

X ———————— X

Q No: 3 (a)

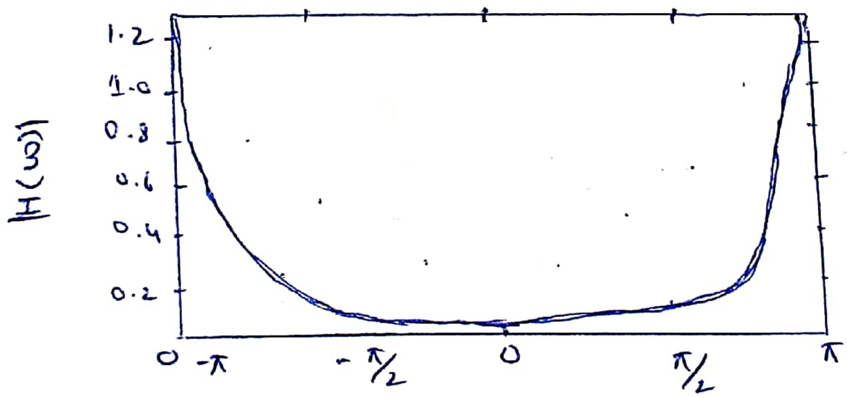
A Two-pole low pass has the system response ----- $H(z) = \dfrac{b_0}{(1 - pz^{-1})^2}$ ----

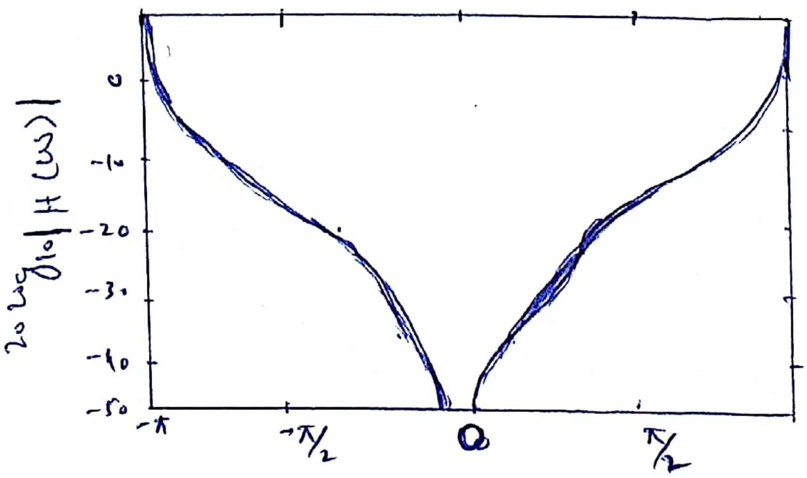
Sol:

At $w = 0$ we have.

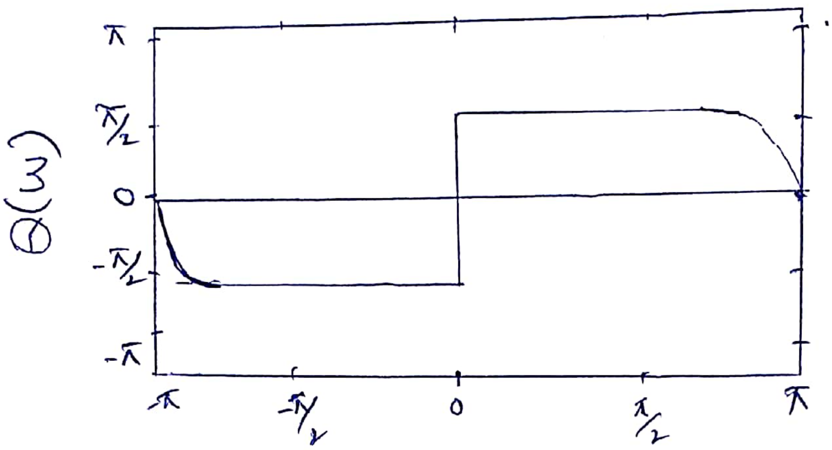
$$H_*(0) = \frac{b_0}{(1-p)^2} = 1$$

$$b_0 = (1-p)^2$$





C(10)



At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-\rho)^2}{(1-\rho e^{-j\pi/4})^2}$$

$$= \frac{(1-\rho)^2}{(1-\rho\cos(\pi/4) + j\rho\sin(\pi/4))^2}$$

$$= \frac{(1-\rho)^2}{(1-\rho/\sqrt{2} + j\rho/\sqrt{2})^2}$$

Hence $= \dfrac{(1-\rho)^4}{\left[(1-\rho/\sqrt{2})^2 + \rho^2/2\right]^2} = \dfrac{1}{2}$

or equivalently:

$$\sqrt{2}(1-\rho)^2 = 1 + \rho^2 - \sqrt{2}\rho$$

The value of $p = 0.32$ satisfy this equation. The system f/n for the desired filter is.

$$H(z) = \frac{0.46}{(1 - 0.32 z^{-1})^2}$$

The same principle can be applied for Design of ~~low~~ band pass filter.

x ————— x

QNo: 3 (b)

Design a Two-Pole bandpass filter...

Solution: Clearly, The filter Must have Poles at $P_{1.2} = re^{\pm j\pi/2}$ and zeros at $z=1$ and $z=-1$. Consequently, The system fn is:

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$

The Gain factor is determined by evaluating The frequency response $H(\omega)$ of The filter at $\omega = \pi/2$. Thus we have.

$$H(\pi/2) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-\gamma^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+\gamma^4+2\gamma^2\cos(8\pi/9)}$$

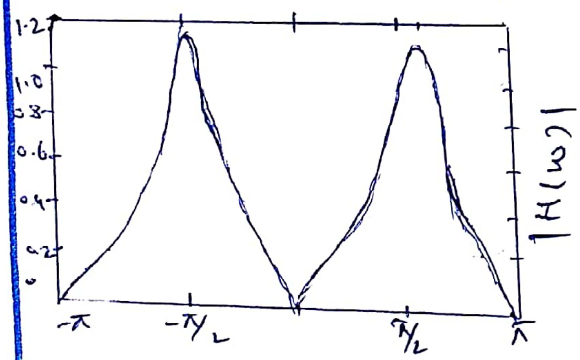
$$= \frac{1}{2}$$

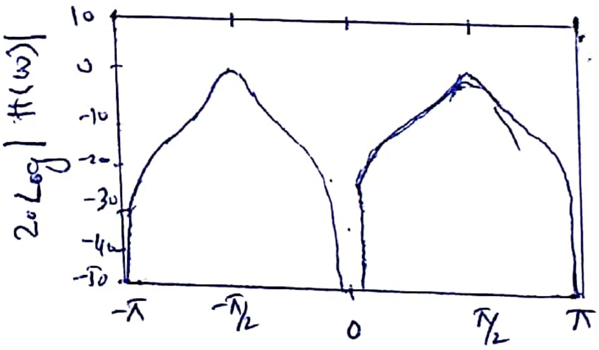
or $1.94(1-\gamma^2)^2 = 1-1.88\gamma^2+\gamma^4$

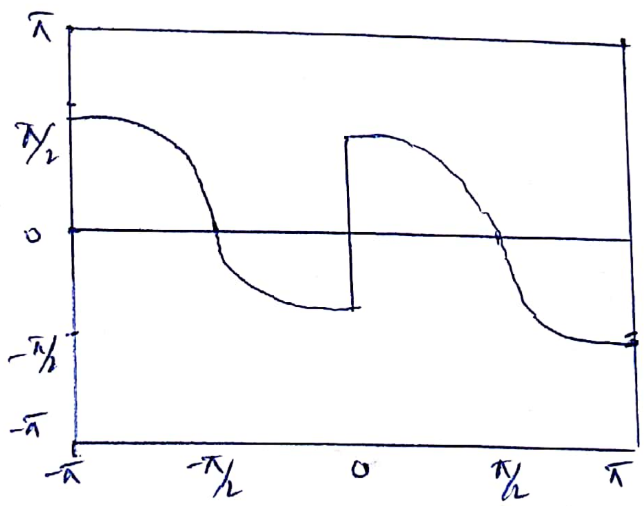
The value of $\gamma^2 = 0.7$ satisfy this equation,

The system fln for The designed filter is.

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$







X —— X

Q No: 4 (a)

A finite duration Sequence of Length L is given as: - - - -

$$X(n) = \begin{cases} 1 & 0 \le n \le L-1 \\ 0 & \text{otherwise.} \end{cases}$$

Determine N point DFT for this Sequence for $N \geq L$.

Sol: The fourier Transform of the Sequence is:

$$X(\omega) = \sum_{n=0}^{L-1} x(n)\, e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}} \Rightarrow$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)}\, e^{-j\omega(L-1)/2}$$

The Magnitude and Phase of $X(\omega)$ are illustrated in figure ①. for $L=10$. The N-point DFT of $x(n)$ is simply $X(\omega)$

evaluated at the set of N equally spaced frequencies $\omega_k = 2\pi k/N$ $k = 0, 1, 2 \cdots N-1$

Hence.

$$X(k) = \frac{1 - e^{-j2\pi kL/N}}{1 - e^{-j2\pi k/N}} \qquad k = 0, 1 \cdots N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$

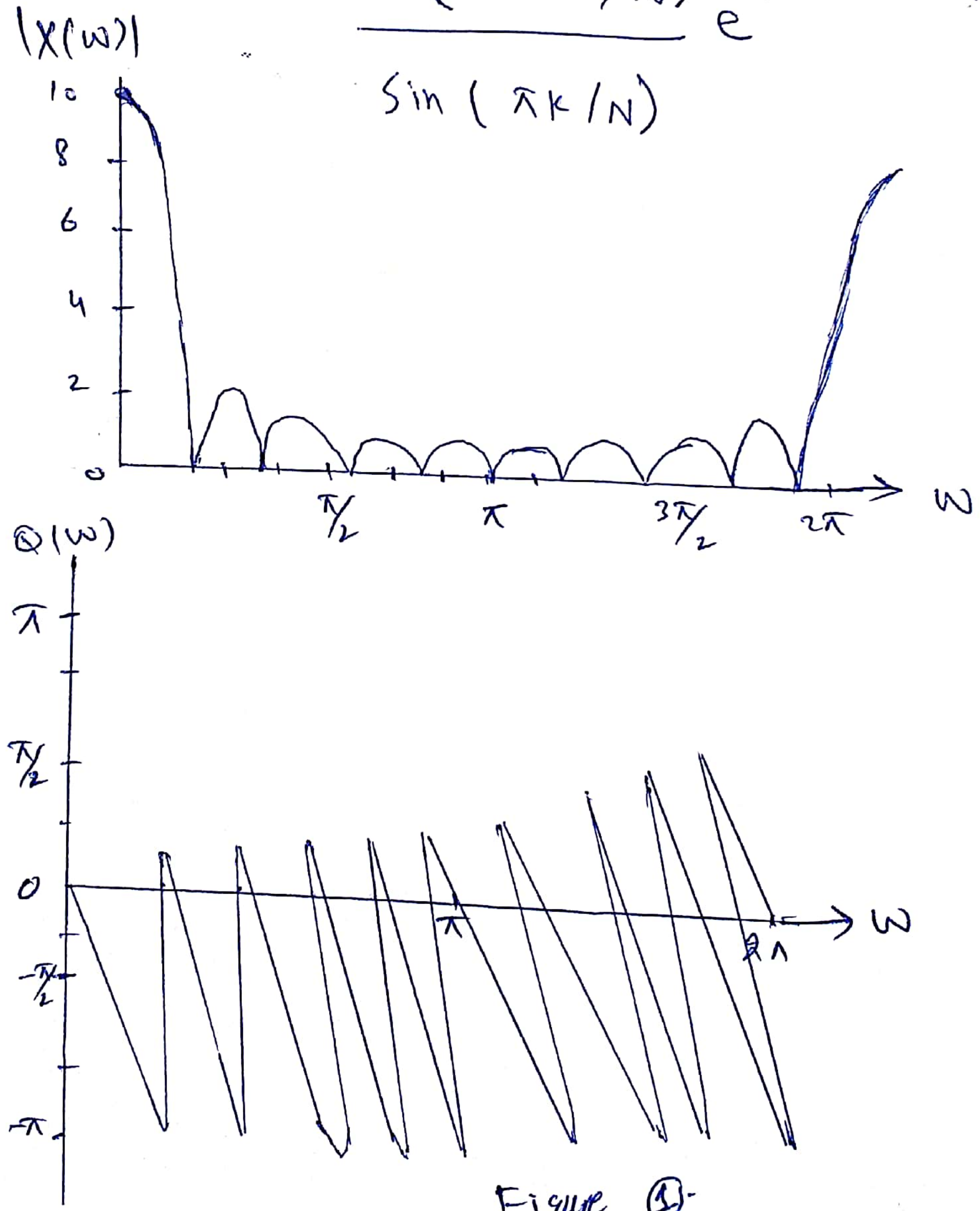


Figure ①.

If N is selected such that N=L
Then DFT becomes.

$$X(k) = \begin{cases} L & k=0 \\ 0 & k=1,2,3\ldots L-1 \end{cases}$$

Thus there is only one non zero value in the DFT. This is apparent from observation of $X(W)$ since $X(W)=0$ at frequencies $W_k = 2\pi k/L$ $k \neq 0$. The reader should verify that $x(n)$ can be recoverd from $X(k)$ by performing an L-point IDFT.

X ———————————— X

QNo: 4(d)

Compute the DFT of the four Point sequence.

$$x(n) = (0\ 1\ 2\ 3)$$

Sol: The First step to determine the Matrix W_4. By exploiting the periodicity property of W_4 and the Summetry property.

$$W_N^{k+N/2} = -W_N^k$$

The Matrix W_4 may be expressed as.

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then.

$$X_4 = W_4 X_1 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

The IDFT of X_4 may be determined by conjugating the elements in W_4 To Obtain W_4^* and then apply froma.

X ——— X