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Q-1 (a) Estimate $\int \theta^4 \sqrt{1-\theta^2}\, d\theta$ (1)

Sol:- $\int \theta \sqrt{1-\theta^2}\, d\theta$

Applying u-Substitude $u = 1-\theta^2$

$$\int \cancel{\sqrt[4]{1-\theta^2}}$$

$$= \int -\frac{\sqrt[4]{u}}{2}\, du$$

Take Constant Out

$$= -\frac{1}{2} \int \sqrt[4]{u}\, du$$

Applying Radical Rule

$$= -\frac{1}{2} \cdot \int u^{1/4}\, du$$

$$= -\frac{1}{2} \frac{u^{1/4+1}}{1/4+1}$$

$$= -\frac{1}{2} \frac{(1-\theta^2)^{1/4+1}}{1/4+1}$$

$$= -\frac{2}{5} (-\theta^2+1)^{5/4}$$

$$\boxed{= -\frac{2}{5} (-\theta^2+1)^{5/4} + C}$$

ANS

Q-1 (b) Estimate $\int_0^1 x^3(1+x^4)^3\,dx$ (2) Using Substitution method

Sol:- $\int_0^1 x^3(1+x^4)^3\,dx$

$x^3(1+x^4)^3 : x^3+3x^7+3x^4+x^{15}$

$= \int_0^1 x^3+3x^7+3x^{11}+x^{15}\,dx$

$= \int_0^1 x^3 dx + \int_0^1 3x^7 dx + \int_0^1 3x^{11} dx + \int_0^1 x^{15} dx$

$\Rightarrow \left[\frac{x^{3+1}}{3+1}\right]_0^1 + 3\left[\frac{x^{7+1}}{7+1}\right]_0^1 + 3\left[\frac{x^{11+1}}{11+1}\right]_0^1 + \left[\frac{x^{15+1}}{15+1}\right]_0^1$

$= \left[\frac{x^4}{4}\right]_0^1 + 3\left[\frac{x^8}{8}\right]_0^1 + 3\left[\frac{x^{12}}{12}\right]_0^1 + \left[\frac{x^{16}}{16}\right]_0^1$

$= \frac{1}{4} + 3\left(\frac{1}{8}\right) + 3\left(\frac{1}{12}\right) + \left(\frac{1}{16}\right)$

$= \frac{1}{4} + \frac{3}{8} + \frac{\cancel{3}^{1}}{\cancel{12}_{4}} + \frac{1}{16}$

$= \frac{1}{4} + \frac{3}{8} + \frac{1}{4} + \frac{1}{16}$

$= \frac{4+6+4+1}{16}$

$= \boxed{\frac{15}{16}}$ Ans

Q=2 The region b/w The curve $y = \sqrt{x}$ $0 \le x \le 4$ & the x-axis is revolved about the x-axis to generate a Solid. Apply the Integration Find The Volume of Solid.

Sol:- Given that $y = \sqrt{x}$

$$0 \le x \le 4 \Rightarrow a \le x \le b$$

As $V = \int_a^b \pi y^2 dx$

$$V = \int_0^4 \pi (\sqrt{x})^2 dx$$

$$V = \pi \int_0^4 x\,dx = \pi \cdot \frac{x^2}{2}\Big|_0^4$$

$$V = \frac{\pi}{2}\left[(4)^2 - 0\right]$$

$$\boxed{V = 8\pi}$$ ANS

Q=3 if $A = 2i - 4j + \sqrt{5}K$ & $B = -2i + 4j - \sqrt{5}K$ then illustrate (i) the Scalar Component of B in the direction of A ii) The vector. $Proje_A B$.

Sol:- (i) Scalar Component of B in Direction of A.

$$B_A = \frac{\vec{B}\cdot\vec{A}}{|\vec{A}||\vec{B}|} \cdot |\vec{A}|$$

$$= \frac{\vec{B}\cdot\vec{A}}{|\vec{B}|}$$

$$\vec{A}\cdot\vec{B} = (2i - 4j + \sqrt{5}K)\cdot(-2i + 4j - \sqrt{5}K)$$

$$= -4 - 16 - 5$$

$$\vec{A}\cdot\vec{B} = -25$$

$$|\vec{B}| = \sqrt{4+16+5} = \sqrt{25}$$

$$|\vec{B}| = 5$$

Putting Values

$$B_A = \frac{-25}{5}$$

$$\boxed{B_A = -5}$$

(ii) $B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$ (5)

$B \cdot A = -4i - 16j - 5k$

$\boxed{B \cdot A = -25}$

$A \cdot A = (2i - 4j + \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$= 4 + 16 + 5$

$\boxed{= 25}$

$\text{Proj}_A B = \left(\frac{B \cdot A}{A \cdot A}\right) A$

$= \left(\frac{-25}{25}\right)(2i - 4j + \sqrt{5}k)$

$= -1(2i - 4j + \sqrt{5}k)$

$\boxed{= -2i + 4j - \sqrt{5}k}$

Q-4 Apply the Fubini's Theorem Calculate $\iint_R f(x,y)\,dA$ for $f(x,y) = 1-6x^2y$ & $R;\ 0 \le x \le 2,\ -1 \le y \le 1$

Sol:- Fubini's theorem

$$= \iint_R f(xy)\,dA$$

$$= \int_0^2\int_{-1}^1 f(xy)\,dA$$

$f(x,y) = 1-6x^2y$

$$= \int_0^2\int_{-1}^1 (1-6x^2y)\,dy\,dx$$

$$= \int_0^2\left(\int_{-1}^1 dy - 6x^2\int_{-1}^1 y\,dy\right)dx$$

$$= \int_0^2 \left(y\Big|_{-1}^1 - \frac{6x^2y^2}{2}\Big|_{-1}^1 \right)dx$$

$$= \int_0^2 (1+1-3x^2-3x^2)\,dx$$

$$= \int_0^2 (2-6x^2)\,dx$$

$$= 2\int_0^2 dx - 6\int_0^2 x^2\,dx$$

$$= 2x\Big|_0^2 - \cancel{6}^{\,2}\,\frac{x^3}{\cancel{3}}\Big|_0^2$$

$$= 4-2(8)$$

$$\boxed{= -12}\ \text{ANS.}$$

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Q=5 (a) Find the maxima, manima of the curve

$$y=-2x^3+6x^2-3$$

Sol:- 1st find $f'(x)$ & $f''(x)$

$$f'(x)=\frac{dy}{dx}=\frac{d}{dx}(-2x^3+6x^2-3)$$

$$=-6x^2+12x-0$$

$$f'(x)=-6x^2+12x$$

Now $f''(x)=\frac{d}{dx}(-6x^2+12x)$

$$f''(x)=-12x+12$$

for maxima & minima

$$f'(x)=0$$

$$-6x^2+12x=0$$

$$6(-x^2+2x)=0$$

$$-x^2+2x=0$$

$$x(-x+2)=0 \qquad -x+2=0 \qquad \boxed{x=2}$$

Put $x=2$ in $f''(x)$

$$f''(x)=(-12(2)+12)$$

$$=-24+12$$

$$f''(x)=-12$$

So $f''(x)<0$

$y=-2x^3+6x^2-3$ is minima at $x=2$

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Q:5 (b) Change into a Spherical Coordinate equation for the Sphere $x^2+y^2+(z-1)^2=1$

Sol:- $x^2+y^2+(z-1)^2=1$

$(\rho \sin\theta \cos\phi)^2+(\rho \sin\phi \sin\phi)^2+(\rho \cos\phi-1)^2=1$

$\rho^2 \sin^2\phi \cos^2\phi+\rho^2 \sin^2\phi \sin^2\theta+\rho^2 \cos^2\phi+1-2\rho \cos\phi=1$

$\rho^2 \sin^2\phi (\cos^2\theta+\sin^2\theta)+\rho^2 \cos^2\theta+1-2\rho \cos\theta = 1$

$\rho^2 \sin^2\theta (\cos^2\phi+\sin^2\phi)+\rho^2 \cos^2\phi+1-2\rho \cos\phi=1$

$\rho^2 (\sin^2\phi)+\rho^2 \cos^2\phi+2\cdot\rho \cos\theta=1-1$

$\rho^2 (\sin^2\phi+\cos^2\phi)-2\rho \cos\phi=0$

$\rho^2=2\rho \cos\phi = 2\cos\phi$