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Section B

Deptt BE(C)

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Subject Differential

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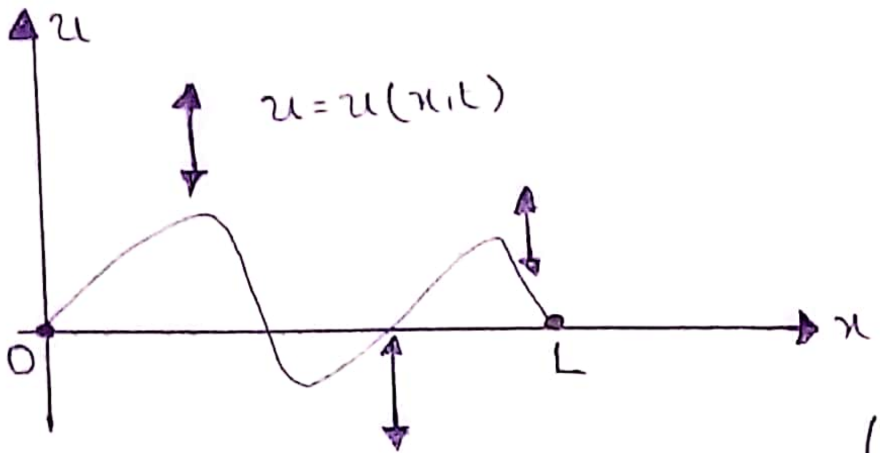
Applications of PDEs:

1: Wave Equation:

The simplest situation to give rise to one-dimensional wave equation is the motion of a stretched string - specifically the transverse vibration of a string such as the string of a musical instrument. Assume that a string is placed along the x-axis, is stretched and then fixed at end $x=0$ and $x=L$; it is then deflected and at some instant, which we call $t=0$, is released and allowed to vibrate. The quantity of interest is the deflection u of string of any point $x, 0 \leq x \leq L$, and at any time $t>0$, we write $u=u(x,t)$. Figure 2 shows a possible displacement of a string at a fixed time t.



(Figure 2)

Subject to various assumptions.

(1) damping forces such as air resistance are negligible.

(2) the weight of the string is negligible.

(3) the tension in the sting is tangential to the curve of the string at any point.

(4) The string perform small transverse oscillations i-e every particle of the string moves strictly vertically and such that its deflection and slope is every point of the string is small.

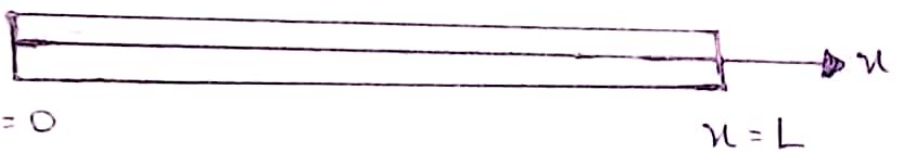
by applying Newton's Law of motion to a small segment of the string, that u satisfies the PDE

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \longrightarrow (1)$$

where $c^2 = \frac{T}{P}$, P being mass per unit length of the string and T being the (constant) horizontal component of the tension in the string. To determine $u(x,t)$ uniquely.

2: HEAT CONDUCTION EQUATION:

Consider a long thin bar, or wire of constant cross section and of homogeneous material oriented along the x-axis



Imagine the bar is thermally insulated laterally and is sufficiently thin that heat flows (by conductio only in the x-direction. Then the temperature u at any point of the bar depends only in the x-koordinate of the point and the time t.

By applying the Principle conservation of energy it can be show that $u(x,t)$ satisfies the PDE

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \qquad 0 \leq x \leq L, \quad t > 0 \longrightarrow (3)$$

where k is a positive constant, Infact k, sometimes called the thermal diffusivity of the bar, is given by

$$K = \frac{k}{s\rho}$$

where k is thermal conductivity of the material of the bar.

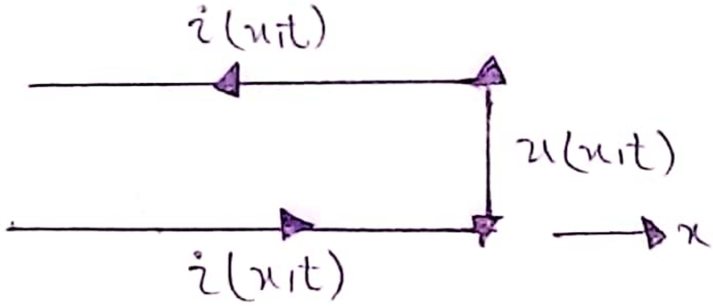
s is the specific heat capicity of the material of the bar.

ρ is the density of material of bar.

The PDE (3) is called the one-dimensional heat conduction equation (or arises) the diffusion equation.

3) TRANSMISSION LINE EQUATIONS:

In a long electrical cable or a telephone wire both the current and voltage depend upon position along the wire as well as the time.



It is possible to show, using basic law of electrical current theory, that the electrical current $i(x,t)$ satisfies the PDE.

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} + (RC+GL)\frac{\partial i}{\partial t} + RGi \longrightarrow (5)$$

Where the constants R, L, G for unit length of cable.

respectively the resistance, induction, capacitance and leakage conductance. The voltage $u(x,t)$ also satisfies (5). Special cases of 5 arises in particular situations. For a submarine cable G is negligible and frequencies are low so inductive effects can also be neglited.

$$\frac{\partial^2 i}{\partial x^2} = RG\frac{\partial i}{\partial t} \longrightarrow (6)$$

which is called submarine equation or telegraph equation. For high frequency alternating currents again with negligeble leakage.

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} \longrightarrow (7)$$

which is called "high frequency line equation".

4) LAPLACE'S EQUATION:

If you look back at the two-dimensional heat conduction equation (4)

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

It is clear that if the heat flow is steady i-e time independent, then $\frac{\partial u}{\partial t} = 0$ so the temperature $u(x,y)$ is solution of

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \longrightarrow (8)$$

(8) is the two-dimensional Laplace equation. Both this and three-deminsional counterpart.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} = 0 \longrightarrow (9)$$

arise in a wide varity of applications. quite part from steady state head conduction theory. Since time does not arise in (8) or (9) it is evident that Laplace equation is always a model for equillibrium situations. In any problem involving Laplace equation we are interested in solving it in a specfic region R for given boundry conductions. Since condition may involve.

(a) u specified the boundry curve C (two-dimensio or boundry surface S (three-dimension) of the region R. Such boundry conditions are called Dirichlet conditions.

(b) The derivative of u normal to the boundry, written $\frac{\partial u}{\partial n}$, specifed on C or S. These are referred to Neumann boundry conditions.

(c) A mixture of (a) and (b).

Such areas where Laplacés equation arises

(a) electrostatics (u being the electrostatics potentic in a charge free region)

(b) Gravitation

(c) Steady state flow of inviscid fluids.

(d) Steady state heat conduction.

(5) OTHER IMPORTANT PDEs IN SCIENCE AND ENGINEERING:

(i) Poisson's equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$$

(two-dimensional form)

where $f(x,y)$ is a given function. This equation arises electrostatics, electricity theory.

(ii) Schroodings equation:

$$-\frac{h^2}{8\pi^2 m}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) = E\psi$$

which arises in quantum mechanics.

(iii) Transverse vibrations equations:

$$a^2 \frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2} = 0$$

for a homogenous rod

(iv) Helmholtz's equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + k^2 u = 0$$

in wave length.