

FINAL EXAM SPRING 2020

NAME : SAIFULLAH JAVE

ID NO : 7156

DEPARTMENT : BE(Civil)

SECTION : 'B'

PAPER : STRUCTURAL ANALYSIS

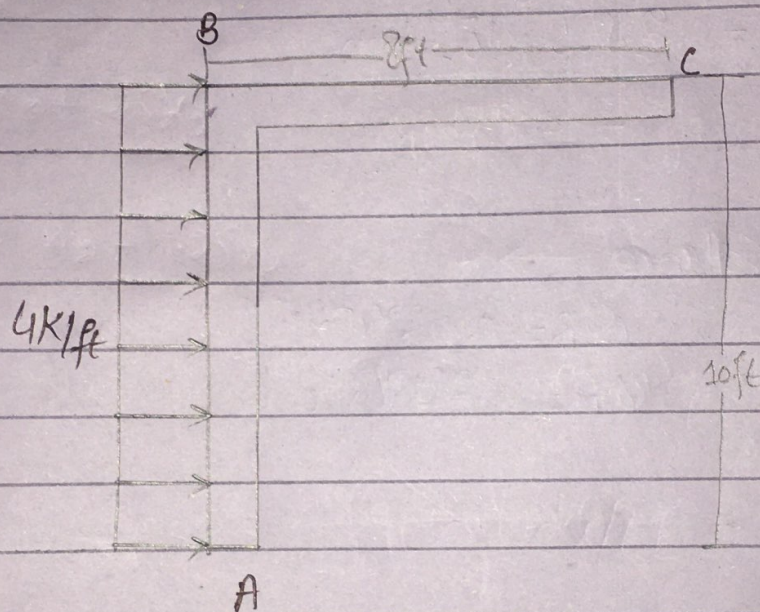
INSTRUCTOR : Engr. AMJID ISLAM

DATE : 26/06/2020

QUESTION No. 1

Determine the vertical Displacement - - -
- - - vertical work.

GIVEN DATA:



$$E = 29 \times 10^3 \text{ ksi}$$

$$I = 600 \text{ in}^4$$

REQUIRED DATA:

vertical displacement = ?

Solution:

FOR REACTION:

$$\sum M_A = 0$$

PAGE 02

$$\Rightarrow -4(10)(5) + C_y(8) = 0$$

$$\Rightarrow C_y = 25 \text{ kips.}$$

$$\sum F = 0 \uparrow +$$

$$25 + A_y = 0$$

$$A_y = -25 \text{ kips.}$$

$$\sum F_x = 0 \rightarrow +$$

$$40 - A_x = 0$$

$$\Rightarrow A_x = 40 \text{ kips.}$$

REAL MOMENTS:

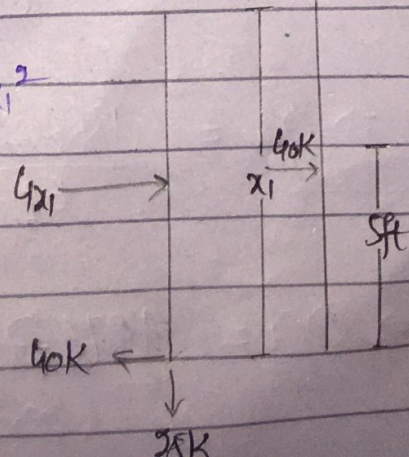
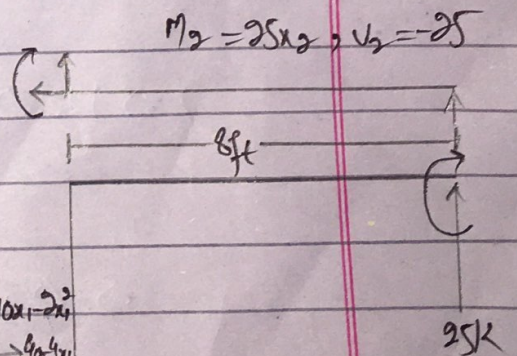
$$\sum M_1 = 0$$

$$\Rightarrow -40(x_1) + 4x_1(x_1/2) + M_1 = 0 \quad M_1 = \frac{40x_1 - 2x_1^2}{1 - 4x_1}$$

$$\Rightarrow M_1 = 40x_1 - 2x_1^2$$

$$\Rightarrow 25x_2 + M_2 = 0$$

$$M_2 = -25x_2$$



VIRTUAL MOMENTS:

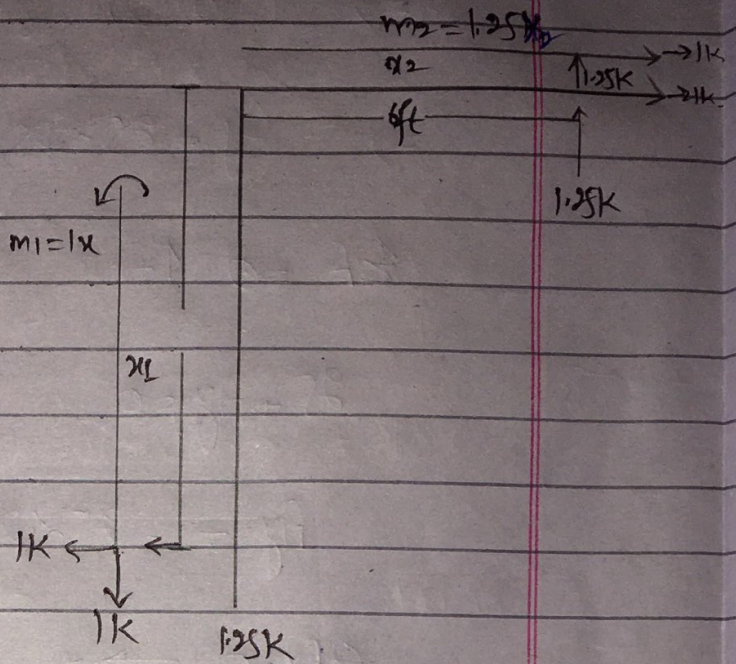
$$\epsilon m_1 = 0$$

$$\Rightarrow -1(x_1) + m_1 = 0$$

$$\Rightarrow m_1 = 1x_1$$

$$-m_2 + 1.25x_2$$

$$\Rightarrow m_2 = 1.25x_2$$



Now from virtual work equation.

$$1k \cdot \Delta C_v = \int_0^L m M dx / EI$$

$$\Rightarrow 1k \cdot \Delta C_v = \int_0^{18} \frac{(40x_1 - 2x_1^2)(1x_1)}{EI} dx$$

$$\Rightarrow \Delta C_v = \frac{85333.3}{EI} + \frac{5555.3}{EI}$$

$$\Rightarrow \Delta C_v = \frac{13666.7}{EI} \text{ k}^2 \cdot \text{ft}^3$$

$$\Rightarrow \Delta C_v = \frac{13666.7 \text{ k}^2 \cdot \text{ft} \times (12^3 \text{ in}^3)}{(29 \times 10^3) (600)}$$

PAGE 04

$$\Rightarrow \boxed{OC_v = 1.357 \text{ inch}}$$

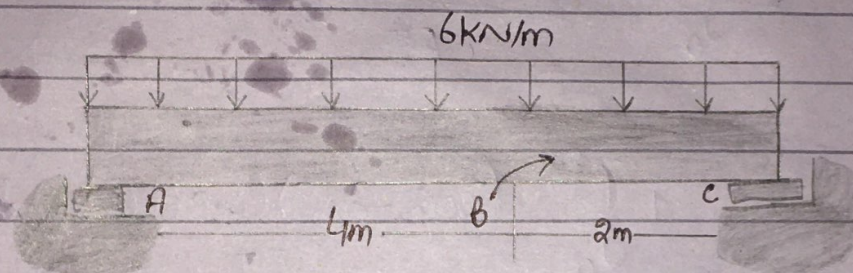
RESULT:

Hence $OC_v = 1.357$ inches.

QUESTION No.2

Determine the slope and displacement -----
----- Use Castigliano's Theorem.

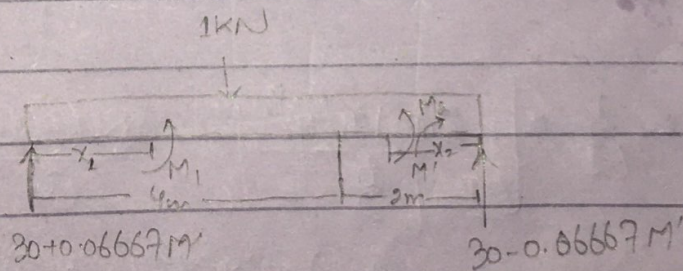
GIVEN DATA:



REQUIRED DATA:

We have to find the
Slope and displacement.

SOLUTION:



$$M_1 = (30 + 0.06667M')x_1 - 2x_1^2$$

$$M_2 = (30 - 0.06667M')x_2 - 2x_2^2$$

$$\text{Now } \frac{\partial M_1}{\partial M'} = 0.06667x_1 \quad \text{and } \frac{\partial M_2}{\partial M'} = 0.06667x_2$$

set $M' = 0$

$$\text{Now } M_1 = (30x_1 - 2x_1^2) \text{ k.ft}$$

$$\& M_2 = (30x_2 - 2x_2^2) \text{ k.ft}$$

Thus

$$Q_B = \int_0^L M \left(\frac{\partial M}{\partial M'} \right) \frac{dx}{EI}$$

$$= \int_0^{4m} \frac{(30x_1 - 2x_1^2)(0.06667x_1) dx_1}{EI}$$

$$+ \int_0^{2m} \frac{(30x_2 - 2x_2^2)(0.06667x_2) dx_2}{EI}$$

$$= \frac{1}{EI} \left[\int_0^{4m} (2.001x_1^2 - 0.1334x_1^3) dx_1 \right.$$

$$\left. + \int_0^{2m} (2.001x_2^2 - 0.1334x_2^3) dx_2 \right]$$

$$= \frac{1}{EI} \left[\frac{2.001x_1^3}{3} \Big|_0^{4m} - \frac{0.1334x_1^4}{4} \Big|_0^{4m} \right.$$

$$\left. + \frac{2.001x_2^3}{3} \Big|_0^{2m} - \frac{0.1334x_2^4}{4} \Big|_0^{2m} \right]$$

$$= \frac{1}{EI} \left[0.667x_1^3 \Big|_0^{4m} - 0.03335x_1^4 \Big|_0^{4m} + 0.667x_2^3 \Big|_0^{2m} - 0.03335x_2^4 \Big|_0^{2m} \right]$$

$$Q_B \Rightarrow \frac{1}{EI} \left[(42.688 - 8.5336) + (5.336 - 0.5336) \right]$$

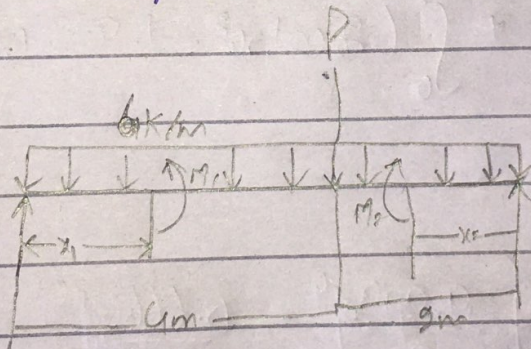
$$\Rightarrow Q_B = \frac{38.9504}{EI}$$

$$\Rightarrow Q_B = \frac{38.9504}{(200 \times 10^9) (60 \times 10^6 \text{ mm}^4)}$$

$$\Rightarrow Q_B = \frac{38.9504}{(200 \times 10^9) (6 \times 10^5)}$$

$$\Rightarrow Q_B = 0.03245 \times 10^{-4} \text{ radian}$$

Now displacement.



$$0.3333P + 30$$

$$0.6667P + 30$$

$$\text{Now } M_1 = (0.3333P + 30)x_1 - 2x_1^2$$

$$M_2 = (0.6667P + 30)x_2 - 2x_2^2$$

$$\text{Now } \frac{\partial M_1}{\partial P} = 0.3333x_1 \quad \& \quad \frac{\partial M_2}{\partial P} = 0.6667x_2$$

$$\text{Set } P = 0$$

$$M_1 = (30x_1 - 2x_1^2) \quad \text{and} \quad M_2 = (30x_2 - 2x_2^2)$$

$$\Delta B = \int_0^l M \left(\frac{\delta M}{\delta P} \right) \frac{dx}{EI}$$

$$\Rightarrow \Delta B = \frac{1}{EI} \left[\int_0^4 (30x_1 - 2x_1^2) \cdot 0.3333x_1 \, dx_1 + \int_0^2 (30x_2 - 2x_2^2) \cdot 0.6667 \, dx_2 \right]$$

$$\Rightarrow \Delta B = \frac{1}{EI} \left[\int_0^4 (9.999x_1^2 - 0.6666x_1^3) \, dx_1 + \int_0^2 (20.001x_2^2 - 1.3334x_2^3) \, dx_2 \right]$$

$$\Rightarrow \Delta B = \frac{1}{EI} \left[\frac{9.999x_1^3}{3} \Big|_0^4 - \frac{0.6666x_1^4}{4} \Big|_0^4 + \frac{20.001x_2^3}{3} \Big|_0^2 - \frac{1.3334x_2^4}{4} \Big|_0^2 \right]$$

$$\Rightarrow \Delta B = \frac{1}{EI} \left[(213.33 - 49.6624) + (53.336 - 5.336) \right]$$

$$OB = \frac{218.6676}{(200 \times 10^9) \times (6 \times 10^{-5})}$$

$$\Rightarrow OB = 0.1822 \times 10^{-4} \text{ ft}$$

$$\Rightarrow \boxed{OB = 2.186 \times 10^{-4} \text{ inch}}$$

RESULT:

Hence

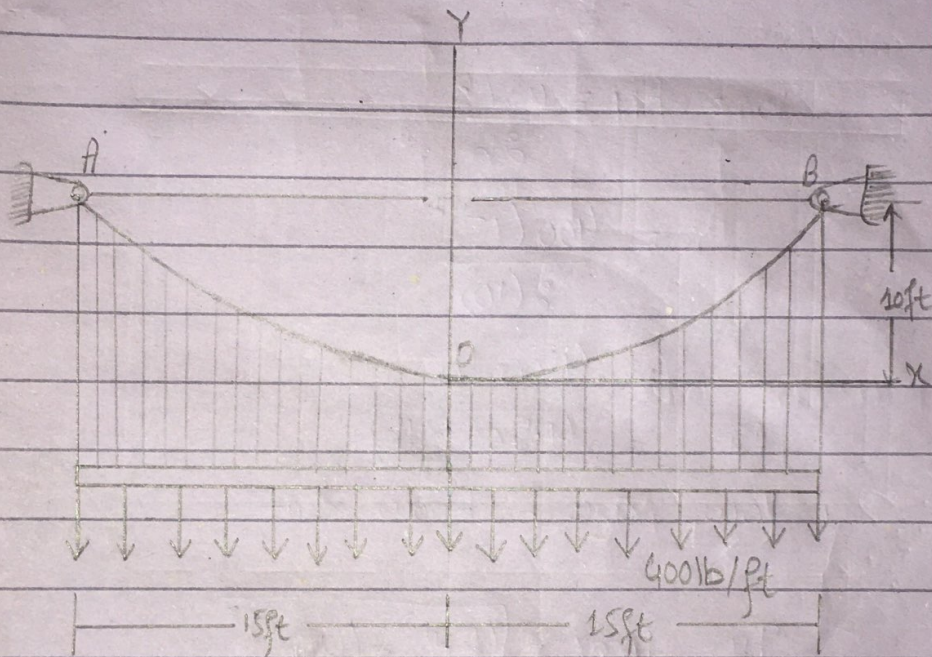
$$OB = 0.02945 \times 10^{-4}$$

$$OB = 2.186 \times 10^{-4}$$

QUESTION No. 3

The Cable is Subjected to uniform -----
----- in the Cable at O and B.

GIVEN DATA:



$$\text{Slope} = 0$$

REQUIRED DATA:

We have to find the equation of the curve and the force in the cable at O and B.

Solution:

As we know that

$$y = \frac{h}{l^2} x^2$$

$$\Rightarrow y = \frac{(10)^2}{(15)^2} x^2$$

$$\Rightarrow y = 0.444 x^2$$

Now we know that

$$\begin{aligned} T_0 = F_H &= \frac{w_0 L^2}{2h} \\ &= \frac{400 (15)^2}{2(10)} \\ &= 4500 \text{ lb} \end{aligned}$$

$$\boxed{F_H = 4500 \text{ Kilb}}$$

Now we know that

$$T_H = T_{\max} = \sqrt{(F_H)^2 + (w_0 L)^2}$$

$$\Rightarrow T_{\max} = \sqrt{(4500)^2 + (400 \times 15)^2}$$

$$\Rightarrow \boxed{T_{\max} = 7500 \text{ lb} = 7.500 \text{ Kilb}}$$

Now we know that

$$T_B = T_{\max} = w_0 L \sqrt{1 + \left(\frac{L}{2h}\right)^2}$$

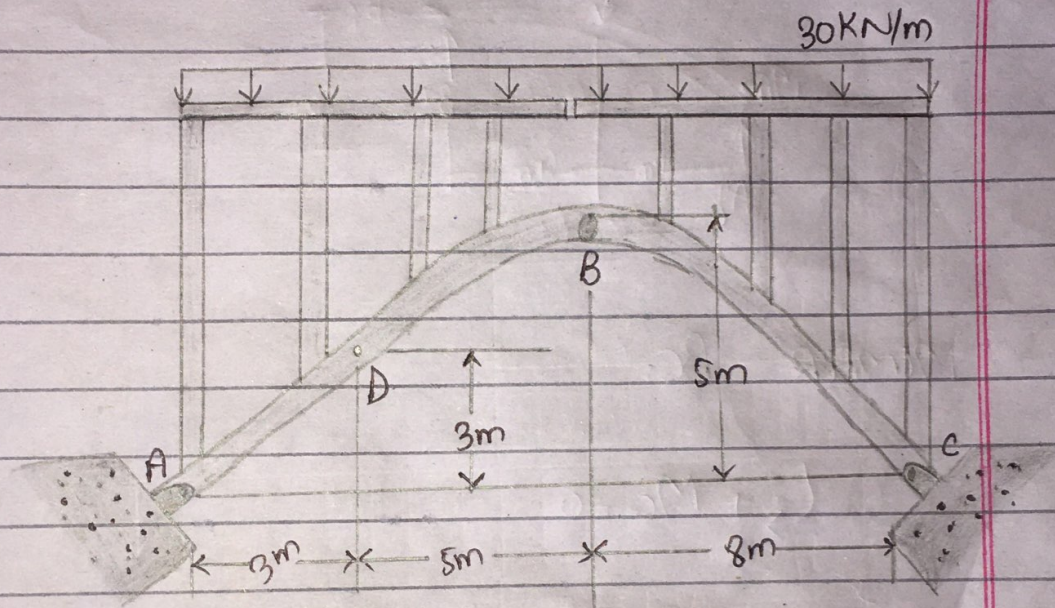
$$\Rightarrow T_{\max} = (400)(15) \sqrt{1 + \left(\frac{15}{10 \times 10}\right)^2}$$

$$\Rightarrow \boxed{T_{\max} = 7500 \text{ lb} = 7.500 \text{ Kilb}}$$

QUESTION No. 4

The three-hinged spandrel arch is subjected to a uniformly distributed load of 30 kN/m . Find the internal moment in the arch at Point D.

GIVEN DATA:



REQUIRED DATA:

We have to find the internal moment in the arch at point D.

Solution:

As we have to find the moment at different members.

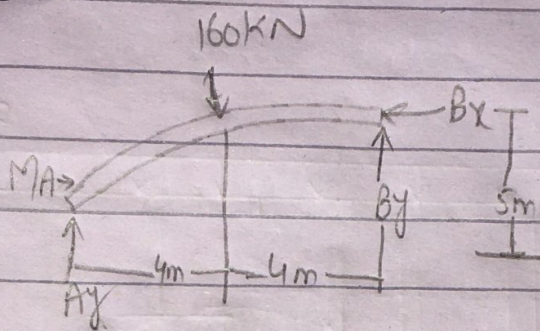
MEMBER AB:

$$\sum + \Sigma M_A = 0$$

$$\Rightarrow B_x(5) + B_y(8) - 240(4) = 0$$

$$\Rightarrow 5B_x + 8B_y = 960$$

MEMBER



MEMBER BC:

$$\hookrightarrow + M_c = 0$$

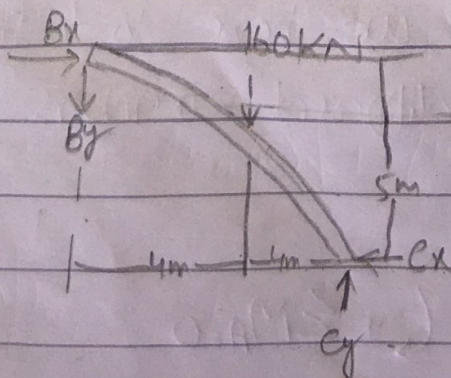
$$\Rightarrow -B_x(5) + B_y(8) + (240)4 = 0$$

$$\text{As } B_y = 0$$

$$\Rightarrow -5B_x + 0 + 960$$

$$\Rightarrow 5B_x = 960$$

$$\Rightarrow \boxed{B_x = 192}$$



NOW SEGMENT BD:

$$\sum M_D = 0$$

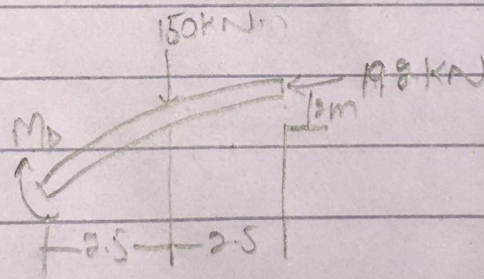
$$\Rightarrow (B_x)(2) - (150 \times 2.5) - M_D = 0$$

$$\Rightarrow (198 \times 2) - (375) - M_D = 0$$

$$\Rightarrow 396 - 375 - M_D = 0$$

$$\Rightarrow 21 - M_D = 0$$

$$\Rightarrow \boxed{M_D = 21 \text{ kNm}}$$



RESULT:

Hence $M_D = 21 \text{ kNm}$.