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Q1(a):→ Define ~~Drag~~ with its components. write down the equations for Friction ~~Drag~~ Drag co-efficient both in laminar and turbulant boundary layer?

Ans:→ (a) Force on Immersed Bodies:→

→ A body which is wholly immersed in a homogenous fluid may be subjected to two kind of forces arising from relative motion between body and fluid these forces are termed as Drag and and lift, depending on force either parallel or at right angle to the motion.

→ Drag force on suberged body can have two components.

(i):→ Pressure Drag.

ii) Friction Drag.

i):→ Pressure Drag:→

→ It is equal to the integration of components in direction of motion of all pressure force exerted on surface of the body.

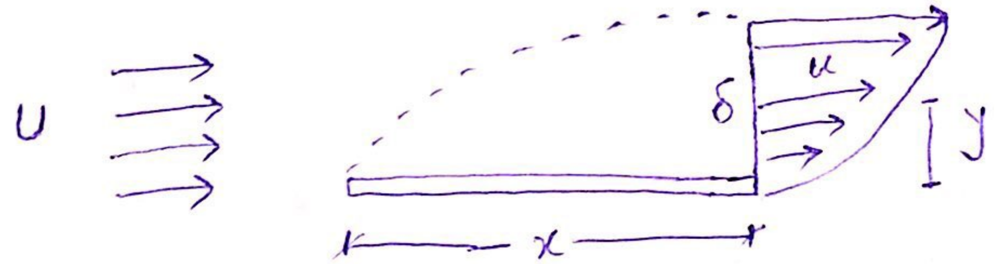
$F_P = C_P \rho \frac{V^2}{2} \times A$ where C_P depends on shape.

ii):→ Friction Drag:→

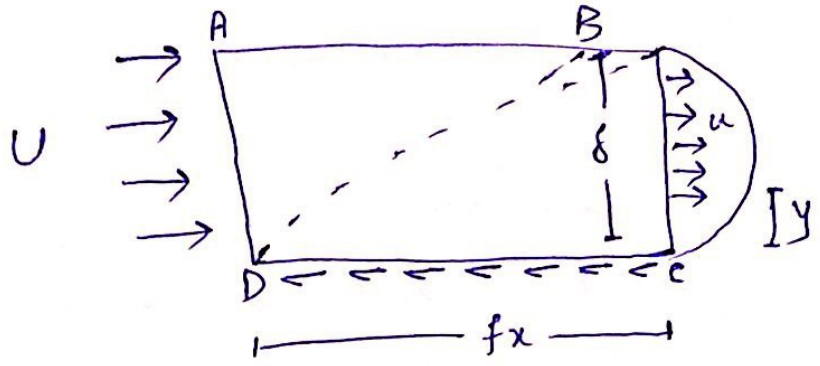
→ It is equal to integration of components of shear stress along the surface of body in direction of motion.

$F_P = C_f \rho \frac{V^2}{2} \times BL$ where C_f depends on viscosity.

*):→ Friction Drag of Boundary layer:→



→ Figure shows the growth of Boundary layer along one side of smooth plate in steady flow consider a control volume where ~~B~~ δ is the thickness of the boundary layer. U is undisturbed velocity and u is distrubed velocity



As $\Sigma fx = 0$

Now according to impulse momentum principle,

$$\Delta F = \frac{\Delta P}{\Delta t}$$

$$= \frac{\Delta (mv)}{\Delta t} \qquad \therefore m = \rho \times \text{volume}$$

$$Fx = \frac{\Delta(\rho \times \text{volume} \times v)}{\Delta t} = \Delta \rho Q v$$

$$F = \rho Q v \longrightarrow (1)$$

$-Fx$ = rate of change of $BC + AB - AD$

$$AD = \rho U (U \delta B)$$

$$CB \rightarrow \rho \times \left(\int_0^\delta u^2 \times dy\right) B$$

$$AB \rightarrow \rho \times U \times \left(U \times B \times \delta - B \int_0^\delta u \, dy\right)$$

Putting values in equ(1)

$$Fx = \rho B \int_0^\delta u(U-u)\, dy$$

→ By solving the equation

$$Fx = \rho B U^2 \delta \alpha$$

→ To find shear stress.

$$\tau_0 = \frac{F_x}{A} = \frac{dF_x}{B \cdot dx} = \frac{\rho B U^2 \alpha \cdot d\delta}{B \cdot dx} = \rho U^2 \alpha \frac{d\delta}{dx}$$

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$

*) :→ Laminar boundary layer :→

$$\frac{u}{U} = f(y/\delta)$$

Assume,

$\eta = \frac{y}{\delta}$ => $y = \delta\eta$ or $dy = \delta\, d\eta$

thus,

$\frac{u}{U} = f(\eta)$ or $u = Uf(\eta)$ => $du = U df(\eta)$

In case of laminar flow,

$$\tau_0 = \mu\, (du/dy)$$

$$\Rightarrow \tau_0 = \frac{\mu}{\delta} \Rightarrow \tau_0 = \mu U \left[\frac{df(\eta)}{d\eta} \right]$$

By solving the equation

$$\tau_0 = \frac{\mu U B}{\delta} \rightarrow (1)$$

As we have $\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

$$\rightarrow \rho U^2 \alpha \frac{d\delta}{dx} = \frac{\mu U B}{\delta}$$

$$\delta\, d\delta = \frac{\mu B}{\rho U \alpha} \cdot dx$$

→ Integrating:→

$$\frac{\delta^2}{2} = \frac{\mu\beta}{\rho U\alpha} \cdot x + c$$

$$x = 0, \quad \delta = 0 \quad \therefore c = 0$$

$$\Rightarrow \frac{\delta^2}{2} = \frac{\mu\beta x}{\rho U\alpha} \quad \text{or} \quad \delta^2 = \frac{2\mu\beta x}{\rho U\alpha}$$

$$\sqrt{\frac{2\beta}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}} = \sqrt{\frac{2\beta}{\alpha}} \cdot \frac{x}{\sqrt{R_x}}$$

$\therefore$ where $R_x = \frac{xU\rho}{\mu}$

$\beta = 1.63$, $\alpha = 0.135$

Thus by putting values

$$\delta = \sqrt{\frac{2 \times 1.63}{0.135}} \times \frac{x}{\sqrt{R_x}}$$

$$\delta = \frac{4.91}{\sqrt{R_x}} \times x$$

↳ This is Thickness of laminer Boundary layer

Now,

$$\tau_0 = \frac{\mu U\beta}{\delta} \quad \therefore \beta = 1.63$$

$$\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

R_x is local Reynold number and is about 500,000.

Now,

$$F_f = B \int_0^L \tau_0 \, dx \qquad \therefore \tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

$$\cdot R_x = \frac{x U \rho}{\mu}$$

Thus

$$F_f = 0.664 B \sqrt{\rho \mu L U^3}$$

$$F.f = C.F \rho \frac{V^2}{2} BL$$

equation on b/s

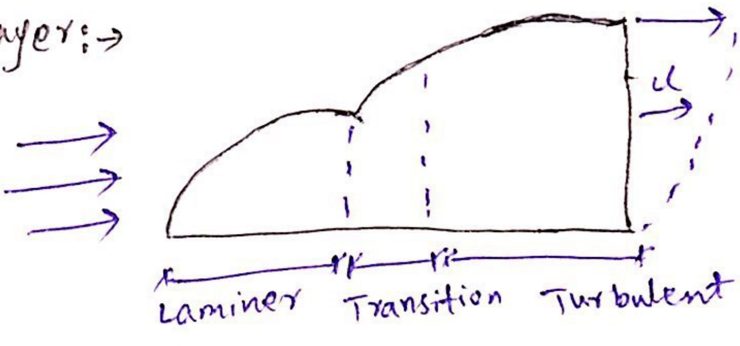
$$C.F = 1.328 \sqrt{\mu / \rho L U}$$

$$C.F = \frac{1.328}{\sqrt{R_x}}$$

for laminar $R < 500,000$.

A):→ Turbulent Boundary Layer:→

→ Figure shows the velocity distribution in Turbulent layer much steeper near wall and flatter throughout the remender layer The shear stress in greater in turbulent layer than in laminer layer.



As we have,

$$\tau_0 = f \rho \frac{V^2}{8}$$

where V is average velocity in pipes.

we may obtain relation between "v" and "U"

$$\frac{v}{U_{max}} = \frac{1}{1+1.33\sqrt{f}}$$ $\therefore$ f = 0.028 from moody's chart

$$\frac{V_{av}}{U_{max}} = \frac{1}{1+1.33\sqrt{0.28}} \Rightarrow U = 1.235\,v_{av}$$

$$v = U/1.235$$

Now,

$$\tau_0 = f\rho\frac{v^2}{8} \text{ and } f = \frac{0.316}{R^{0.25}} \text{ and } R = \frac{Dv}{\nu}$$

$$\Rightarrow \tau_0 = \frac{0.316}{(Dv/\nu)^{1/4}} \times \rho \times \frac{v^2}{8}$$ where $v = U/1.235$

and $D = 2\delta$

$$\Rightarrow \tau_0 = \frac{0.316}{\left(\frac{D(U/1.235)}{\nu}\right)^{1/4}} \times \frac{\rho}{8} \times \left(\frac{U}{1.235}\right)^2$$

$$\tau_0 = \frac{0.23\rho U^2}{(\delta U/\nu)^{1/4}}$$

Equating both equation for τ_0 and applying boundary conditions of

$$x = 0 \,, \quad \delta = 0$$

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \cdot \left(\frac{\nu}{ux}\right)^{1/5} \cdot x$$

For $\alpha = 0.0972$

$$\delta = 0.377 \times (\nu/Ux)^{1/5} \cdot x$$

or $$\frac{\delta}{x} = \frac{0.77}{(Rx)^{1/5}}$$

$$\tau_0 = 0.0587\, \rho \frac{U^2}{2} (\nu/Ux)^{1/5}$$

$$df = B \int_0^L \tau_0\, dx = 0.735\, \rho \frac{U^2}{2} \left(\frac{\nu}{UL}\right)^{1/5} \times BL$$

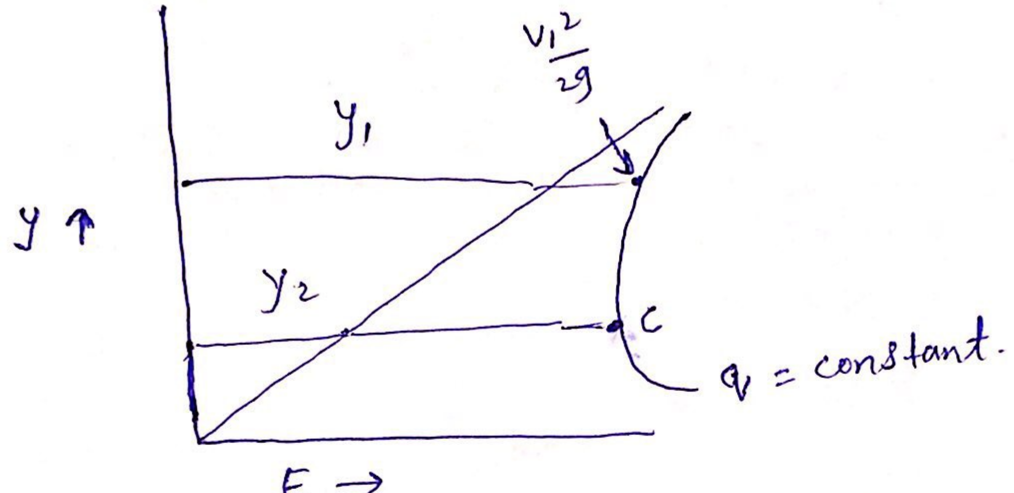
As we have $F.f = C_f \cdot \rho \cdot \frac{U^2}{2} \times BL$

Thus,
$$C.f = \frac{0.0735}{(R)^{1/5}} \qquad (500{,}000 < R < 10^7)$$

For $R > 10^7$, $$C.f = \frac{0.4555}{(\log R)^{2.58}}$$

Q1(b) :→ Dervie equation for critical depth, critical velocity of rectangular section of a channel?

Ans:→



→ This is specific energy equation 4 :-

For particular q, There will be two kind of possible value of y, For given E. The equation is cubic with three rods with third being negative giving no values Thus two alternative depths represents two totally different flow regimes - slow and deep an upper portion and shallow on lower ~~po~~ portion.

→ Point represent dividing point between two regima of flow. Thus for given "q", value of E is minimum and flow at this point is critical flow Depth of flow at this point is critical depth b/c and velocity at this point is critical velocity.

→ Thus relation of critical depth can be found as,

$$E = y + \frac{1}{2g}\left(q^2/y^2\right)$$

For minimum specific energy.

$$\frac{dE}{dy} = 0$$

$$\frac{dE}{dy} = 1 - \frac{2}{2g}\left(q^2/y^3\right)$$

$$\frac{dE}{dy} = 1 - q^2/gy^3$$

$$1 = \frac{q^2}{gy^3} = q^2 = gy^3$$

$$y_{cr} = \left(q^2/g\right)^{1/3}$$

As $q = Vy$, $V_c^2 = gy^3$

or $V_c = \sqrt{g y_c}$

$$y_c = \frac{V_c^2}{g}$$

Now

$$\frac{y_c}{2} = \frac{V_c^2}{2g}$$

$$E_{min} > y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2}$$

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$\frac{3}{2} y_c$ OR $y_{cr} = \frac{2}{3}$

	subcritical	critical	super critical
Depth of Flow	$y > y_c$	$y = y_c$	$y < y_c$
velocity slop	$V < V_c$ mild slop $S_o < S_c$	$V = V_c$ critical slop	$V > V_c$

Q2:→

Given Data

Depth of Rectangular channel (d) = ?

Flow rate (Q) = 3.5 m^3/sec

slope (S_o) = 0.0008

$n = 0.0219$

width of Bed = 7.885

Required = ?

critical depth = ?

Flow sub critical or super critical = ?

Sol:→

$$Area = 7.885 \times d$$
$$= 7.885d$$

$$Perimeter = d + 7.885 + d$$
$$= 7.885 + 2d$$

$$\text{Hydraulic Radius } (Rh) = A/P$$
$$= \frac{7.885d}{7.885 + 2d}$$

By using manning equation

$$Q = \frac{1}{n} A Rn^{2/3} . S_o^{1/2}$$

Putting values

$$3.5 = \frac{1}{0.0219} \times 7.885d \times \left(\frac{7.885d}{2d+7.885}\right)^{2/3} \times (0.0008)^{1/2}$$

$d = 0.5$

Area $= 7.885\,(0.554)$

$= 4.37\,m^2$

Perimeter $= 7.885 + 2\,(0.554)$

$= 8.993\,m$

Hydraulic Radius (Rh) $= \frac{\text{Area}}{\text{Perimeter}}$

$= \frac{4.36}{8.993}$

Hydraulic Radius $= 0.84\,m$

*) :→ Critical depth :→

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

$q = Q/B$

$= 3.5/7.885 \Rightarrow 0.4438\,m^2/sec$

$$\Rightarrow y_{cr} = \left(\frac{(0.443)^2}{9.81}\right)^{1/2} \Rightarrow 0.271$$

As $y > y_{cr}$

$0.554 > 0.271$

So flow is sub-critical.

Q3:→

Given Data

Friction Drag $(F_D) = ?$

Width $(B) = 200mm = 0.2m$

Length $= 800mm = 0.8m$

Specific Gravity $(S) = 0.89$

Undisturbed velocity $(V) = 5\ m/sec$

Kinematic viscosity $(\nu) = 0.93 \times 10^{-4} m^2/sec$

Sol:→ Checking whether flow is laminar or not By Reynold Number,

$$R = \frac{DV}{\nu}$$

For smooth flat plate,

$$D = L \quad , V = U$$

So $$R = \frac{LU}{\nu}$$

$$= \frac{0.8 \times 5}{0.93 \times 10^{-4}} = 43010$$

$43010 < 500,000 \rightarrow$ Laminar

By using formula,

$$F.f = C_f \cdot \rho \cdot \frac{V^2}{2} \cdot BL$$

where

$$C.f = \frac{1.328}{\sqrt{R}} = \frac{1.328}{\sqrt{43010}} = 0.0064$$

$$S = \frac{\rho_{oil}}{\rho_{water}} \Rightarrow 0.89 = \frac{\rho_{oil}}{1000}$$

$$\rho_{oil} = 0.89 \times 1000$$

$$\rho_{oil} = 890 \ kg/m^3$$

$$\Rightarrow F_f = C_f \cdot \rho \cdot \frac{U^2}{2} \cdot BL$$

$$= 0.0064 \times 890 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$F_f = 11.39 N$$