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Degree	Bs (cs)
Exam	Mid
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Course	Differential Equations.

Q No. 1:

(a) Define differential eauation along with 2 examples?

Ans:

A differential eqjuation is an equation involving and its derivitives.
The solution to a differential equation is in the form of a function / Class of function.

Examples of Differential equation:

(i)

$$\left(\frac{dy}{dx}\right)^3 + \frac{d^4y}{dx^4} + y = 2\sin(x) + \cos^3(x)$$

This is a differential equation of order 4.

(ii)

$$y'' + 2y' = 3y$$

or

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 3y$$

This is a differential equation of order 2.

Q No 1:

(B)

Define a ~~Separable~~ Separable Differential equation (DE)?

Ans:

A differential equation that can be factored into two parts is called a separable differential equation.

i.e

$$G(x,y) = M(x)\,N(y)$$

$$\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$$

We can write this equation as

$$y\,dy = -x e^{-x}\,dx$$

Hence this is a separable differential equation.

i):- Solve the following IVP using separable D.E & find the interval of validity of the solution

a:- $y' = \frac{xy^3}{\sqrt{1+x^2}}$ $\quad y(0) = -1$

Solution:-

$y^{-3} \, dy = x(1+x^2)^{-\frac{1}{2}} dx$

$\int y^{-3} dy = \int x (1+x^2)^{-\frac{1}{2}} dx$

$\frac{1}{-2y^2} = \sqrt{1+x^2} + c$

$\frac{-1}{2} = \sqrt{1} + c$

$c = -\frac{3}{2}$

$\frac{1}{-2y^2} = \sqrt{1+x^2} - \frac{3}{2}$

$\frac{1}{y^2} = 3 - 2\sqrt{1+x^2}$

$y^2 = \frac{1}{3-2\sqrt{1+x^2}}$

$y(x) = \frac{-1}{\sqrt{3-2\sqrt{1+x^2}}}$

Now finding interval of validity.

$3-2\sqrt{1+x^2}>0$

$3>2\sqrt{1+x^2}$

$9>4(1+x^2)$

$\frac{9}{4}>1+x^2$

$\frac{5}{4}>x^2$

$-\frac{\sqrt{5}}{2}<x<\frac{\sqrt{5}}{2}$

$x=0$

Interval of validity.

Ans

$y' = e^{-y}(2x-4).\ y(5)=0$

Step 1 #.

Multiplying by e^y & by dx.

$e^y\,dy = (2x-4)\,dx$

$\int e^y\,dy = \int (2x-4)\,dx$

$e^y = x^2+4x+c$

Natural log

$y = \ln(x^2 + 4x + c)$

Finding c

$y(5) = \ln((5)^2 - 4(5) + c)$

$\ln(5 + c) = 0$

$5 + c = 1$

$c = -4$

$y = \ln(x^2 - 4x - 4)$ Ans.

Q2: Solve the following IVP using linear D.E method.

(i):- Explain the steps for using linear D.E

Ans: Substitute $y = uv$

Factor the parts involving v. Put the v term equal to zero.

Solve using separation of variable to find v.

Substitute v back into the equation we got at step 2.

Solve that to find u.

Finally substitute u & v in to $y = uv$ to get our solution.

ii) $\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin(x) - 1 \quad y\left[\frac{\pi}{4}\right] = 3\sqrt{2},$

$0 \le x \le \frac{\pi}{2}$

Ans: Sol:-

$$y' + \frac{\sin(x)}{\cos(x)}y = 2\cos^2(x)\sin(x) - \frac{1}{\cos x}$$

$$y' + \tan(x)y = 2\cos^2(x)\sin x - \sec(x).$$

$$M(t) = e^{\int \tan(x)dx} = e^{\ln|\sec(x)|} = e^{\ln \sec(x)}$$

$$\sec(x)\, y^3 + \sec(x)\tan(x)y = 2\sec(x)\cos^2(x)\sin(x) - \sec^2(x)$$

$$(\sec(x)y)^2 = 2\cos(x)\sin(x) - \sec^2(x).$$

$$\int (\sec(x)\, y(x))'\, dx = \int 2\, 2\cos(x)\sin(x) - \sec^2(x)$$

$$\sec(x)y(x) = \int \sin(2x) - \sec^2(x)dx$$

$$\sec(x)\, y(x) = -\frac{1}{2}\cos(2x) - \tan(x) + C$$

$$y(x) = -\frac{1}{2}\cos(x)\cos(2x) - \cos(x)\tan x + C\cos(x)$$

$$= -\frac{1}{2}\cos(x)\cos(2x) - \sin(x) + C\cos(x).$$

Put the value of "y" & "x"

$$3\sqrt{2} = y\frac{\pi}{4} = -\frac{1}{2}\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right) + C\cos(x).$$

$$3\sqrt{2} = -\frac{\sqrt{2}}{2} + c\frac{\sqrt{2}}{2}$$

$$c = 7.$$

$$y(x) = -\tfrac{1}{2}\cos(x)\cos(2x) - \sin(x) + 7\cos(x).$$

Ans.

iii) x'

Q3: Solve the following IVP for the exact equation and find the the in of validity for the solution.

i) $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0, \ y(0) = -3$

Ans: Sol:-

$M = 2xy - 9x^2 \ , \quad M_y = 2x$
$N = 2y + x^2 \ ; \quad N_x = 2x$

Now, how do we actually find

$\psi(x, y)$?

$\psi_x = M$
$\psi_y = N$

$\psi = \int M dx$ or $\psi = \int N dy$.

$\psi_y = x^2 + h'(y) = 2y + x^2 + 1 = N.$

$h'(y) = 2y + 1.$

$h(y) = \int 2(y) + 1 dy = y^2 + y + K$

$\psi(x, y) = x^2 y - 3x^2 + y^2 + y^2 + y + K = y^2 + (x^2+1) - 3 + K.$

$y^2 + (x^2+1)y + 3x^2 + K = C$

$y^2+(x^2+1)\,y-3x^2=c-k$

$y^2+(x^2+1)\,y-3x^2=c.$

Initial condition to find c.

$(-3)^2+(0+1)(-3)\ -3(0)^2=c \quad \therefore c=6$

put the value of c.

$y^2+(x^2+1)\,y-3x^2-6=0.$

using Quadratic Formula.

~~$y(x)=-(x^2+1)\pm x^2+12x^3+2x^2+2$~~

$$y(x)=\frac{-(x^2+1)\pm\sqrt{(x^2+1)^2-4(1)(-3x^2-6)}}{2(1)}$$

$$=\frac{-(x^2+1)\pm\sqrt{x^2+12x^3+2x^2+25}}{2}$$

$$-3=y(0)=\frac{-1\pm\sqrt{25}}{2}=\frac{-1\pm5}{2}\pm-3,2$$

$$y(x)=\frac{-(x^2+1)-\sqrt{x^4+12x^2+25}}{2}$$

$x^4+12x^3+2x^2+25=0.$

Ans

ii) $\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2+1))\, y' = 0, \quad y(5) = 0$

Ans:- Sol:-

$M = \frac{2ty}{t^2+1} - 2t \qquad M_y = \frac{2t}{t^2+1}$

$N = \ln(t^2+1) - 2 \qquad N_y = \frac{2t}{t^2+1}$

$\int$ first one.

$\psi(x,y) = \int \frac{2ty}{t^2+1} - 2t \, dy = y\ln(t^2+1) - t^2 + h(y)$

Now differentiate.

$\psi_y = \ln(t^2+1) + h'(y) \quad \ln(t^2+1) - 2.$

$h'(y) = -2 \Rightarrow h(y) = -2y.$

$\psi(t,y) = y\ln(t^2+1) - t^2 - 2y$

$y\ln(t^2+1) - t^2 - 2y = c$

$c = -25$

$y(\ln(t^2+1) - 2) - t^2 = -25$

$y(t) = \frac{t^2 - 25}{\ln(t^2+1) - 2}$

$\ln(t^2+1) - 2 = 0$

$\ln(t^2+1) = 2$

$t^2 + 1 = e^2$

$t = \pm\sqrt{e^2 - 1}$ Ans