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Q NO. 1(a)

Estimate $\int \theta^4 \sqrt{1-\theta^2}\, d\theta$

Solution:: Given That

$$\int \theta^4 \sqrt{1-\theta^2}\, d\theta$$

So

Let $1-\theta^2 = u$

$$\frac{d}{d\theta}(1-\theta^2) = \frac{d}{d\theta}.u$$

$$-2\theta = \frac{du}{d\theta}$$

$$\theta . d\theta = \frac{-1}{2} du.$$

Now

$$= \int (u)^{1/4} . \left(\frac{-1}{2}\right) du$$

$$= \frac{-1}{2} \int u^{1/4}\, du \qquad \because \frac{1}{4}+1$$

$$= 5/4$$

$$= \frac{-1}{2} . \frac{4^2}{5} . u^{5/4} + c$$

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$$= \frac{-2}{5} \cdot 4^{5/4} + c$$

By back substitution.

$$= \frac{-2}{5} (1-x^2)^{5/4} + c$$

Result =

$$\frac{-2}{5} (1-x)^{5/4} + c$$

Q NO. 1 (b)

Estimate $\int_0^1 x^3 (1+x^4)^3 dx$ using substitution method:-

Solution:- Given That

$$\int_0^1 x^3 (1+x^4)^3 dx$$

Let $1+x^4 = u$. ← (1)

$$\frac{d}{dx}(1+x^4) = \frac{d}{dx}u.$$

$$4x^3 = \frac{du}{dx}.$$

$$x^3 = \frac{1}{4}\cdot\frac{du}{dx}$$

Now put $\boxed{x=0}$ in eq. (1).

$$1+(1)^4 = 2$$

$$\boxed{u=2}$$

$$= \int_1^2 (u)^3 \frac{1}{u}\, du.$$

$$= \frac{1}{4} \int_1^2 u^3\, du$$

$$= \frac{1}{4} \left. \frac{u^4}{4} \right|_1^2$$

$$= \frac{1}{4} \left(\frac{(2)^4}{4} - \frac{(1)^4}{4} \right)$$

$$= 3/8$$

Result :-

$$\boxed{= 3/8}$$ Ans. Ans.

Q No. 2 (a)

Find The centre of Sphere and radius:

$$x^2+y^2+z^2+3x-4z+1=0$$

Solution:- Given

$$x^2+y^2+z^2+3x-4z+1=0$$

$$(x^2+3x)\ y^2+z^2-4z+1=0$$

$$\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)+\left((y-0)^2+z^2-4z+\left(\frac{-4}{2}\right)^2\right)$$

$$=-1+\left(\frac{3}{2}\right)^2+\left(\frac{-4}{2}\right)^2$$

$$\left(x+\frac{3}{2}\right)^2+(y+0)+(z-2)^2=\frac{21}{4}$$

So

$$(x_0, y_0, z_0)\ \text{centre} = \left(\frac{-3}{2}\right), 0, 2)$$

$$\text{Radius} = r_0 = \sqrt{\frac{21}{4}}$$

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Q No. 2 (b)

Solution:-

Given That

$y = \sqrt{x}$

$0 \le x \le 4 \Rightarrow a \le x \le b$

As we know That

$V = \int_0^4 \pi y^2 dx$

$V = \int_0^4 \pi (\sqrt{x})^2 dx$

$V = \int_0^4 \pi x . dx$

$V = \pi \int_0^4 x dx$

$V = \pi \frac{x^2}{2} \Big|_0^4$

$V = \frac{\pi}{2} ((4)^2 - 0)$

$V = \frac{\pi}{\cancel{2}} (\cancel{16})^8$

$V = \frac{8\pi}{\cancel{1}} = \boxed{V = 8\pi}$

Ans.

Q NO. 3

If $A = 2i - 4j + \sqrt{5}k$ & $B = -2i + 4j - \sqrt{5}k$ then illustrate the vector $\text{project}_A B$.

Solution:- We have projection formula:

$$\text{Proj}_A B = \frac{B \cdot A}{||A||^2} A \rightarrow (1)$$

$$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (+2i - 4j + \sqrt{5}k)$$

$$B \cdot A = (-4(i \cdot i) - 16(j \cdot j) - 5(k \cdot k))$$

We know that

$$B \cdot A = (-4(1) - 16(1) - 5(1))$$

$$= -4 - 16 - 5$$

$$\boxed{B \cdot A = -25} \rightarrow (a)$$

$$||A||^2 = \left(\sqrt{(2i)^2 + (-4j)^2 + (\sqrt{5}k)^2}\right)^2$$

$$= \left(\sqrt{4 + 16 + 5}\right)^{\cancel{2}}$$

$$\boxed{||A||^2 = 25} \rightarrow (b)$$

$$A = 2i - 4j + \sqrt{5}k \rightarrow (c)$$

Put (a), (b) & (c) in eq. (1)

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We got

$$\text{Project}_A B = \frac{-25}{25}(2i - 4j + \sqrt{5}k)$$

$$\text{Project}_A B = -1(2i - 4j + \sqrt{5}k)$$

$$\Rightarrow \text{Proj}_A B = -2i + 4j - \sqrt{5}k$$

This is required Answer.

Q NO. 4

Find The area of the region between The graph and x-axis.

where

$$y = -x^2 + 5x - 4 \quad [0,2]$$

Solution:- Given That

$$y = -x^2 + 5x - 4 \quad [0,2]$$

Required :- Area = A = ?

So

As $a = 0, \; b = 2$

$$A = \int_a^b f(x)\, dx.$$

$$A = \int_0^2 f(-x^2 + 5x - 4)\, dx$$

$$A = \left(-\frac{x^3}{3} + \frac{5}{2}x^2 - 4x \right)\Big|_0^2$$

$$A = -\frac{1}{3}(2)^3 + \frac{5}{2}(2)^2 - 4(2) - 0$$

$$A = \left(-\tfrac{1}{3}(8) + \tfrac{5}{2}(4) - 8 \right)$$

$$A = \frac{-8}{3} + \frac{\cancel{20}^{10}}{\cancel{2}} - 8$$

$$A = -8/3 + 10 - 8$$

$$A = -8/3 + 2$$

$$A = \frac{-8+6}{3}$$

$$A = -2/3$$

$$\boxed{A = -0.6}$$

As area is never in negative, so we take the value of area positive.

Result : $\boxed{A = +0.6}$ Ans.

Q NO. 5(a)

Estimate the Angle b/w

$$A = i - 2j - 2k$$
$$B = 6i + 3j + 2k$$

Solution:-

$$A = i - 2j - 2k$$

$$|A| = \sqrt{(1)^2 + (-2)^2 + (-2)^2}$$

$$|A| = \sqrt{1 + 4 + 4}$$

$$|A| = \sqrt{9}$$

$$\boxed{|A| = 3}$$

$$B = 6i + 3j + 2k$$

$$|B| = \sqrt{(6)^2 + (3)^2 + (2)^2}$$

$$|B| = \sqrt{36 + 9 + 4}$$

$$|B| = \sqrt{49}$$

$$\boxed{|B| = 7}$$

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$$\theta = \cos^{-1}\left(\frac{A.B}{|A||B|}\right)$$

$$\theta = \cos^{-1}\left(\frac{(i-2j-2k)(6i+3j+2k)}{3 \times 7}\right)$$

$$\theta = \cos^{-1}\left(\frac{(1)(6)+(-2)(3)+(-2)(2)}{21}\right)$$

$$\theta = \cos^{-1}\left(\frac{6-6-4}{21}\right)$$

$$\theta = \cos^{-1}\left(\frac{-4}{21}\right)$$

$$\theta = 100.98^{\circ}$$

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Q5 (b)

Change into a spherical co-ordinate equation for the $x^2 + y^2 + (z^2 - 1) = 1$

Solution:- Given That

$$x^2 + y^2 + (z^2 - 1) = 1$$

$$(\rho \sin\phi \cos\theta)^2 + (\rho \sin\phi \sin\theta)^2 + (\rho\cos\phi - 1)^2 = 1$$

$$\rho^2 \sin^2\phi \cos^2\theta + \rho^2 \sin^2\phi \sin^2\theta + \rho^2\cos^2\phi + 1 - 2\rho\cos\phi = 1$$

$$\rho^2 \sin^2\phi (\cos^2\theta + \sin^2\theta) + \rho^2 \cos^2\theta + 1 - 2\rho\cos\phi = 1$$

$$\rho^2 (\sin^2\phi) + \rho^2 \cos^2\phi - 2\rho \cos\phi = 1 - 1$$

$$\rho^2 (\sin^2\phi + \cos^2\phi) - 2\rho\cos\phi = 0$$

$$\rho^2 = 2\rho\cos\phi$$

$$\boxed{\rho = 2\cos\phi}$$ Ans.