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Sec : B

Calculus

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QNo i

$$g(t) = \begin{cases} 0 & t<0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3<t \le 4 \\ 12 & t>4 \end{cases}$$

a) find point of discontinuity

b) find $\lim_{t \to 3} g$

SOLUTION:

a) At

$t=0$

$t=3$

$t=4$

Now at $t=0$

$$g(0) = 0$$

$$\lim_{t \to 0} g(t) = \lim_{t \to 0} 0 = 0$$

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and

$$\lim_{t \to 0} g(t) = \lim_{t \to 0} t^2 = 0$$

$$\lim_{t \to 0} g(t) = (0)^2 = 0$$

g is continuous at $t = 0$

for $t = 3$

$$\lim_{t \to 3} = t^2$$

$$= (3)^2$$

$$= 9$$

$$\lim_{t \to 3} g(t) = \lim_{t \to 3} (2t + 3) = 2(3) + 3 = 9$$

g is continuous at 3

for $t=4$

$$g(4) = 2(4)+3 = 11$$

and

$$\lim_{t \to 4} g(t) = \lim_{t \to 4} 12 = 12$$

So $\lim_{t \to 4} g(t)$ does not exist

It is discontinuous at $t=4$

b) find $\lim_{t \to 3}$

As we solve in part 1st

$g(t)$ exist

for $t=3$

$$\lim_{t \to 3} = t^2$$

$$= (3)^2$$

$$= 9$$

So

$$\lim_{t \to 3} g(t) = 9$$

$$\lim_{t \to 3} g(t) = 9$$

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QNo 2

Find the Maclaurin's series for

$$Y(x) = x^2 + \sin x$$

SOLUTION:

$$y(x) = x^2 + \sin x$$

$$y(0) = (0)^2 + \sin(0) = 0$$

$$y'(x) = 2x + \cos x \quad , \quad y'(0) = 2(0) + \cos(0) = 0+1 = 1$$

$$y''(x) = 2 - \sin x \quad , \quad y''(0) = 2 - \sin(0) = 2 - 0 = 2$$

$$y'''(x) = -\cos x \quad , \quad y'''(0) = -\cos(0) = -1$$

$$y''''(x) = \sin x \quad , \quad y''''(0) = \sin(0) = 1$$

Maclaurin Series:

$$y(x) = y(0) + y'(0)x + \frac{y''(0)x^2}{2!} + \frac{y'''(0)x^3}{3!} + \frac{y''''(0)x^4}{4!} \dots\dots$$

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$$y(x) = 0 + 1x + \frac{2(x^2)}{2!} + \frac{(-1)(x^3)}{3!} + \frac{(1)(x^4)}{4!} + - - - - -$$

Q NO 3(i)

Find y''

$$1+xy = x^2+y^2$$

SOLUTION:

$$1+xy = x^2+y^2$$

Diff w.r.t x

$$\emptyset + x\cancel{\frac{dy}{dx}}$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}(x^2) + \frac{d}{dx}(y^2)$$

$$0+\left\{y(1) + x\frac{dy}{dx}\right\} = 2x + 2y\frac{dy}{dx}$$

$$0 + y + x\frac{dy}{dx} = 2x + 2y\cdot\frac{dy}{dx}$$

$$2x - y = (x-2y)\frac{dy}{dx}$$

$$\frac{2x - y}{x - 2y} = \frac{dy}{dx}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{2x - y}{x - 2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{(x - 2y)(2 - dy/dx) - (2x - y)\left(1 - 2\frac{dy}{dx}\right)}{(x - 2y)^2}$$

$$= y'' = \frac{2x - xy' - 4y - 2yy' - 2x' + 4xy' + y - 2yy'}{(x - 2y)^2}$$

$$y'' = \frac{3xy' - 3y}{(x - 2y)^2}$$

Q NO 3 (ii)

$$Y = x^3 (1+x)^9 e^{6x}$$

SOLUTION:

$$Y = x^3 (1+x)^9 e^{6x}$$

Taking log on both sides

$$\ln y = \ln (x^3 (1+x)^9 e^{6x})$$

$$\ln y = \ln x^3 + \ln (1+x)^9 + \ln (e)^{6x}$$

$$\ln y = 3 \ln x + 9 \ln (1+x) + 6x$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(3 \ln x) + \frac{d}{dx}(9 \ln (1+x)) +$$

$$\frac{d}{dx}(6x)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+x}(1) + 6$$

$$\frac{dy}{dx} = y\left(6 + \frac{3}{x} + \frac{9}{1+x}\right)$$

$$\frac{dy}{dx} = x^3 (1+x)^9 e^{6x} \left(6 + 3/x + \frac{9}{1+x}\right)$$