Final Term Paper

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Subj Multi Variate calculas

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9th Semister.

Q6 $f(x,y,f) - x^2 + y^2 - 14$ and point $(1,-2,3)$

Sol:- $f = x^2 + y^2 + z^2 - 14$

$\vec{n} = \nabla f(1,-2,3) = (f_x . f_y . f_z)$

$f_x = 2x$ and $f_z = 2(1) = 2$

$f_y = 2y$. $f_y(2-(2)) = -4$

$f_z - 2z$ $f_z = 2(3) = 6$

Required equation of tangent prime is.

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$2(x-1) + (-4)(y-(-2)) + 6(z-3) = 0$

$2(x-1) - 4(y+2) + 6(z-3) = 0$

$2x - 2 - 4y - 8 + 6z - 18 = 0$

$2x - 4y + 6z - 28 = 0$

$2x - 4y + 6z = 28$.

(P-9 (2))

Q3) $f(x, y) = e^x \sin y + e^y \cos x$

Solution:-

Laplace Equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 \;(a)$

$f = e^x \sin y + e^y \cos x$

$\frac{\partial f}{\partial x} = e^x \sin y + e^y(-\sin x)$

$= e^x \sin y - e^y \sin x$

$\frac{\partial^2 f}{\partial x^2} = e^x \sin - e^y \cos x$

$\frac{\partial f}{\partial y} = e \cos y + e^y \cos x$

$\frac{\partial^2 f}{\partial y^2} = e^x(-\sin y) + e^y \cos x$

$= e^x \sin y + e^y \cos x$

Now putting values in (a)

$(e^x \sin y - e^y \cos x) + (-e^x \sin y + e^y \cos x) = 0$

$e^x \cancel{\sin y} - e^y \cancel{\cos x} - e^x \cancel{\sin} y + e^y \cancel{\cos} x = 0$

Hence, Satisfied Laplace's equation

Q3: $f(x,y) = x^3 e^{-y} + y \sec u$

Ans

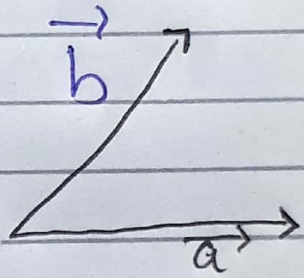
$$\frac{\partial f}{\partial x} = 3xe^{2-y} + y \sec u \tan x$$

$$\frac{\partial f}{\partial y} = x^3(-1)e^{-y} + \sec u$$

$$= -x^3 e^{-y} + \sec x.$$

Q4: Find the vector projection of $b = 6i + 3j + 2k$ onto $a = i - 2j - 2k$.

Sol:- $\text{Proj}_{\vec{a}} \vec{b} = \left(\frac{\vec{b} \cdot \vec{a}}{\| a \|^2} \right) \vec{a}$



$$= \frac{(6\hat{i} + 3\hat{j} + 2x) \cdot (\hat{i} - 2\hat{j} - 2k}{\left(\sqrt{1 + (-2)^2 + (-2)^2}\right)^2} \cdot (\hat{i} - 2\hat{j} - 2\hat{k})$$

$$= \frac{\cancel{6} - \cancel{6} - 4}{(1 + 4 + 4)} \cdot (\hat{i} - 2\hat{j} - 2\hat{k})$$

$$= \frac{-4}{9} (\hat{i} - 2\hat{j} - 2\hat{k})$$

(P.g (4))

Find the directional derivative of $f(xy) = xe^y + \cos(xy)$ at the point $(2,0)$ in the direction of $a = 3\hat{i} - 4\hat{j}$.

$f(x,y) = xe^y + \cos(xy)$ at point $(2,0)$

$\vec{a} = 3_2 - 4_j$

The partial derivation of f at point $(2,0)$ i,

$\frac{\partial f}{\partial x}(x,y) = e^y + (-\sin(4y)\cdot y)$

$= ey = y\sin(xy)$

$\frac{\partial f(2,0)}{\partial x} = e^0 - 0\sin(2.0)$

$= 1$

$\frac{\partial f}{\partial f} = (4,y) = xe^y + (-\sin(xy)\cdot y)$

$= xe^y - S\,x\sin y$

$\frac{\partial f}{\partial y}(2,0) = 2e^0 - 2\sin(2.0)$

$= 2 - 2.0$

$= 2$

Therefore gradient is

$$\nabla f(2,0) = 1\hat{i} + 2\hat{j} = (1,2) \quad (i)$$

The directional derivative at (2,0) in direction of

$$D_u f(2,0) = \nabla f(2,0)\, a \quad -(ii)$$

$$a = \frac{3\hat{i} - 4\hat{j}}{\sqrt{3^2 + (-4)^2}} = \frac{3i - 4j}{\sqrt{9+16}} = \frac{3i - 4j}{\sqrt{25}}$$

$$= \frac{3i - 4j}{55}$$

Putting values in (ii)

$$D_u f(2,0) = (\hat{i} + 2\hat{j})\left(\frac{3i - 4j}{5}\right) = \frac{3-8}{5}$$

$$= \frac{-5}{5} = -1.$$

—x—x—

Q1) find $\frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y \partial x}$ for

$z = \arcsin\left(\frac{x}{y}\right)$

Sol *

$$\frac{\partial}{\partial x}\left(\frac{\partial x}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{-x}{y^2\sqrt{y^2-x^2}}\right)$$

$$\Rightarrow \frac{-1}{y}\frac{\partial}{\partial x}\left[\frac{+x}{\sqrt{y^2-x^2}}\right.$$

$$\Rightarrow \frac{-1}{y}\left[\frac{1 \cdot (\sqrt{y^2-x^2}) - x + \frac{1}{2}(y^2-x^2)^{-1/2} \cdot -2x}{(y^2-x^2)}\right]$$

$$\Rightarrow -\frac{1}{y}\frac{\left(\sqrt{y^2-x^2} + \frac{x^2}{\sqrt{y^2-x^2}}\right]}{y^2-x^2}$$

$$\frac{1}{y}\left[\frac{y^2-x^2+x^2}{(y^2-x^2)(y^2-x^2)^{1/2}}\right]$$

$=$

$= \cancel{\frac{1}{y}}$

$= \frac{-y}{(y^2-x^2)^{3/2}}$

$\frac{\partial^2 z}{\partial y \partial x}$

$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\sin^{-1}\frac{x}{y}\right)\right)$

$= \frac{\partial y}{\partial y}\left(\frac{1}{\sqrt{y^2-x^2}}\right)$

$= \frac{0 - \sqrt{y^2-x^2} - 1 \cdot \frac{1}{\cancel{2}}(y^2-x^2)^{-1/2} \cdot \partial y}{(\sqrt{y^2-x^2})^{\cancel{2}}}$

$\frac{-y}{(y^2-x^2)(y^2-x^2)^{1/2}}$

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$$\frac{-y}{(y^2-x^2)^{3/2}}$$

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~~P.g (8)~~ P.g (9)

Q7: Evluante the double Integent?

Sol:- $= \int_0^1 \int_0^1 xy + y^2 \, dx \, dy$

$= \int_0^1 \left[\frac{x^2 y}{2} \Big|_0^1 + y^2 x \Big|_0^1 \right] dy$

$= \int_0^1 \left(\left(\frac{1^2 \cdot y}{2} - \cancel{\frac{0^2 \cdot y}{2}} \right) + \left[y^2 (1) - y^2 (0) \right] \right) dy$

$= \int_0^1 \frac{y^2}{2} + y^2 \, dy$

$= \frac{y^3}{2.3} \Big|_0^1 + \frac{y^3}{3} \Big|_0^1$

$= \frac{y^3}{6} \Big|_0^1 + \frac{y^3}{3} \Big|_0^1$

$= \left[\frac{1^3}{6} - \frac{0^3}{6} \right] + \left[\frac{1}{3} - \frac{0}{3} \right]$

$= \frac{1}{6} + \frac{1}{3} - \frac{1+2}{6} = \frac{3}{6} = \frac{1}{2}$