IQRA NATIONAL UNIVERSITY
Peshawar

SALMAN FIDA

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Registration#7168

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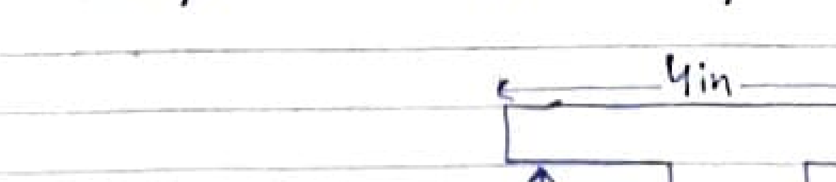
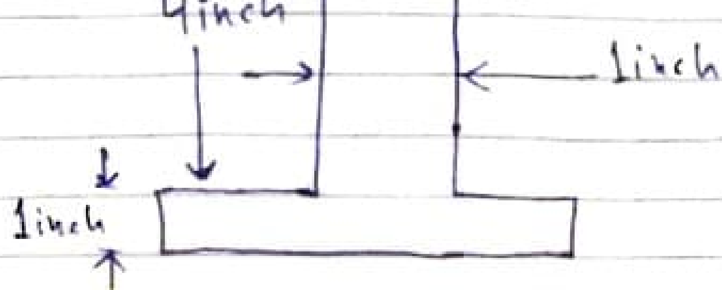
Q: Construct the Mohr's Circle diagram and find the principle stress and maximum in plane shear stress for the stress state of a point C located at the center of uniformly distributed load and 1 inches below the top fiber of beam cross section shown in figure. However to contruct the Mohr's Circle it is necessary to draw the shear stress and flexural stress variation diagram for maximum shear force and bending moment respectively. Compare the results obtained from the Mohr's circle with stress transformation equations.

Hint: To calculate the stress in the beam cross section the moment of inertia must be known.

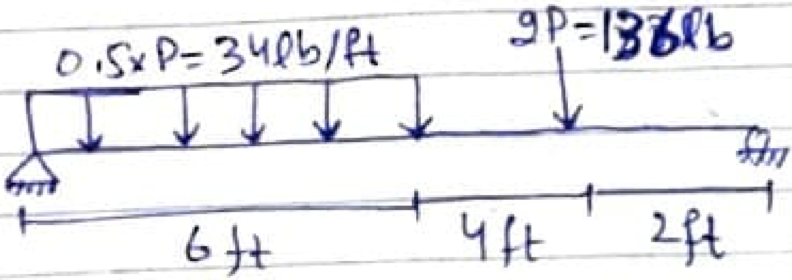
Where P is the last two digits of your class registration number in pounds.

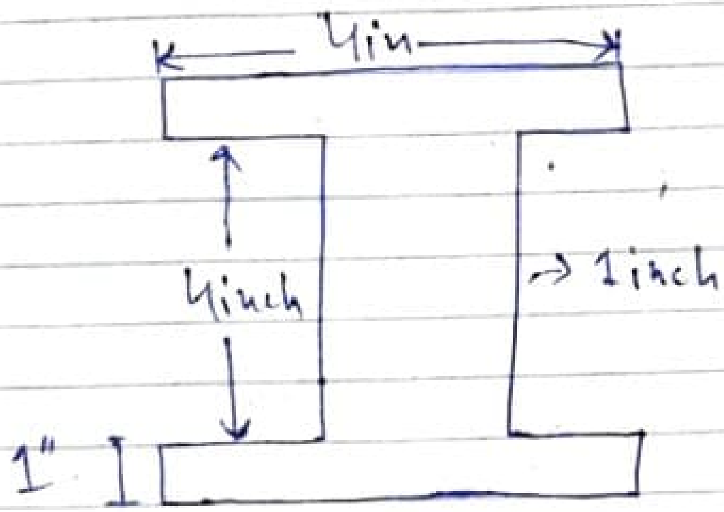




Registration # 7168 So, load $P = 68\,lb$

$$2P = 2 \times 68 = 136\,lb$$
$$0.5P = 0.5 \times 68 = 34\,lb/ft^2$$





Solution:-

First we have to find Reaction forces R_A & R_B

$$R_A + R_B = 34 \times 6 + 136$$
$$\boxed{R_A + R_B = 340\,lb} \quad \text{--- } (1)$$

Now we will take moment about "A"

$$\circlearrowleft + \sum M_A = 0$$

$$-(34 \times 6 \times 3) - (136 \times 10) + 12 R_B = 0$$

$R_B \times 12 - 1972 = 0$

$R_B = \frac{1972}{12}$

$\boxed{R_B = 164.33\ lb}$

Put this in Eq. (1)

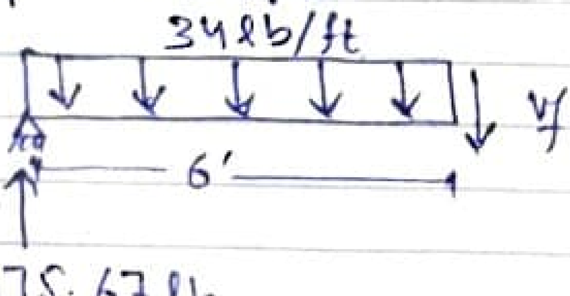
$R_A + R_B = 340\ lb$

$R_A = 340 - R_B$

$R_A = 340 - 164.33$

$\boxed{R_A = 175.67\ lb}$

Now we have to take Shear force at change point of beam.



Shear force at $A = 175.67\ lb$

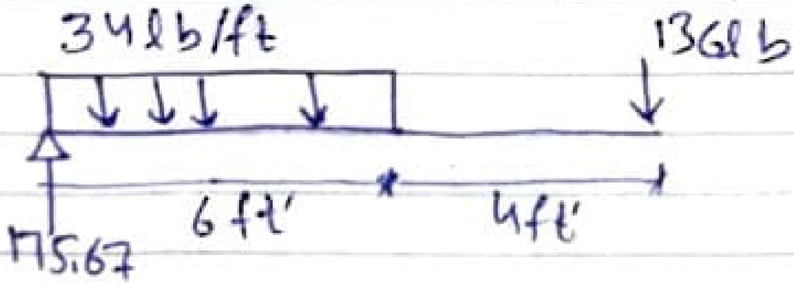
$\sum f_y = 0 \uparrow_+$

$175.67 - (34 \times 6) - V_{f6'} = 0$

$175.67 - 204 = V_{f6}$

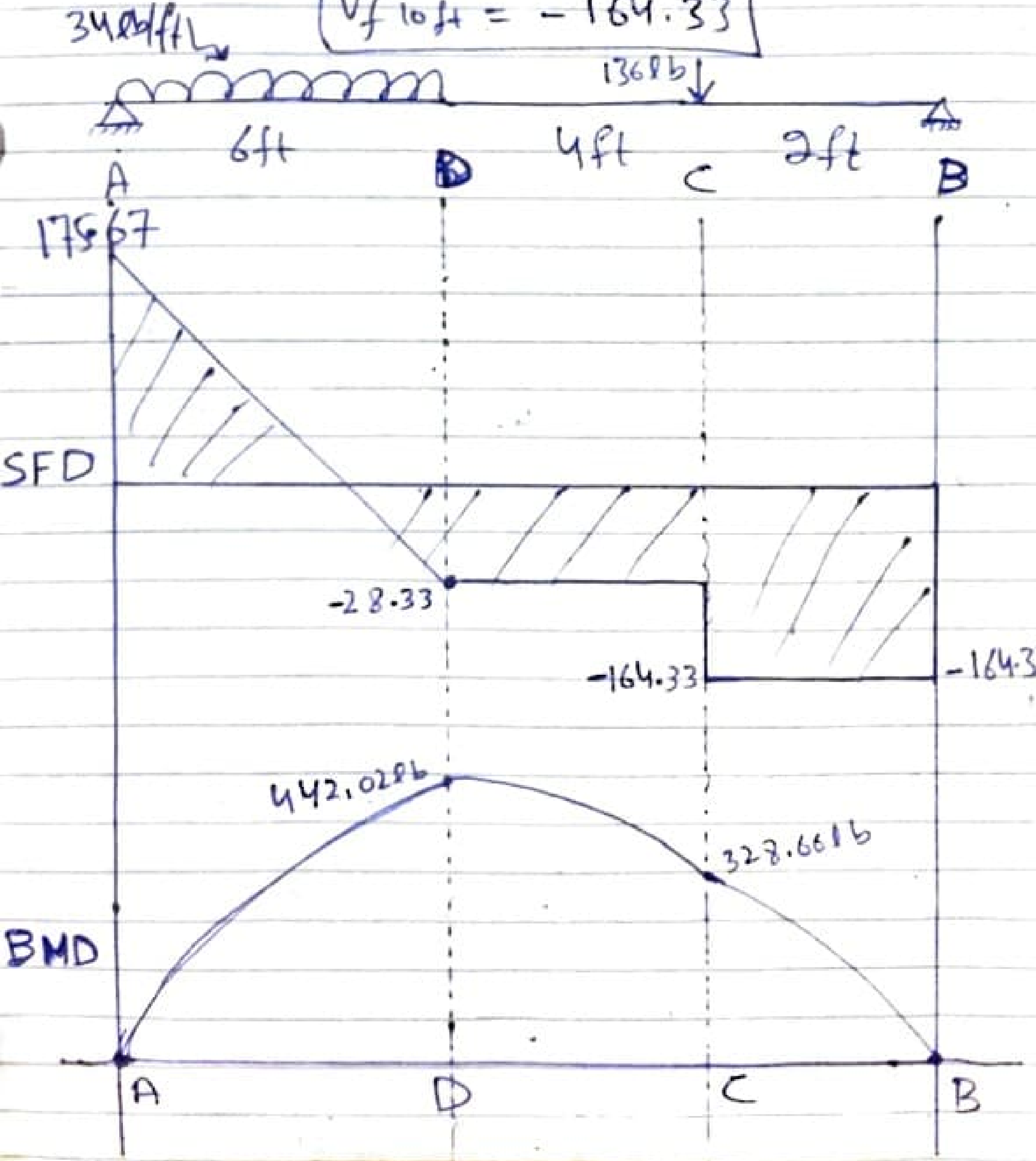
$\boxed{V_{f6ft} = -28.33\ lb}$

Now shear at 10ft means left from point load.



$\Sigma F_y = 0 \; +\uparrow \qquad 175.67 - (34 \times 6) - 136 - V_{f\,10'} = 0$

$$\boxed{V_{f\,10ft} = -164.33}$$



Shear force at A = 175.67 lb
Shear force at D = 175.67 - 204 = -28.33 lb
Shear force just left to C = -28.33 lb
Shear force just right to C = -28.33 - 136 = -164.33 lb
Shear force just left to B = -164.33 lb

Bending Moment at A = 0
Bending moment at D = 175.67 × 6 = 442.02 lb
Bending moment at C = 164.33 × 2 = 328.66 lb
Bending moment at B = 0

Shear stress:- As per the Question the maximum shear stress $\tau = \frac{VQ}{It}$ occurs where the maximum shear force lies in above diagram
max - shear force is ~~[illegible]~~ 73.67 lb

$+\uparrow \Sigma F_y \Rightarrow 175.67 - 34 \times 3 - V = 0$

$$\boxed{V = 73.67 \text{ lb}}$$

* Find moment of inertia:-

$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$ from given Questions

$I_{xx_1} = \frac{1}{12}(4)(1)^3 + 4(2.5)^2 = 25.33$

$I_{xx_2} = \frac{1}{12}(4)^3(1) + 4(0)^2 = 5.33$

$I_{xx_3} = \frac{1}{12}(1)^3(4) + (3-5.5)^2 4 = 25.33$

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$I_{xx} = 25.33 + 5.33 + 25.33$$

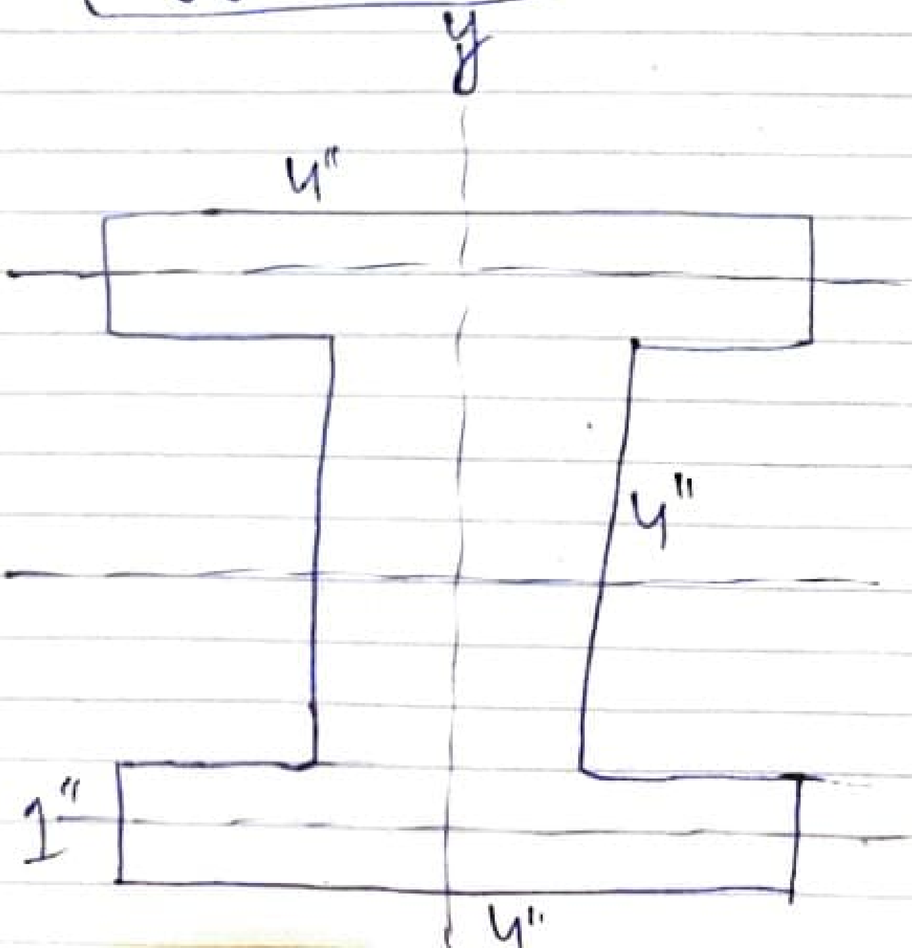
$$\boxed{I_{xx} = 56\,in^4}$$

$$I_{yy} = I_{yy_1} + I_{yy_2} + I_{yy_3}$$

$$= \frac{bh^3}{12} + \frac{bh^3}{12} + \frac{bh^3}{12}$$

$$= \frac{4^3 \times 1}{12} + \frac{1^3 \times 4}{12} + \frac{1 \times 4^3}{12}$$

$$\boxed{I_{yy} = 11\,in^4}$$



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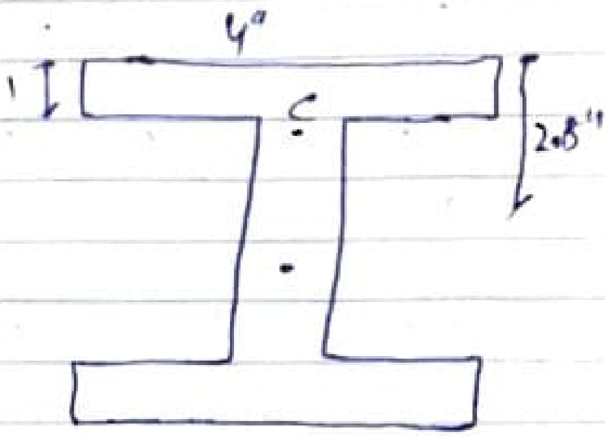
* Finding Shear stress at point "c" (1" below from top fibre)



$\tau_{xy} = \tau_{yx} = \frac{VQ}{IB}$

$Q = Ay$

$A = 1 \times 4$

$\boxed{A = 4 in^2}$

$Q = 1 \times 4 \times 2.5$

$\boxed{Q = 10 in}$

$\tau_{xy} = \tau_{yx} = \frac{47.66 \times 10}{56 \times 4}$

$\boxed{\tau_{xy} = 2.12 \ lb/in^2}$

$\sigma_x = \frac{My}{I}$

$= \frac{12 \times 242.61 \times 2}{56}$

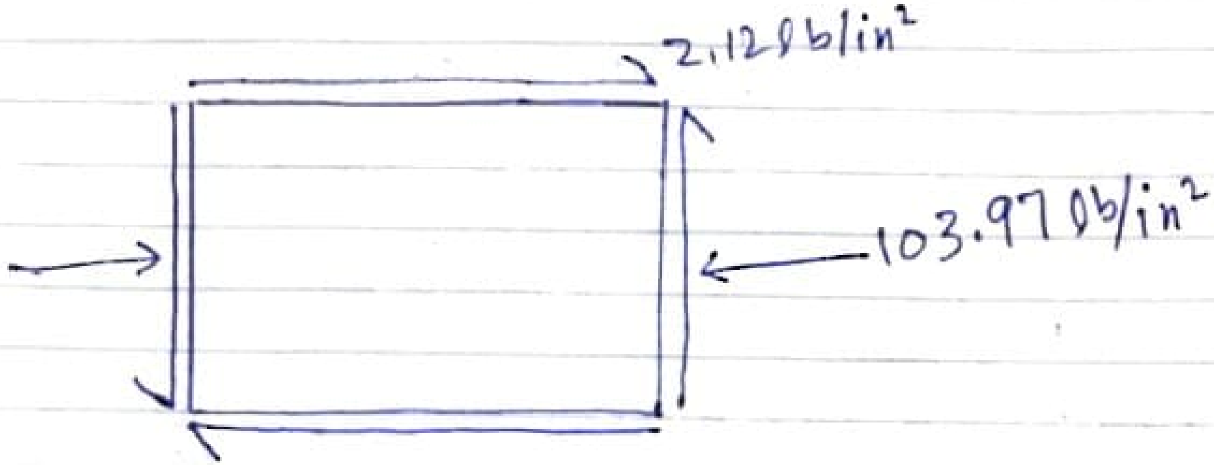
$\boxed{\sigma_x = 103.97 \ lb/in^2}$

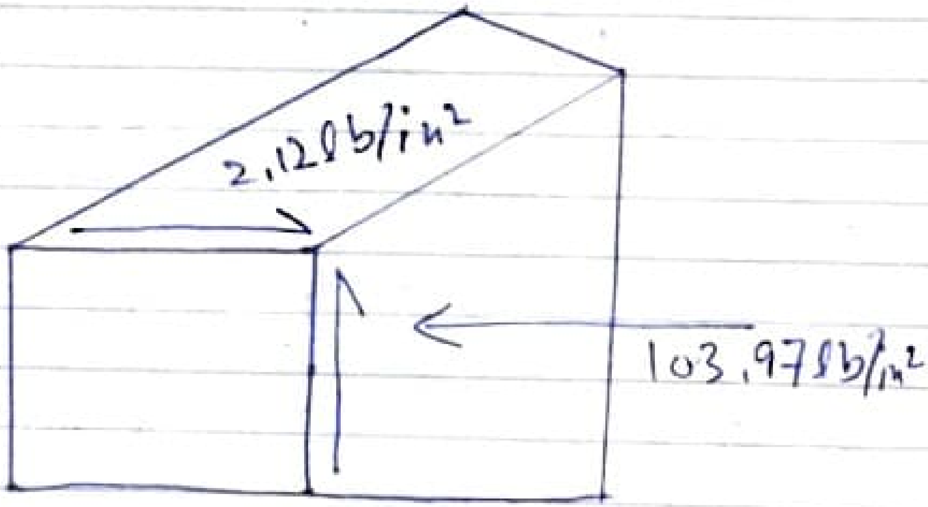
flurure stress at point c

$\sigma = 103.97 \ lb/in^2$

Stress at point C

$\tau = 2.12\ lb/in^2$





*Assume that element rotates at 30° rotation

* As we drive following equation for stress transformation

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{(\sigma_x + \sigma_y)}{2}(\cos 2\phi) + \tau_{xy}\sin 2\phi$$

$$\sigma_x' = \frac{-103.97+0}{2} + \left(\frac{-103.97+0}{2}\right)(\cos 2\times 30^\circ) + 2.12\sin 60^\circ$$

$$\boxed{\sigma_x' = \cancel{37.46\ lb/in^2}} \quad \boxed{-192.44\ lb/in^2}$$

for $\sigma y'$

$$\sigma y' = \frac{\sigma x + \sigma y}{2} - \frac{(\sigma x - \sigma y)}{2}(\cos 2\theta) - \tau xy \sin\theta$$

$$\sigma y' = \frac{-103.96+0}{2} - \left(\frac{-103.97-0}{2}\right)(\cos 60^\circ) - 2.12 \sin 60^\circ$$

$$\boxed{\sigma y' = -121.43 \ lb/in^2}$$

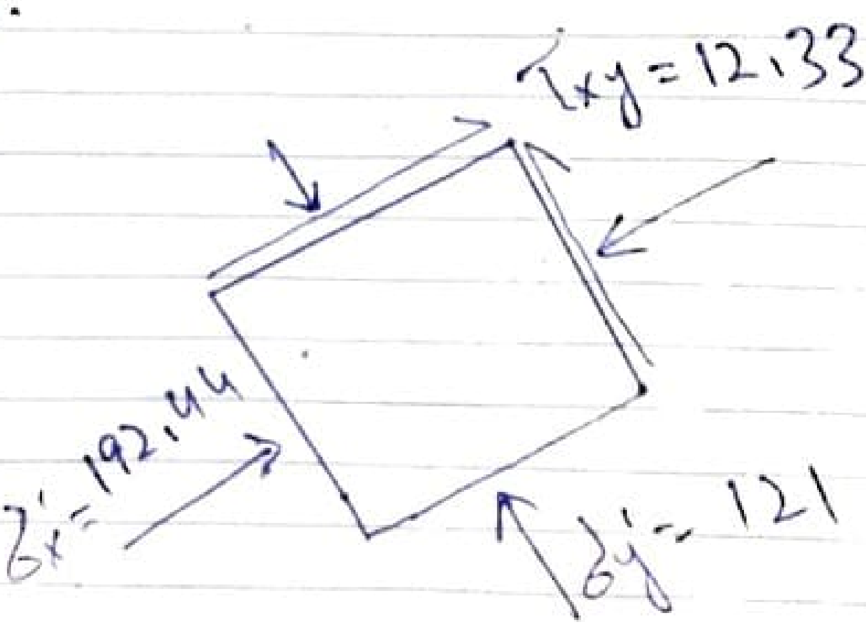
for,

$$\tau xy' = \frac{-\sigma x - \sigma y}{2} \sin 2\theta + \tau xy \cos 2\theta$$

$$\tau x'y' = \frac{-103.6-0}{2} \sin 60 + 2.12 \cos 60$$

$$\boxed{\tau x'y' = 12.33 \ lb/in^2}$$

Now stress state after 30° clockwise orientation is shown.



* Find Principle Stress:-
We know that the principle stress eq. as

$$\sigma_{1,2} = \frac{\sigma x + \sigma y}{2} \pm \sqrt{\frac{(\sigma x - \sigma y)}{2} + \tau x^2 y}$$

$$\sigma_{1,2} = \frac{-103.97+0}{2} \pm \sqrt{\frac{(-103.97-0)^2}{2} + (2.21)^2}$$

$$\sigma_{1,2} = -51.98 \pm 73.55$$

$$\boxed{\sigma y = \sigma_1 = 21.72 \ lb/in^2}$$

$$\sigma x = \sigma_2 = -51.98 - 73.55$$

$$\boxed{\sigma x = \sigma_2 = -125.53 \ lb/in^2}$$

OR

first find $\theta p = ?$

$$\tan 2\theta p = \frac{\tau xy}{(\sigma x - \sigma y/2)} = \frac{2.21}{(-103.97-0)}$$

$= -7.0$ clockwise

Put in general equation

$$\sigma_{x'max} = \frac{-103.97+0}{2} + \frac{(-103.97-0)}{2}\cos(2)(-2.70)$$
$$+ 2.21 \sin 2(-2.70)$$

$$\boxed{\sigma_{x'max} = 88.08}$$

* Max in plane shear Stress in this case

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tan 2\theta_s = 23.21$$

$$\theta = 175.11 \text{ Anti-clockwise}$$

Now Put this in the general eq. for $\tau_{x'y'}$

$$\tau_{x'y'} = -\frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{x'y'} = -\left(\frac{-103.97-0}{2}\right)\sin 2(175.11) + 2.21\cos 2(175)$$

$$\tau_{x'y'} = -6.57 \text{ lb in}$$

[Max in plane shear stress]

Mohr's Circle :-

Center Coordinate

$$(h, h) = \left[\frac{-103.97 + 0}{2}, 0\right]$$

$$= [-51.98, 0]$$

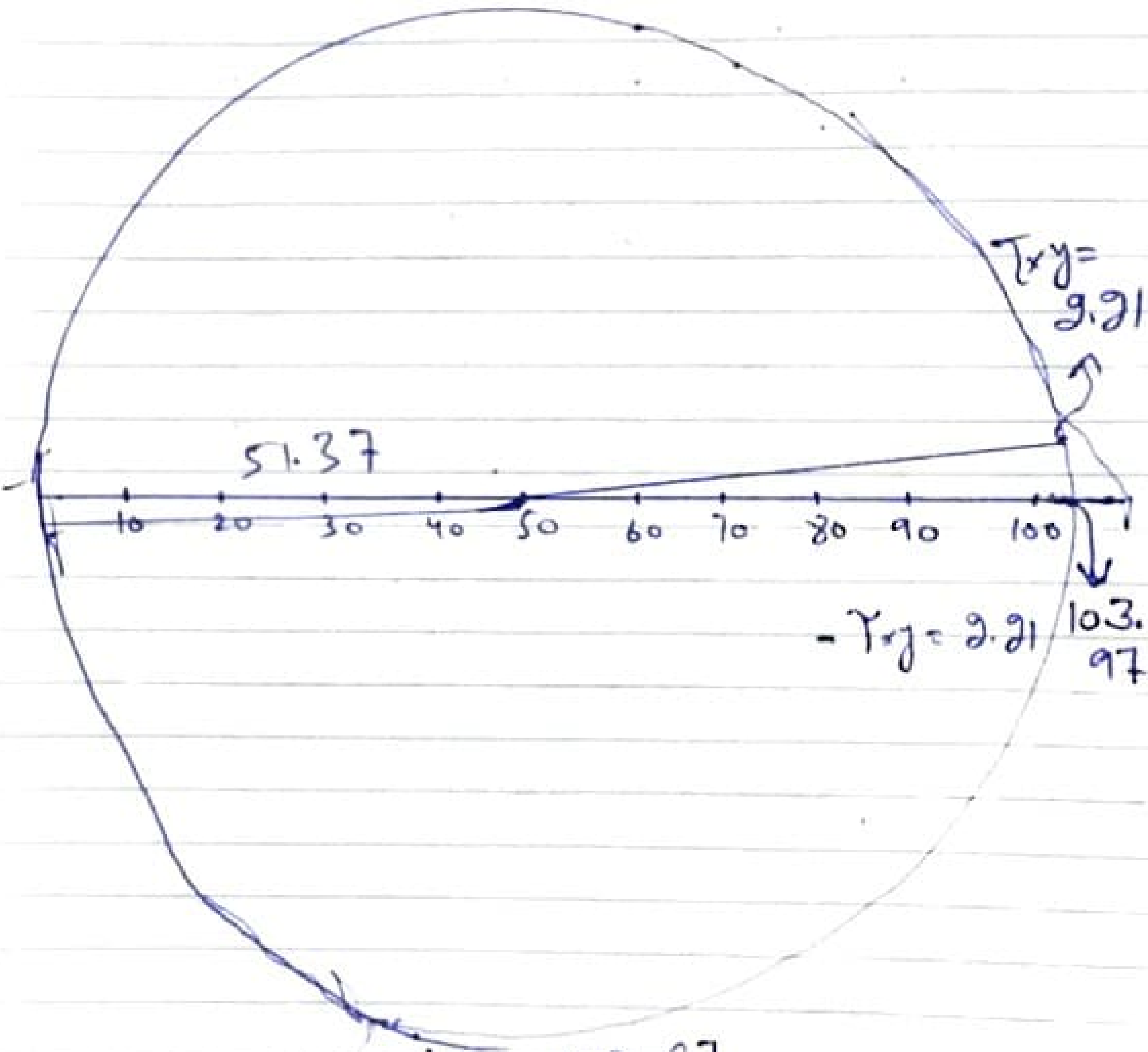
Radius of Mohr's circle is

$$r = \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau_{xy}^2}$$

$$r = \sqrt{\left(\frac{-103.97 - 0}{2}\right)^2 + (2.21)^2}$$

$$\boxed{r = 52.03 \text{ in}}$$

Mohr's Circle



$\sigma_{x_1} = -103.97$

$\sigma_{x_2} = 0$

* As shown from Mohr's Circle:
The value obtain that of principle stress and maximum shear stress are almost same with value obtained from transformation equation.