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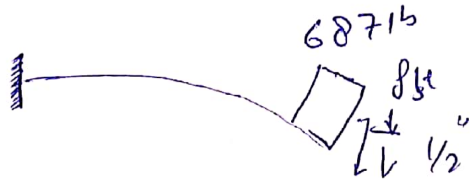
Subject # INTRO- Structure & - Earthquake engg.

Dep # BE Civil

Section # "C"

Semester # 08th

Problem; 01



Given data

$E = 29000\,ksi$

$I = 150\,in^4$

$\delta_{st} = 6871b$

Solution

The general E.O.M for "SDOF" System is

$Ku + C\dot{u} + m\ddot{u} = P(t)$

In our Case System is undamped $(c=0)$ undergoing free vibration $(P(t)=0)$ Hence general "E.O.M" become

$Ku + m\ddot{u} = 0 \longrightarrow$ "A"

Now

$$K = \frac{3EI}{l^3} = \frac{3 \times 29000\, K/in^2 \times 150\, in^4}{(16 \times 12\, in)^3}$$

$$K = 7.56\, K/in \times 26.455$$

or $K = 199.7321\, K/ft$

Now

$$m = \frac{6871\, lb.sec^2}{32.2\, ft} = \cancel{234}\, 213.41\ slug$$

As $$\omega_n = \sqrt{K/m} = \sqrt{\frac{199.735}{213.412}}$$

$$\omega_n = 0.964\ rad/sec$$

As $$T_n = 2\pi/\omega_n = \frac{2(3.14)}{0.967}$$

$$T_n = 6.494\ sec$$

Now by putting values in eq "A"

$$199.735u + 213.41\ddot{u} = 0$$

where

$k = lb/ft \quad m = lb\,sec^2/ft$

- General Solution to the "E.O.M" for undamped free vibration is

$$u(t) = u(0) \cdot \cos(\omega_n \cdot t) + \frac{\dot{u}(0)}{\omega_n} \cdot \sin(\omega_n t)$$

- But

$$u(0) = 1/2'' = 1/2 \times 1/12 = 1/24 \text{ ft}$$

and $\dot{u}(0) = 0$

$$-u(t) = (1/24) \cdot \cos(0.967) + 0$$

$$= (1/24)(\cos(0.967))$$

- equivalent Static force at any time "t" is

$$f_{(s)}(t) = k \cdot u(t) = \frac{199.735 \cdot \cos(0.967)}{24}$$

$$f_{(s)}(t) = 8.322 \cos(0.967)$$

- Amplitude of dynamic displacement u_o for undamped free vibration is

$$u_o = \sqrt{\left[(u(0))^2 + \left(\dot{u}(0)/\omega_n\right)^2\right]}$$

$$u_o \sqrt{\left(1/24\right)^2 + 0}$$

$$u_o = 1/24\ ft$$

- Amplitude of equivalent Static force:

$$ku_o = 199.735 \times 1/24$$

$$\boxed{ku_o = 8.321}$$

Q No.2

Given data

distance = 1/2"

E = 29000 ksi

δ = 7863 lb?

I = 107?

Damping ratio. ζ = 4%

Required

(i) Develope and Solve the equation of motion for vibration at free end.

(ii) also develop equation showing vibration in the equivalent of static force with time?

(iii) draw graph variation of displacment with Time and the variation of equalent static force with time =?

Soloution

For part (1) and (2)

Now we know that from equation of motion for damped vibration

$Ku + c\dot{u} + m\ddot{u} = 0$ — (i)

we know that

$K = 3EI/L^3 = \frac{3 \times 29000 \times 150}{(12)^3}$

$= 7.55$ K in

$= 90625$ lb/ft

Now mass

$M = F/g = \frac{7683}{32.2} = 238.64$ slug

Now we find "c"

we know that

$c = \zeta \times 2 m \omega_n$ — (ii)

$4/100 \times 2 \times 238.64 \times \omega_n$

First we find ω_n

first we find wn

$$w_n = \sqrt{K/m} = \sqrt{\frac{96025}{238.64}}$$

$= 20.1$ rad/sce

Now put in eq (11)

$$c = 4/100 \times 2 \times 238.64 \times 20.1$$

$$= 383.74 \text{ lb.sce/ft}$$

Now put value in eq (1)

$$Ku + cu' + mu'' = 0$$

$$96025u + 383.74u' + 238.64u'' = 0$$

Now Solution to the E.O.M for damped free vibration

$$u(T) = e^{-\xi w_n T}\left[u(0)\cos(w_{DT}) + \frac{1}{w_D}\left(u'(0) + u(0)\xi w_n \times \sin w_{DT}\right)\right]$$

$w_D = 20.1$ rad/sce

$$\omega(t) = e^{-0.04 \times 20.1t}\left[\frac{1}{24} \times \cos(20.15) + \frac{1}{20.1}\left[0 + \frac{1}{24} \times 0.04 \times 20.1 \times \sin(20.10)\right]\right]$$

$$e^{-0.804}\left[0.042\cos(20.1t) + 0.0017\sin(20.240)\right]$$

$$\boxed{e^{0.804}\left[0.042\cos(20.1t) + 0.0017\sin 20.1t\right]}$$

Now we know that

$$f(s) = k\,y(t)$$

$$K = 90625$$

$$f(s) = 90625\left(e^{-0.804} \times 0.042\cos(20.1t) + 0.0017\sin 20.1t\right)$$

$= e^{-0.804}$)

$= e^{-0.804}(90625 \times 0.042 (\cos 20.1t)$

$+ 90625 \times 0.0017 \sin 20.1)$

$= e^{-804} \left[3806.25 \cos(20.1t) + 154.0625 (\sin 20.1t) \right]$

$e^{-804} (3806.25 \cos(20.1t) + 154.0.625 \sin(20.1t))$

"problem 03"

Given data:

Force = 60 kips

$u_1 = 6871/1000 = 6.862$ in

After, $j = 7$ (cycles)

completed = 3.57 sec

$u_{J+1} = 2.286$ cm $= 0.9$ in

Note : Ignore verticle vibration

Required:→

(a) Damping ratios

(b) Natural period of undamped vibration.

(c) Stiffness and Structure

(d) Weight of tank

(E) Damping Co-efficient

(f) Number of cycle to reduce Displacment amplitude to 0.5"

Soloution:

"a" ζ = Damping ratio = ?

As;

$$\delta = \frac{1}{2\pi\zeta}$$

$$7 = \frac{1}{2(3.14)\zeta} \ln\left[\frac{6.871}{0.9}\right]$$

$$\zeta\,(7 \times 2 \times 3.14) = 2.032$$

$$\zeta\,(43.96) = 2.032$$

$$\zeta = 2.032/43.96$$

$$= 0.0462$$

$$\zeta = 4.62\%$$

(b) $T_n = ?$

As Seven cycles are completed in "3.57" Sec

Thus required to complete one cycle $= 7/3.57 = 1.96$ sec

$$\boxed{T_D = 1.96 \text{ sec}}$$

Now

$$\omega_D = \omega_n \sqrt{1-\zeta^2}$$

$$\Rightarrow \frac{2\pi}{\omega_D} = \frac{2\pi}{\omega_n \sqrt{1-\zeta^2}}$$

As $T_D = \frac{T_n}{\sqrt{1-\zeta^2}}$

$$\Rightarrow T_n = T_D\left(\sqrt{1-\zeta^2}\right)$$

~~$= 1.96 (\sqrt{(0.0462)^2}$~~

$$= 1.96 \left(\sqrt{1-(0.0462)^2}\right)$$

$$\boxed{T_n = 1.957 \text{ sec}}$$ "Natural period undamped vibration"

"c" Stiffness of Structure "K = ?"

As $K = \frac{F \cdot \cos\theta}{2}$

$K = \frac{60 \cdot \cos(60^\circ)}{2}$ | $F = 60kip$

$= 15\,K/in$ | $\theta = 60$

$\boxed{K = 18000\ lb/ft}$

"d" weight of tank $W = ?$

$w_n = \sqrt{K/m} = \sqrt{K/(w/g)} = \sqrt{K \cdot g/w}$

$\Rightarrow w_n^2 = K \cdot g/w \Rightarrow (W = K \cdot g/w_n^2)$

By putting values of $w_n = 2\pi/T_n$

$W = K \cdot g/(4\pi^2/T_n^2) = Kg\left(\frac{T_n^2}{4\pi^2}\right)$

$W = \frac{18000\,lb}{ft} \cdot \frac{32.2\ ft}{sec^2} \left(\frac{(1.957)^2}{4(3.14)^2}\right)$

$W = 56284.75\,lb.$

$\approx \boxed{56.281\ kip}$

"e" Damping co-efficient "c = ?"

it is know that $\zeta = c/2m\omega_n$

$\Rightarrow c = \zeta(2m\omega_n) = \zeta(2m(2\pi/T_n))$

By putting values

$$c = \frac{0.0462\,(2(\frac{56284}{32.2}))(2(3.14))}{1.957}$$

$$\boxed{c = 518.2861\,lb/sec}$$

"f" No. of cycles displacement altitude from 6.871 in to 0.5 in

j = ?

$$j = \frac{1}{2\pi\zeta}\ln\left(\frac{y_1}{y_{j+1}}\right)$$

$$= \frac{1}{2(3.14)(0.0462)}\ln\left[\frac{6.871}{0.9}\right]$$

$= 7.001$ or

$$\boxed{J = 7 \text{ cycles}}$$