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Section: A
Semester: 2
(S.E)

Q1 ans] $x_1 - 7x_2 + x_3 = 0$
$2x_2 - 8x_3 = 8$
$5x_1 - 5x_3 = 10$

Solution

$$\begin{bmatrix} 1 & -7 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 5 & -5 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -8 \\ 0 & 5 & -10 \end{bmatrix} \quad R_3 + (-5 \cdot R_1)$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -8 \\ 0 & 1 & -2 \end{bmatrix} \quad R_3/5$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -4 \\ 0 & 1 & -2 \end{bmatrix} \frac{R_2}{2}$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 1 & -4 \\ 0 & 0 & +2 \end{vmatrix}$$

Rank of $A = 3$

Now AB

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 2 & 8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 2 & 8 & 8 \\ 0 & 5 & -b & 10 \end{array}\right] R_3 - 5R_1$$

rank of aB is 3

its consistant.

Q2]

Solution

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 3 \\ 5 & -2 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} Adj(A) \quad [|A| \neq 0]$$

$$|A| = 3(-7+6) - 4(14-18) + 5(-4+5)$$
$$= -3 + 4 + 5$$
$$|A| = 6$$

$$adj\,A = \begin{bmatrix} -1 & 1 & 1 \\ -38 & -4 & -26 \\ 17 & -2 & -11 \end{bmatrix}$$

$$A^{-1} = \frac{adj\,A}{A|}$$

$$= \frac{\begin{bmatrix} -1 & 1 & 1 \\ -38 & -4 & -26 \\ 17 & -2 & -11 \end{bmatrix}}{6}$$

$$A^{-1} = \begin{bmatrix} -1/6 & 1/6 & 1/6 \\ \frac{-38}{6} & \frac{-4}{6} & \frac{-26}{6} \\ \frac{17}{6} & \frac{-2}{6} & \frac{-11}{6} \end{bmatrix}$$

Q3) $2x + 2y + 4z = 18$
$x + 3y + 2z = 13$
$3x + 2y - 3z = 14$

Solution

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 2 & 2 & 4 & 18 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 0 & -4 & 0 & -8 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 0 & -4 & 0 & -8 \\ 0 & -7 & -9 & -25 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 0 & -4 & 0 & -8 \\ 0 & -7 & -9 & -25 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 0 & -4 & 0 & -8 \\ 0 & 0 & -9 & -11 \end{array}\right]$$

$z = 11/9$

$y = -2$

$x = 41/9$

Q4) $\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

First find eigen ~~values~~ vectors

a $\lambda = 5$ $\begin{bmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix} = \begin{bmatrix} -1 & 2 & -2 \\ -5 & -2 & 2 \\ -2 & 4 & -4 \end{bmatrix}$

preform row operation

$\begin{bmatrix} -1 & 2 & -2 \\ -5 & -2 & 2 \\ -2 & 4 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

now solve matrix equation $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} \begin{matrix} 0 \\ =0 \\ 0 \end{matrix}$

if we take $v_3 = t$, then $v_1 = 0$, $v_2 = t$ $v_3 = t$

$v = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

eigen vector 5 $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

eigen vector 2, $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{bmatrix}$

eigen vector 1 = $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{4} \\ 1 \end{bmatrix}$

from matrix P, whose i-th column is the i-th egnvector:

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{4} \\ 1 & 1 & 1 \end{bmatrix}$$

form diagonal matrix D, whose element at row i, column i is the i-th eigenvalue:

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

these matrices have property that $A = PDP^{-1}$

$V_1 \quad 0$

$V_2 = 0$

$V_3 \quad 0$

$V_3 = 1$

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{4} \\ 1 & 1 & 1 \end{bmatrix}$$

$$\text{Decimal } P = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.25 \\ 1 & 1 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\overset{1}{3}6$
$\underline{\times 3}$
108

$\overset{2}{2}7$
$\underline{\times 4}$
108

5

Q25) $3x_1 + 5x_2 - 4x_3 = 0$
$-3x_1 - 5x_2 + 4x_3 = 0$
$6x_1 + 1x_2 - 8x_3 = 0$

Solution

$$|A| = \begin{vmatrix} 3 & 5 & -4 \\ -3 & -5 & 4 \\ 6 & 1 & -8 \end{vmatrix}$$

$$= 3(40-4) - 5(+24-24) - 4(-3+30)$$

$$= 3(36) - 5(0) - 4(27)$$
$$= 108 - 0 - 108$$
$$= 0$$

So thereby this has trivial solution

Q6) $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$

Solution

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -3 \end{bmatrix} \begin{matrix} R_2 - 3R_1 \\ R_3 - R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} -2R_3 + R_2$$

$$\begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} C_2 - 3C_1$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} C_3 - 4C_1 \\ C_3 - 3C_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} -\tfrac{1}{6} C_4$$

$$\boxed{\text{Rank} = 2}$$