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Question 1

Augment matrix System equation

$$= \begin{bmatrix} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ -4 & 6 & -1 & 7 \end{bmatrix}$$

Sol:

$$\begin{bmatrix} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ -4 & 6 & -1 & 7 \end{bmatrix}$$

$R_2 - 3R_1$

$R_3 + 4R_1$

$$= \left|\begin{array}{ccc:c} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & -6 & 15 & -9 \end{array}\right|$$

$-2R_1 + R_3$

$R_3 - (2R_1)$

$$= \left|\begin{array}{ccc:c} -2 & 0 & 7 & -1 \\ 0 & 2 & -5 & 4 \\ 0 & 0 & 7 & -1 \end{array}\right|$$

$R_1 - R_3$

$$= \left|\begin{array}{ccc:c} 2 & 0 & 0 & 0 \\ 0 & 2 & -5 & 4 \\ 0 & 0 & 7 & -1 \end{array}\right|$$

$7R_2 + 5R_3$

$$= \left|\begin{array}{ccc:c} 2 & 0 & 0 & 0 \\ 0 & 14 & 0 & 23 \\ 0 & 0 & 7 & -1 \end{array}\right|$$

$\frac{1}{2} R_1$

$\frac{1}{14} R_2$

$\frac{1}{7} R_3$

$$= \left|\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 23/14 \\ 0 & 0 & 1 & -1/7 \end{array}\right|$$

$x = 0$

$y = 23/14$

$z = -1/7$ Answer

Question 2

Find inverse of Matrix

$$: \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{bmatrix}$$

let:

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$2(0-2) + 1(0-2) + 0$$

$$= -4 - 2$$

$$= -6$$

$$= \begin{vmatrix} +\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} \\ +\begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} \end{vmatrix}$$

$$|A| = \begin{vmatrix} -2 & 2 & -1 \\ 0 & 0 & -1 \\ -2 & -4 & 2 \end{vmatrix}$$

$$A^{-1} = \frac{1}{2} \begin{vmatrix} -2 & 2 & -1 \\ 0 & 0 & -1 \\ -2 & -4 & 2 \end{vmatrix}$$

$$A^{-1} = \begin{vmatrix} \frac{-2}{2} & 2/2 & -1/2 \\ 0 & 0 & -1/2 \\ -2/2 & -4/2 & 2/2 \end{vmatrix}$$

$$A^{-1} = \begin{vmatrix} -1 & 1 & -1/2 \\ 0 & 0 & -1/2 \\ -1 & -2 & 1 \end{vmatrix}$$

Answer

Question 3

Find Eigen value and Eigen vector

$$A: \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Sol:

$$A = \det(A - \lambda I)$$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 8-\lambda & -6 & 6 \\ -6 & 7-\lambda & -4 \\ 6 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$= (8-\lambda)\begin{vmatrix} 7-\lambda & -4 \\ -4 & 3-\lambda \end{vmatrix} + 6\begin{vmatrix} -6 & -4 \\ 2 & 3-\lambda \end{vmatrix} + 2\begin{vmatrix} -6 & 7-\lambda \\ 2 & -4 \end{vmatrix} = 0$$

$$= (8-\lambda)[(7-\lambda)(3-\lambda) - 16] + 6[-6(3-\lambda) + 8] + 2[24 - 2(7-\lambda)] = 0$$

$$= (8-\lambda)[21 - 7\lambda - 3\lambda + \lambda^2 - 16] + 6[-18 + 6\lambda + 8] + 2[24 - 14 + 2\lambda] = 0$$

$$= (8-\lambda)[21 - 10\lambda + \lambda^2 - 16] + 6[6\lambda - 10] + 2[2\lambda + 10] = 0$$

$$= (8-\lambda)[\lambda^2 - 10\lambda + 5] + 6(6\lambda - 10) + 2(2\lambda + 10) = 0$$

$$= 8\lambda^2 - 80\lambda + 40 - \lambda^3 + 10\lambda^2 - 5\lambda + 36 - 60 + 4\lambda + 20 = 0$$

$$-\lambda^3 + 18\lambda^2 - 45\lambda = 0$$

$$-\lambda(\lambda^2 - 18\lambda + 45) = 0$$

$$\boxed{\lambda = 0}$$

$$\lambda^2 - 18\lambda + 45 = 0$$

$$\lambda^2 - 15\lambda - 3\lambda + 45 = 0$$

$$\lambda(\lambda - 15) - 3(\lambda - 15) = 0$$

$$(\lambda - 15)(\lambda - 3) = 0$$

$$\boxed{\lambda = 15} \qquad \boxed{\lambda = 3}$$