IQRA NATIONAL UNIVERSITY

Differential Equations

Mid Term Assignment Summer 2020

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=> Question No (1)

=> Part (A)

Estimate the genral Solution of $y' = (x+2)y^2$.

(Solution) $y' = (x+2)y^2$

$$\Rightarrow \frac{dy}{dx} = (x+2)y^2$$

Take Integration

$$\int \frac{1}{y^2}\, dy = \int (x+2)\,dx$$

$$\Rightarrow \int y^{-2}\,dy = \int (x+2)\,dx$$

$$\frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + 2x + c$$

$$\frac{y^{-1}}{-1} = \frac{x^2}{2} + 2x + c$$

Multiplying both side by -1

$$y^{-1} = -\left(\frac{x^2}{2} + 2x + c\right)$$

$$\Rightarrow \boxed{y = -\left(\frac{1}{x^2/2 + 2x + c}\right)}$$ Answer.

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⇒ Question No (1)

⇒ Part (B)

Estimate the genrall solution of $y' = (y+9x)^2$

(Solution) $y' = (y+9x)^2 \rightarrow (1)$

Let $y + 9x = u$

$$\frac{dy}{dx} + 9 = \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{du}{dx} - 9$$

So eq (1) becomes

$$\frac{du}{dx} - 9 = u^2$$

$$\frac{du}{dx} = u^2 + 9$$

$$\Rightarrow \int \frac{1}{u^2+9} du = \int dx$$

$$\int \frac{1}{(3)^2+(4)^2} du = \int dx$$

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$$\Rightarrow \boxed{\frac{1}{3}\tan^{-1}(4/3) = x + c_1}$$

$$\Rightarrow \tan^{-1}(4/3) = 3x + 3c_1$$

$$4/3 = \tan(3x + c)$$

$$u = 3\tan(3x + c)$$

$$y + 9x = 3\tan(3x + c)$$

$$\Rightarrow \boxed{y = -9x + 3\tan(x + c)}$$

Answer.

$\longleftrightarrow$

⇒ Question No (2)

⇒ Part (A)

Estimate the genrall Solution of $x^3dx+y^3dy=0$

(Solution) $x^3dx+y^3dy=0$

Let $M=x^3$, $N=y^3$

$\Rightarrow \frac{dM}{dy}=0$, $\frac{dN}{dx}=0$

Now equation will be exact

$u=\int Mdx+k(y)$

$u=\int x^3dx+ky \longrightarrow (1)$

Now, $\frac{du}{dy}=0+d/dy\,ky$

But $\frac{du}{dy}=N$

So, $N=d/dy\,ky$

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$\Rightarrow \quad \int dK(y) = \int y^3 dy$

$K(y) = y^4/4 + C_1$

So equation 1 becomes

$u = x^4/4 + y^4/4 + C_1$

$C_2 = x^4/4 + y^4/4 + C_1$

$\Rightarrow \quad \boxed{C = \frac{x^4}{4} + y^4/4}$ Answer.

$\longleftrightarrow$

=> Question No (3)

=> part (A)

Find the genrall solution $4y'' - 20y' + 25y = 0$

(Solution)

$$4y'' - 20y' + 25y = 0$$

The Auxiallary eq is

$$4\lambda^2 - 20\lambda + 25 = 0$$

$$\Rightarrow 4\lambda^2 - 10\lambda - 10\lambda + 25 = 0$$

$$2\lambda(2\lambda - 5) - 5(5\lambda - 5) = 0$$

$$\Rightarrow (2\lambda - 5)(2\lambda - 5) = 0$$

$$2\lambda - 5 = 0 \quad , \quad 2\lambda - 5 = 0$$

$$\Rightarrow 2\lambda = 5 \quad , \quad 2\lambda = 5$$

$$\Rightarrow \lambda_1 = 5/2 \quad , \quad \lambda_2 = 5/2$$

$$\text{As } \lambda_1 = \lambda_2 = 5/2$$

So roots are real and equal

$$\boxed{y = (c_1 + c_2 x)e^{\lambda x} = (c_1 + c_2 x)e^{5/2 x}}$$

Answer.

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=> Question No (3)

=> Part (B)

Estimate generall solution of $4y''-6y'-7y=0$

(Solution) $4y''-6y'-7y=0$

Auxilary eq is

$$4\lambda^2-6\lambda-7=0$$

So by quadratic formula

Here,

$$a=4, \quad b=-6, \quad c=-7$$

$$\lambda=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\lambda=\frac{-(-6)\pm\sqrt{(-6)^2-4(4)(-7)}}{2(4)}$$

$$\lambda=\frac{6\pm\sqrt{36+112}}{8}$$

$$\lambda=\frac{6\pm\sqrt{148}}{8}$$

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=> Taking 2 as Common

$$\lambda = \frac{3 \pm \sqrt{37}}{4}$$

So, $\lambda_1 = \frac{3+\sqrt{37}}{4}$

$\lambda_2 = \frac{3-\sqrt{37}}{4}$

Roots are real, So

$$y = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

$$\boxed{y = c_1 e^{\frac{3+\sqrt{37}}{4} t} + c_2 e^{\frac{3-\sqrt{37}}{4} t}}$$ Answer.

$\longleftrightarrow$

Thank You