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Summer Final Term

Structural Analysis II

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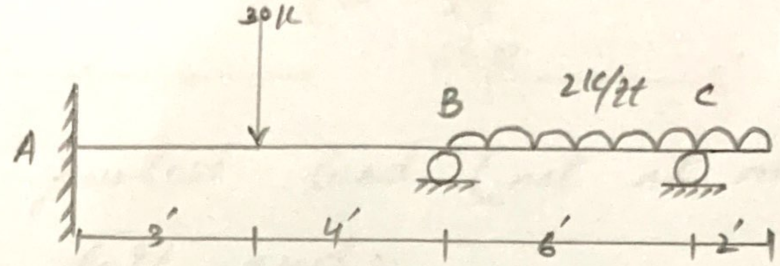
Ans 01:

Analyze the beam shown in FIG 1 by stiffness method. Assume EI is constant

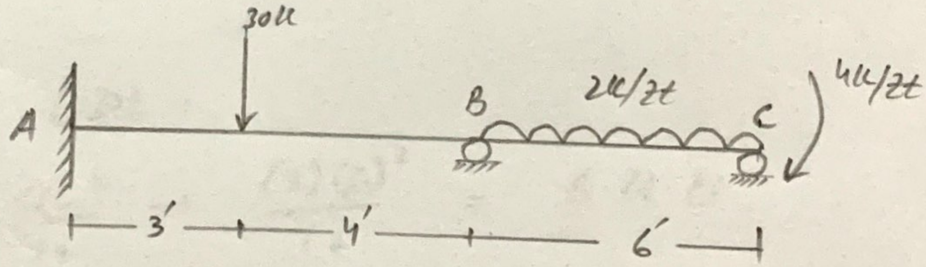


Step #1:-

Determining Kinematic Indeterminacy

$K.I = 5^{\circ}$

So we have to reduce the extended portion.

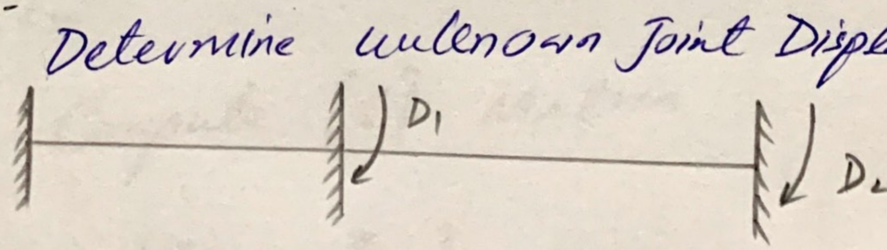


$$= \frac{2(2)}{1} = 4\ k.ft$$

Now

$K.I = 2^{\circ}$

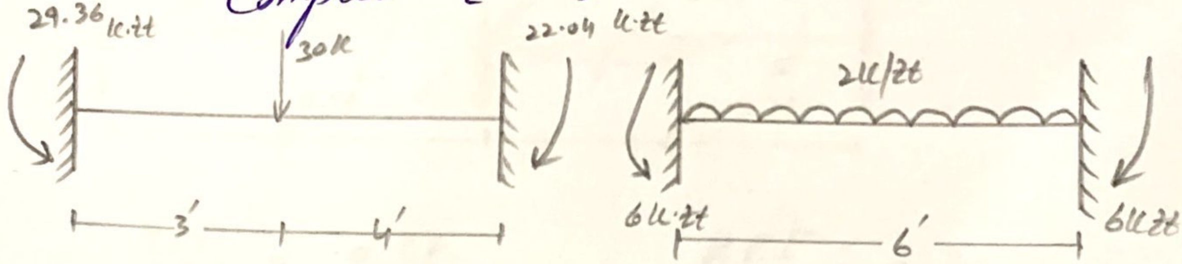
Step #2:- Determine unknown Joint Displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \quad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step # 03:-

Compuet [ADL] Matrix.



→ For pointed load (not at mid)

→ For left end:-

$$= \frac{Pab^2}{L^2} = \frac{(30)(3)(4)^2}{(7)^2} = 29.38 \text{ k.ft}$$

→ For Right end:-

$$= \frac{Pa^2b}{L^2} = \frac{(30)(3)^2(4)}{7^2} = 22.04 \text{ k.ft}$$

⇒ For UDL :

$$\frac{wL^2}{12} \rightarrow \frac{(2)(6)^2}{12} = 6 \text{ k.ft}$$

$ADL_1 = +22.04 - 6 \quad = 16.04$ k.ft

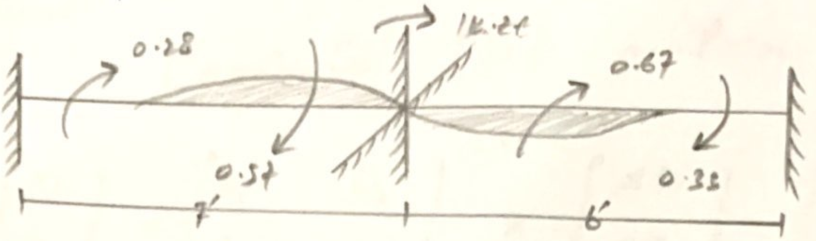
$ADL_2 = 6$ k.ft

Step # 04:-

Compute [S] Matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

(a) $D_1 = 1k$ $\qquad$ $D_2 = 0$



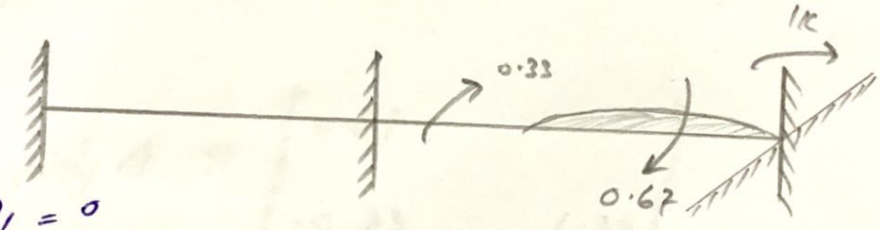
$$\frac{4EI}{7} = 0.57 \qquad \frac{2EI}{6} = 0.33$$

$$\frac{4EI}{6} = 0.67 \qquad 2EI = 0.28$$

$$S_{11} = 0.57 + 0.67$$
$$= 1.24\ EA$$

$$S_{21} = 0.33\ EA$$

(b)



$D_1 = 0$
$D_2 = 1k$

$$\frac{4EI}{6} = 0.67$$

$$\frac{2EI}{6} = 0.33$$

$S_{12} = 0.33$
$S_{22} = 0.67$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step # 5 :-

Compute [D] matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$= \frac{1}{\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}} \times \text{Adj } A$$

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$$

$$= 0.8308 - 0.1089$$

$$|S| = 0.7219$$

$$\text{Adj } A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

Now

$$\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{\begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}}{0.7219} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{\begin{bmatrix} (0.67 \times -16.04) + (-0.33 \times -2) \\ (-0.33 \times -16.04) + (1.24 \times -2) \end{bmatrix}}{0.7219}$$

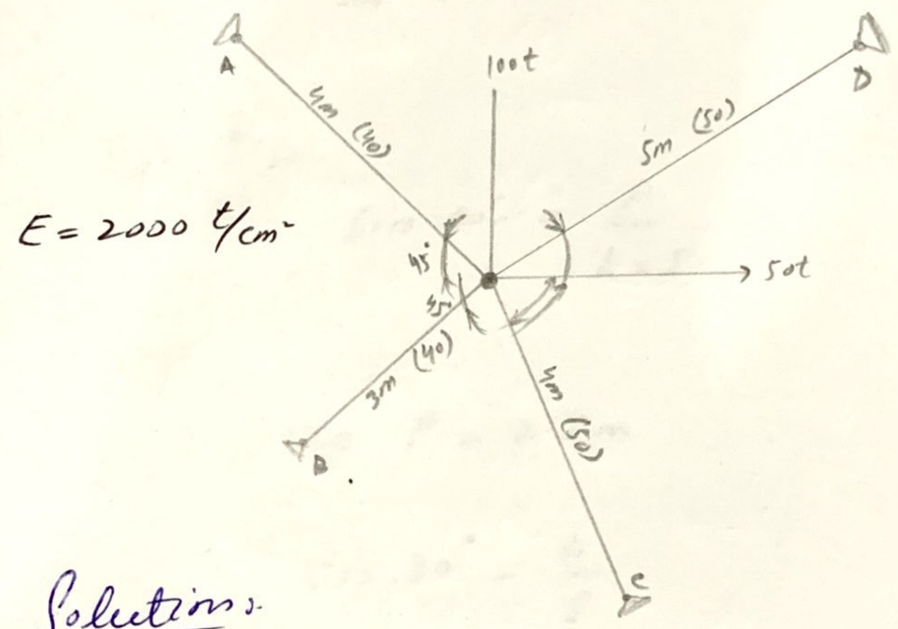
$$= \frac{\begin{bmatrix} -10.7468 + 0.66 \\ 5.2932 - 2.48 \end{bmatrix}}{0.7219}$$

$$= \frac{1}{0.7219} \begin{bmatrix} -10.0868 \\ 2.8132 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} -13.973 \\ 3.897 \end{bmatrix}$$ Answer

QNo 2:-

Analyze the pin-joined frame shown by stiffness method. Length of the members in 'm' and cross-sectional area of the members in cm^2 are shown in FIG-3 Take $E = 2000\ t/cm^2$.



Solution:

For A :-

$$\sin 45^\circ = \frac{P}{h} = \frac{P}{4}$$

$$\rightarrow \quad P = 2.828\ m$$

$$\cos 45^\circ = \frac{b}{4}$$

$$\rightarrow \quad b = 2.828\ m$$

Now

For B :-

$$\sin 45^\circ = \frac{P}{3}$$

$$\rightarrow P = 2.12\ m$$

$$\cos 45^\circ = \frac{b}{h}$$

$$\rightarrow b = 2.12\ m$$

For C :-

$$\sin 30^\circ = \frac{P}{h=5}$$

$$\rightarrow P = 2.5\ m$$

$$\cos 30^\circ = \frac{b}{5}$$

$$b = 4.33\ m$$

Now $EA_{(A)} = 2000 \times 40 = 80{,}000\ t$

$EA_{(B)} = 2000 \times 40 = 80{,}000\ t$

$EA_{(C)} = 2000 \times 50 = 100{,}000\ t$

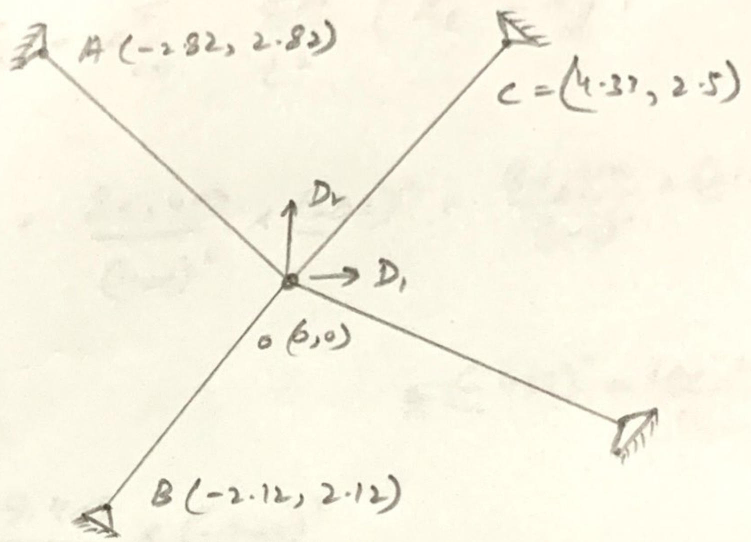
$EA_{(d)} = 2000 \times 50 = 100{,}000\ t$

Step # 01 :-

$$K.I = 2j - r$$
$$= 2(5) - 8 = 2^{\circ}$$

Step # 2 :-

select unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} , \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step # 3 :-

$$[AMD]_{4\times 2} \quad \& \quad [S]_{2\times 2}$$

(i) $D_1 = 1$, $D_2 = 0$

$$AMD = \frac{EA}{L^2} (X_k - X_j)$$

$$AMD_{11} = \frac{80,000}{(400)^2} \times (0 + 282) = 141$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0 + 212) = 188.44$$

$$AMD_{31} = \frac{100,000}{(500)^2} \times (0-433) = -173.2$$

$$AMD_{41} = \frac{100,000}{(400)^2} \times (0-200) = -125$$

Now $S_{11} = \sum_{i=1}^{m} \frac{EA}{L^2} (x_k - x_j)^2$

$$= \frac{80,000}{(400)^2} \times (282)^2 + \frac{80,000}{(300)^3} \times (212)^2 + \frac{100,000}{(500)^2} \cancel{\times (433)^2}$$

$$\times (-433)^2 + \frac{100,000}{(400)^2} \times (-200)^2$$

$$S_{11} = \frac{99.405}{(400)^2} \times (-300)^2$$

$$S_{11} = 99.405 + 133.107 + 149.991 + 62.5$$

$$\boxed{S_{11} = 445.063}$$

$$S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (x_k - x_j)(y_k - y_j)$$

$$= \frac{80,000}{(400)^3} \times (282)(-282) + \frac{80,000}{(300)^3} \times (212)(212)$$

$$+ \frac{100,000}{(500)^3} \times (-433)(0-250) + \frac{100000}{(400)^2} \times (-200)(0+346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

(ii) $D_1 = 0 \quad D_1 = 1K'$

$$AMD = \frac{EA}{L^2}(Y_K - Y_j)$$

$$AMD_{12} = \frac{80,000}{400^2}(-282) = -141$$

$$AMD_{22} = \frac{80,000}{300^2}(212) = 188.44$$

$$AMD_{32} = \frac{100,000}{500^2}(-250) = -100$$

$$AMD_{42} = \frac{100,000}{400^2}(346) = 216.25$$

Now,

$$S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3}(Y_K - Y_j)^2$$

$$= \frac{80,000}{400^3}(-282)^2 + \frac{80,000}{300^3}(212)^2 + \frac{100,000}{500^3}(-250)^2 + \frac{100,000}{400^3}(346)^2$$

$$\boxed{S_{22} = 469.628}$$

Step # 04:-

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.063 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

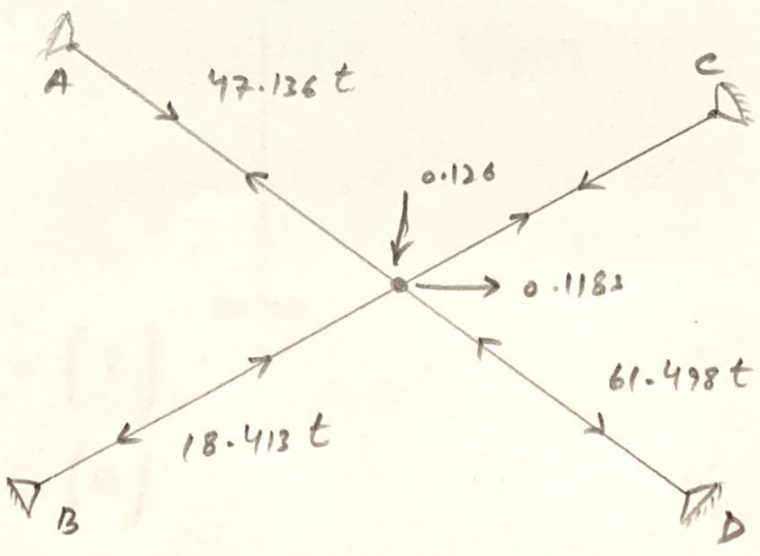
Step # 05 :- $[AM]$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times 0.216 \\ 188.44 \times 0.1183 + 188.44 \times (0.126) \\ -173 \times 0.1183 + (-100) \times (0.216) \\ -125 \times 0.1183 + 216.25 \times (0.126) \end{bmatrix}$$

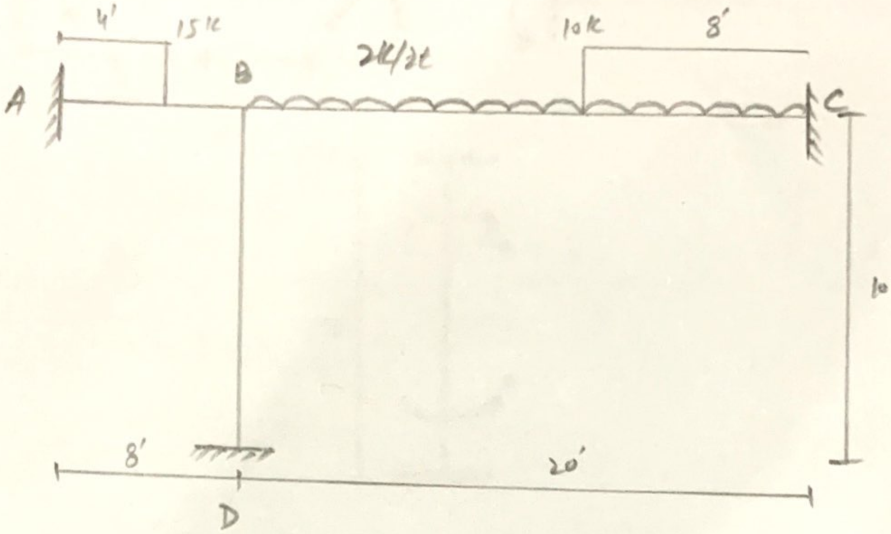
$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 - 46.71 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_1 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$



Q.No3:-

Analyze the rigid-joint frame shown in Fig-2 by stiffness method Assume EI is constant.



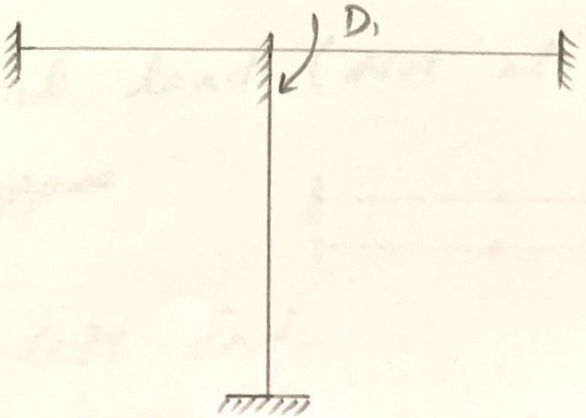
Step # 1:

Determine Kinematic Indeterminacy

$K.I = 1°$

Step # 2:

Determine Unknown Joint Displacement

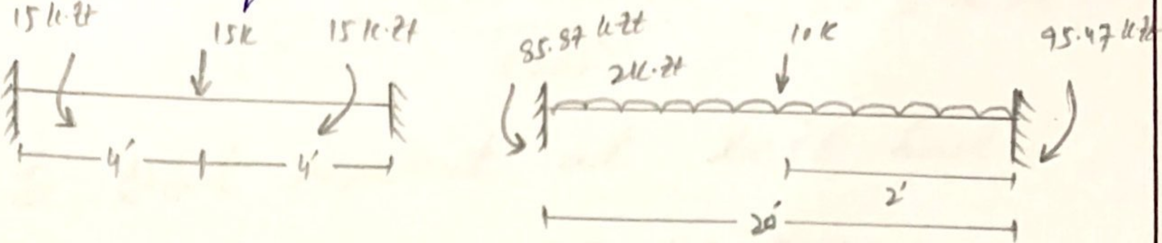


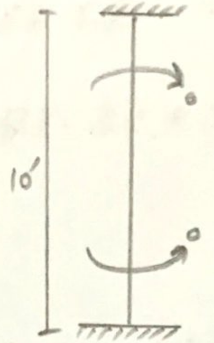
$[D] = [?]$

$[AD] = [0]$

Step # 03 :-

Comput [ADL] Matrix





→ Point load at Centre :-

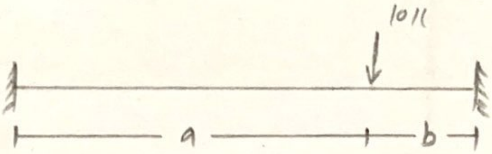
$$\frac{PL}{8} = \frac{(15)(8)}{8} = 15 \text{ kip·ft}$$

→ Uniformly Distributed load :-

$$\frac{wL^2}{12} = \frac{(2)(20)^2}{12} = 66.67 \text{ k·ft}$$

→ Point load (Not at mid) :-

Suppose



For left End :-

$$\frac{Pab^2}{L^2} = \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ k·ft}$$

For right End :-

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ k-ft}$$

So total Moment at left End :-

$28.8 + 66.67 = 95.47$ k-ft

So [AD] $-85.87 + 15 = -70.87$ k-ft

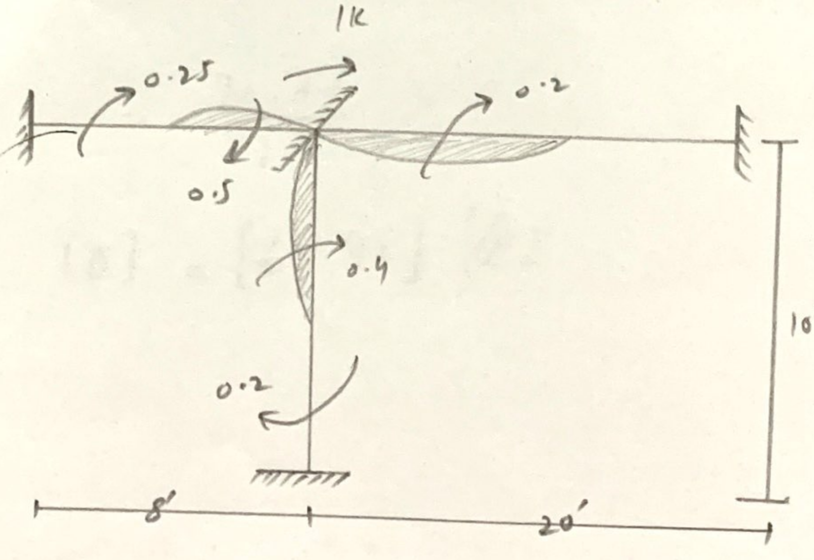
Step # 04 :-

Determine [S] Matrix

$[S] = [S_{11}]$

Now

$D = 1k$



$\rightarrow \frac{4EI}{8} = 0.5$ $\qquad \frac{2EI}{8} = 0.25$

$\rightarrow \frac{4EI}{20} = 0.2$ $\qquad 2EI = 0.1$

Scanned with CamScanner

$\rightarrow \frac{4EI}{10} = 0.4 \qquad 2EI = 0.2$

$[S] = (0.5 + 0.4 + 0.2)\,EI$

$= 1.1\,EI$

$[S] = 1.1\,EI$

Step # 5 :-

Compute [D] Matrix

$[D] = [S]^{-1} \times [AD] - [ADL]$

$[D] = \frac{1}{1.1} \times [0] - [-70.87]$

$= \frac{70.87}{1.1}$

$[D] = [64.42]\ \frac{1}{EI}$