

CALCULUS AND ANALYTIC GEOMETRY

Mid Term Assignment

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Q1(a) Differentiate $\frac{2x^3 - 3x^2 + 5}{x^2 + 1}$ with respect to x .

(a) Sol: $\frac{d}{dx} \left(\frac{2x^3 - 3x^2 + 5}{x^2 + 1} \right)$

$$\Rightarrow \frac{(x^2 + 1) \frac{d}{dx} (2x^3 - 3x^2 + 5) - (2x^3 - 3x^2 + 5) \frac{d}{dx} (x^2 + 1)}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{(x^2 + 1)(6x^2 - 6x) - (2x^3 - 3x^2 + 5)(2x)}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{(x^2 + 1)6x(x^2 - 1) - (2x^3 - 3x^2 + 5)(2x)}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{6x(x^2 + 1)(x^2 - 1) - (2x^3 - 3x^2 + 5)(2x)}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{2x [3x(x^2 + 1)(x^2 - 1) - (2x^3 - 3x^2 + 5)]}{x^4 + 2x^2 + 1}$$

Q1(b) Differentiate $\frac{(x^2+1)^2}{x^2-1}$ with respect to x .

(b) Sol:- $y = \frac{(x^2+1)^2}{x^2-1}$

$$\Rightarrow \frac{dy}{dx} = \frac{(x^2+1)^2}{x^2-1}$$

$$\Rightarrow \frac{(x^2-1) \frac{d}{dx} (x^2+1)^2 - (x^2+1)^2 \frac{d}{dx} (x^2-1)}{(x^2-1)^2}$$

$$\Rightarrow \frac{(x^2-1) 2(x^2+1) \frac{d}{dx} (x^2+1) (x^2+1)^2 (2x)}{(x^2-1)^2}$$

$$\Rightarrow \frac{(x^2-1) [2(x^2+1) 2x] - (x^2+1)^2 (2x)}{(x^2-1)^2}$$

$$\Rightarrow \frac{(x^2-1) [4x(x^2+1) - (x^2+1)^2 (2x)]}{(x^2-1)^2}$$

$$\Rightarrow \frac{x^2+1 [(x^2-1) 4x - (x^2+1) 2x]}{(x^2-1)^2}$$

$$\Rightarrow \frac{(2x)(x^2+1) [2x(x^2-1) - x^2+1]}{x^4+2x^2+1}$$

Q2 (a) Find $\frac{dy}{dx}$ if $y = (1+2\sqrt{x})^3 \cdot x^{2/3}$ using chain rule.

(a) Sol: $\frac{dy}{dx} = (1+2\sqrt{x})^3 \cdot x^{2/3}$ let $x=u$

$$\Rightarrow \frac{dy}{dx} = (1+2\sqrt{u})^3 \frac{2}{3} u^{-1/3} + u^{2/3} [3(1+2\sqrt{u})^2 \cdot \frac{1}{u}]$$

$$\Rightarrow \frac{dx}{du} = 1 \quad \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\Rightarrow (1+2\sqrt{u})^3 \frac{2}{3} u^{-1/3} + u^{2/3} [3 \frac{1}{2} (1+2\sqrt{u})^2] \times 1$$

$$\Rightarrow \frac{dy}{dx} = (1+2\sqrt{u})^2 \left[(1+2\sqrt{u}) \frac{2}{3} u^{-1/3} + 3 \frac{u^{2/3}}{u} \right]$$

$$\Rightarrow (1+2\sqrt{u})^2 \left[(1+2\sqrt{u}) \frac{2}{6\sqrt{u}} + 3u^{-1/3} \right]$$

$$\Rightarrow (1+2\sqrt{u})^2 \left[(1+2\sqrt{u}) \frac{2}{6\sqrt{u}} + \frac{3}{3\sqrt{u}} \right]$$

use $u=x$

$$\Rightarrow (1+2\sqrt{x})^2 \left[(1+2\sqrt{x}) \frac{2}{6\sqrt{x}} + \frac{3}{3\sqrt{x}} \right]$$

Q2(b) Find $\frac{dy}{dx}$ if $y = \frac{\sqrt{1-x}}{1+x}$ using chain rule:

(b) Sol: Let $\frac{1-x}{1+x} = u$

$$\Rightarrow \frac{du}{dx} = \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2}$$

$$\Rightarrow \frac{-(1+x) - (1-x)}{(1+x)^2}$$

$$\Rightarrow \frac{-1 - \cancel{x} - 1 + \cancel{x}}{(1+x)^2} \Rightarrow \frac{-2}{(1+x)^2}$$

$$\frac{dy}{du} = \sqrt{u} \Rightarrow \frac{1}{2} u^{-1/2} \Rightarrow \frac{1}{2\sqrt{u}} \text{ using chain rule}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\Rightarrow \frac{1}{2\sqrt{u}} \times \frac{-(1+x)^2}{2}$$

$$\Rightarrow \frac{-(1+x)^2}{2\sqrt{u}} \Rightarrow \frac{-(1+x)^2}{4 \frac{\sqrt{1-x}}{1+x}}$$

$$\Rightarrow \frac{-(1+x)^2 (1+x)^{1/2}}{4\sqrt{1-x}}$$

$$\Rightarrow \frac{-(1+x)^{3/2}}{4\sqrt{1-x}}$$

Q3: (a) Find the integration of $\int \frac{1}{\sqrt{x^3}} dx$.

(a) sol: $\int \frac{1}{\sqrt{x^3}} dx$

$$\Rightarrow \int \frac{1}{(x^3)^{1/2}} dx$$

$$\Rightarrow \int x^{-3/2} dx$$

use formula $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$$= \frac{x^{-3/2+1}}{-3/2+1} + C$$

$$\left\{ \frac{-3+2}{2} \right\}$$

$$= \frac{x^{-1/2}}{-1/2} + C$$

$$\int \frac{1}{\sqrt{x^3}} dx = \frac{-2}{\sqrt{x}} + C$$

Q3(b) Find the Integration of $\int \frac{1}{(6x+7)^6} dx$

(b) Sol: $\int \frac{1}{(6x+7)^6} 6 dx$

$$= \int (6x+7)^{-6} dx$$

use again $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$$= \frac{1}{6} \int (6x+7)^{-6} (6) dx \quad \left(\frac{1}{6} \times 6\right) = 1$$

$$= \frac{1}{6} \frac{(6x+7)^{-6+1}}{-6+1} + C$$

$$= \int \frac{1}{(6x+7)^6} dx = -\frac{1}{30} (6x+7)^{-5} + C$$