Differential Equation

SAAD KHAN
7300

Submitted to: Ma'am SHOMAILA MAZHAR

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Q.1) Solve the following objectives

i)

order of $A = m \times p$

order of $B = p \times n$

$\therefore$ order of $AB = m \times n$

ii) The number of non-zero rows in Echelon form is called rank of the matrix.

For e.g;

$$A = \begin{bmatrix} 1 & 0 & -2 & 5 & 3 \\ 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since it contains three (03) non-zero row

$\therefore$ Rank is 3

iii) Sol:-

We know that

For singular matrix

$|B| = 0$

$\therefore$

$$|B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} = 0$$

$\Rightarrow 1 \times a - 2 \times 4 = 0$

$\Rightarrow a - 8 = 0$

$a = 8$

iv) Sol:-

$$A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$$

taking modulus of matrix A

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$|A| = [2i \times (-i)] - [i \times i]$$

$$|A| = -2i^2 - i^2 \qquad (\because i^2 = -1)$$

$$|A| = -2(-1) - (-1)^2$$

$$= 2 + 1$$

$$|A| = 3$$

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v) Diagnols are same therefore it is scalar matrix

$$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

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vi) Solution of $\frac{dy}{dx} + 2xy = y$?

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$dy = y(1-2x) \cdot dx$$

$$\frac{dy}{y} = (1-2x)\, dx$$

applying integration

$$\int \frac{dy}{dx} = \int (1-2x)\,dx$$

$$= \int 1\,dx - 2\int x\,dx$$

$$= x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$= x - x^2 + c$$

$$= e(x-x^2)\, ec$$

$$y = ce^{x-x^2}$$

vii)

~~Orde~~ $\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$

order = 1

degree = 3

viii) $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$

order = 2

degree = 1

ix) The differential eq. $2\frac{dy}{dx} + x^2y = 2x+3, y(0) = 5$ is

This is not possible

x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Sol:- expanding with first row

$\Rightarrow 1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} - a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$

$\Rightarrow 1(bc^2 - cb^2) - a(c^2-b^2) + a^2(c-b)$

$\Rightarrow bc(c-b) - a(c-b)(c+b) + a^2(c-b)$

$\Rightarrow (c-b)\left[bc - a(c+b)+a^2\right]$

$(c-b)\left[bc-ac-ab+a^2\right]$

$$\Rightarrow (c-b)\left[c(b-a)-a(b-a)\right]$$

$$=(c-b)(b-a)(c-a)$$

Q.2

i) Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factors which are linear in a, b, c.

Sol:-

$$=\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

taking 'a' common from C_1

" 'b' " " C_2

" 'c' " " C_3

expanding by R_1

$$\Rightarrow abc\left[1\begin{vmatrix} b & c \\ b^2 & c^2 \end{vmatrix} - 1\begin{vmatrix} a & c \\ a^2 & c^2 \end{vmatrix} + 1\begin{vmatrix} a & b \\ a^2 & b^2 \end{vmatrix}\right]$$

$$= abc\left[bc^2-b^2c-ac^2+a^2c+ab^2-a^2b\right]$$

$$= abc\left[bc^2-ac^2-b^2c+a^2c+ab^2-a^2b\right]$$

$$= abc\left[c^2(b-a)-c(b^2-a^2)+ab(b-a)\right]$$

$$= abc\left[c^2(b-a)-c(b+a)(b-a)+ab(b-a)\right]$$

$$= abc\,(b-a)\left[c^2-c(ba)+ab\right]$$

$= abc(b-a)[c^2-cb-ca+ab]$

$= abc(b-a)[c(c-b)-a(c-b)]$

$= abc(b-a)(c-b)(c-a)$

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2 ii) Find Eigen value

Solution :-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

For eigen values,
we consider

$|A-2I| = 0$

$\therefore$

$$A-2I = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$R_3 - R_1$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ 0 & -4+\lambda & 4-\lambda & 0 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$\begin{array}{r} -1-3+\lambda \\ 3-\lambda+1 \\ \hline \end{array}$$

$-1-3+\lambda$	$-1-3+\lambda$
$3-\lambda+1$	$-4+\lambda$
4λ	$3-\lambda^2$

expanding by column first

$$2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -4+\lambda & 4-\lambda & 0 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & 0 \\ -4+\lambda & 4-\lambda & 0 \\ -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by C_3 Expand by C_3

$$= 2-\lambda \left\{ -1 \begin{vmatrix} -4+\lambda & 4-\lambda \\ -1 & -1 \end{vmatrix} + (2-\lambda) \begin{vmatrix} 3-\lambda & -1 \\ -4+\lambda & 4-\lambda \end{vmatrix} \right\} + 1 \left\{ (2-\lambda) \begin{vmatrix} -1 & -1 \\ -4+\lambda & 4-\lambda \end{vmatrix} \right\} = 0$$

$$\Rightarrow -(2-\lambda)(4-\lambda+4-\lambda) + (2\lambda)^2 \left[(3-\lambda)(4-\lambda) - 4 - \lambda \right] + 1 \left[(2-\lambda)(\lambda-4-4+\lambda) \right] = 0$$

$$= (\lambda-2)(8-2\lambda) + \left[(\lambda-2)^2 (12-7\lambda+\lambda^3-4+\lambda) \right] + \left[(2-\lambda)(2\lambda-8) \right] = 0$$

$$\Rightarrow (\lambda-2)(8-2\lambda) + (\lambda-2)^2(\lambda-6\lambda+8) + (\lambda-2)(8-2\lambda) = 0$$

$$= (\lambda-2)\{\cancel{16}-4\lambda+\lambda^3-6\lambda^2+8\lambda-2\lambda^2+12\lambda-\cancel{16}\} = 0$$

$\lambda - 2 = 0$; $\lambda^3 - 8\lambda^2 + 16\lambda = 0$

$\lambda = 2$; $\lambda(\lambda^2 - 8\lambda + 16) = 0$

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda-4) - 4(\lambda-4) = 0$

$(\lambda-4)(\lambda-4) = 0$

$\lambda = 4, 4$

Answer $\Rightarrow 0, 2, 4, 4$

3) $(x^2+3y^2)dy - 2xy\,dy = 0$

Find general solution at $x=2$ and $y=6$

Sol :-

$$(x^2+3y^2)dx = 2xy\,dy \quad , \; y(2)=6$$

$$2xy\,dy = (x^2+3y^2)dx$$

$$2xy\frac{dy}{dx} = x^2+3y^2$$

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy} \text{ ——— } ① \quad \text{Homogenous}$$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2x\cdot vx}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{x^2}(1+3v^2)}{2\cancel{x^2}v}$$

$$v + x\frac{dv}{dx} = \frac{1+3v}{2v}$$

$$\text{eq (A)} \Rightarrow \frac{x^2+y^2}{x^3} = 5$$

$$x^2+y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y = \sqrt{5x^3-x^2}$$