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ID NUMBER : 7930

SECTION : "A"

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SUBJECT : Mechanic's of Solid II

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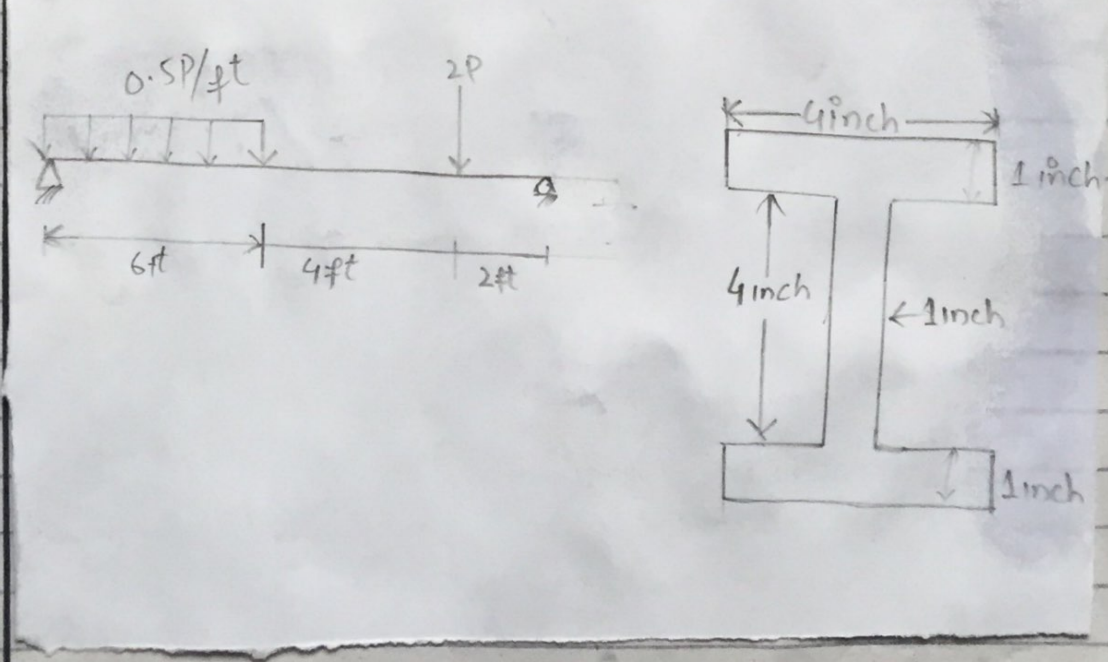
# Question

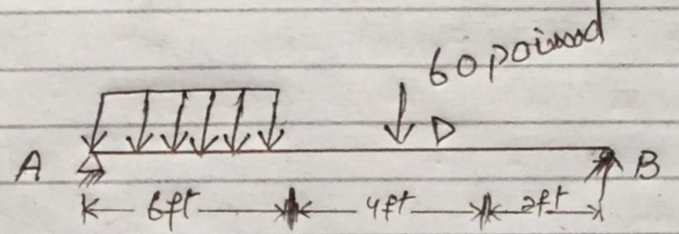
Construct the Mohr's Circle diagram and find the principle stress and maximum in the plane shear stress for the stress state of a point C located at the centre of uniformly distributed load and 1 inches below the top fiber of beam cross section shown in figure. However to construct the Mohr circle it is necessary to draw the shear stress and flexural stress variation diagram for maximum shear force and binding moment respectively. Compare the results obtained from the Mohr's Circle with the stress transformation equition.



# Solution:

In the given diagram the value of p=30. So the 2p load becomes 60 pounds and 0.5 p/ft becomes 15

The free body diagram for given beam is




Now!

We will find the Support reactions at point A and B Taking moment at A

$$\Sigma M_A = 0$$

$$(R_B \times 12) - (60 \times 10) - (15 \times 6 \times 3) = 0$$

$$12 R_B - 600 - 270 = 0$$

$$R_B = 870/12$$

$$\boxed{R_B = 72.5 \text{ lb}}$$

Now!

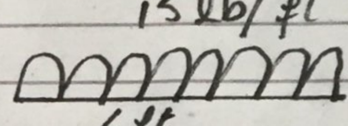
Taking $\Sigma F_y = 0$

$$R_A + R_B - 15 \times 6 - 60 = 0$$

$$R_A + 72.5 - 90 - 60 = 0 \Rightarrow \boxed{R_A = 77.5 \text{ lb}}$$

## Shear force at beam change point.

Shear force at 6ft from left support.



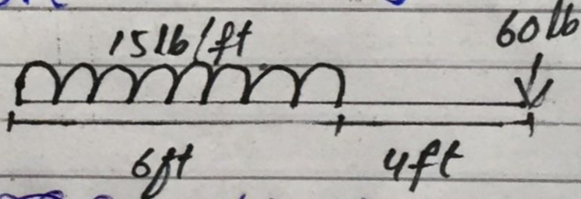

$\Sigma F_y = 0 \quad \begin{pmatrix} \uparrow +ive \\ \downarrow -ive \end{pmatrix}$

$-V_{6ft} + 77.5 - (15 \times 6) = 0$

$V_{6ft} = 77.5 - 90$

$\boxed{V_{6ft} = -12.5 \text{ lbs}}$

## Shear force at 10ft from left support.

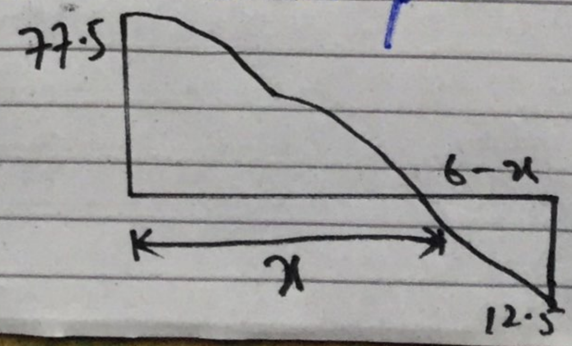



$-V_{10ft} + 77.5 - 90 - 60 = 0$

$V_{10ft} = 77.5 - 30$

$\boxed{V_{10ft} = 47.5 \text{ lb}}$

## Moment at zero shear point:



We Know That

$$\frac{77.5}{x} = \frac{12.5}{(6-x)}$$

$$= 77.5(6-x) = x \times 12.5$$

$$465 - 77.5x = 12.5x$$

$$465 = 77.5x + 12.5x$$

$$= 465 = 90x$$

$$x = 465/90$$

$$\boxed{x = 5.166 ft}$$

Now Take section a 5.166 ft from left support

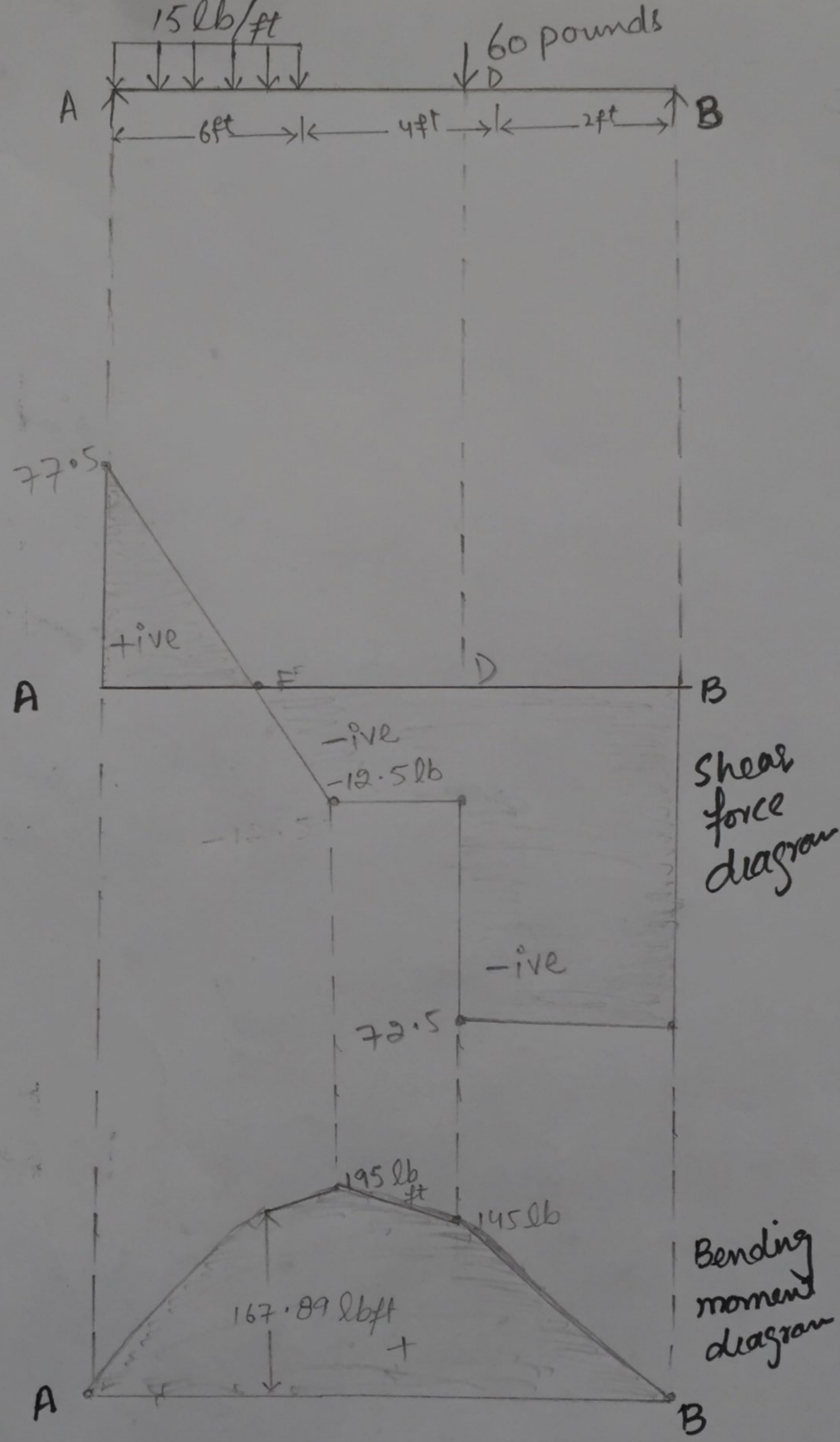
$\Sigma M_{5.166} = 0$ (Anticlock is +ive)

$$M_{5.166} - (77.5 \times 5.166) + (15 \times 6)\frac{5.166}{2} = 0$$

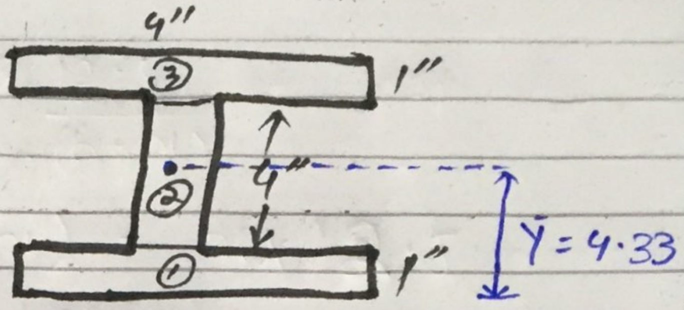
$$M_{5.166} - 400.4 + 232.5 = 0$$

$$M_{5.166} - 632.9 = 0$$

$$\boxed{M_{5.166} = 632.9}$$


15 lb/ft
60 pounds
A
D
B
6ft
4ft
2ft
77.5
+ive
A
F
D
B
-ive
-12.5 lb
Shear force diagram
-ive
72.5
195 lb ft
145 lb
167.89 lbft
+
A
B
Bending moment diagram

# MOMENT OF INERTIA




The centroid of section about y axis

$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{(0.5)(4) + (3)(4) + (9.5)(4}{4+4+4}$$

$$\boxed{\bar{y} = 4.33}$$

**NOW!**

**Moment of inertia of whole section is give by!**

$$I_y = I_1 + I_2 + I_3$$

$$I_1 = \frac{Bh^3}{12} + Ad_1^2 \qquad \left(\text{but } d = \bar{y} - y\right)$$

$$I_1 = \frac{4 \times 1^3}{12} + 4(3 - 0.5)^2 = 0.33 + 25$$

$$\boxed{= 25.33 \text{ inch}^4}$$

$I_2 = ?$

$$I_2 = \frac{1 \times 4^3}{12} + 4(0)^2 = 5.33 + 0 = \boxed{5.33\ in^4}$$

$I_3 = ?$

$$I_3 = \frac{4 \times 1^3}{12} + 4(3-5.5)^2 = 0.33 + 25 = \boxed{25.33\ inch^4}$$

$$I_y = I_1 + I_2 + I_3 = 25.33 + 5.33 + 25.33$$

$$I_y = 55.99\ inch^4 = 56\ inch^4$$

$$\boxed{I_y = 56\ inch^4}$$

## Shear stress:

$$\tau = \frac{VQ}{Ib}$$

Now! Shear stress at top fibre

$$\tau = \frac{47.5(3 \times 4)}{56 \times 4}$$

$$\boxed{\tau = 0}$$

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Shear Stress at point point C:

$$\tau_c = \frac{47.5\,(3\times4)}{56\times4} = \frac{570}{224}$$

$$\boxed{\tau_c = 2.544}$$

Shear Stress at centroid axis (Maxi)

$$\tau_{\substack{centroid\\axis}} = \frac{VQ_T}{Ib}$$

$$Q_{Total} = Q_1 + Q_2 + Q_3$$

$$Q_1 = y_1 A_1 = (0.5)(4) = 2$$

$$Q_2 = y_2 A_2 = (3)(4) = 12$$

$$Q_3 = y_3 A_3 = (9.5)(4) = 38$$

$$Q_T = Q_1 + Q_2 + Q_3$$

$$Q_T = 2 + 12 + 38$$

$$\boxed{Q_T = 52}$$

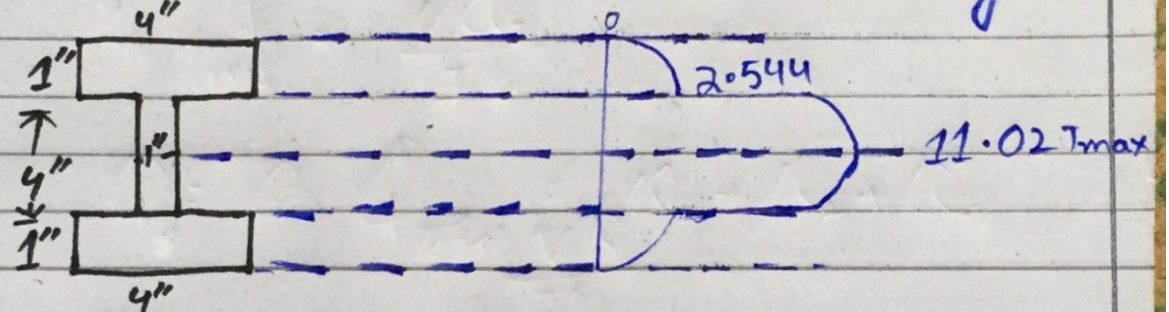
$$\tau_{centroid} = 47.5 \times 52 / 56 \times 4$$

$$\tau_{centroid} = \cancel{\phantom{00}}^{2470} / 224 = 11.02$$

$$\tau_{centroid} = 11.02 \ lb/in^2 = \tau_{max}$$

* Shear stress is maximum at Centroid.

# * Shear stress variation diagram




# * Flexure stress :-

We know that flexural stress: is given by

$$\sigma = My / I$$

- M → Max bending moment.
- I → Moment of inertia
- y → Centroid
-

Flexural stress at top Fibre.

$$\delta_{Top} = \frac{(632.9)(3)}{56}$$

$$\boxed{\delta_{Top} = 33.90} \ lb/inch^2$$

Flexural stress at point c

$$\delta_c = \frac{(632.9)(2)}{56}$$

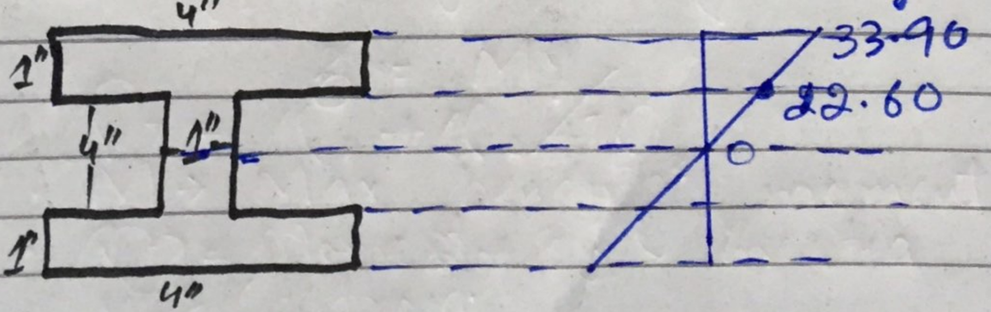
$$\boxed{\delta_c = 22.60} \ lb/in^2$$

Flexural stress at centroid.

$$\delta_{centroid} = (632.9)(0)$$

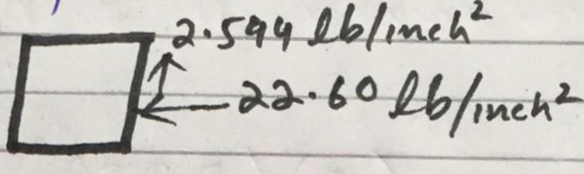
$$\boxed{\delta_{centroid} = 0}$$

Flexural stress variation diagram.



# Stress State of point C:

Shear stress at point C = $\tau_c = 2.544\ lb/inch^2$
flexural " at point C = $\delta_c = 22.60\ lb/inch$
stress is compresive b/c point C lies in compresive zone of beam cross section




Now! Shear stress condition at point C, Assume at 20° Clock wise orientation (-ive)

$\delta_x = -22.60\ lb/in^2$ $\qquad \delta_x' = ?$

$\delta_y = 0$ $\qquad \delta_y' = ?$

$\tau_{xy} = 2.544$ $\qquad \tau_{x'y'} = ?$

$\theta = 20^\circ$

**SO!**
Stress transformation equation.

$\cancel{\delta_x' = \frac{\delta_x + \delta_y}{2}} +$

$$\delta_x' = \frac{\delta_x + \delta_y}{2} + \frac{\delta_x - \delta_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{-22.60 + 0}{2} + \left(\frac{-22.60 - 0}{2}\right) \cos -2(20) + (2.544) \sin -2(20)$$

$$= -9.395 + 7.1960 - 7.822$$

$$= \delta x' = -10.02 \ lb/inch^2$$

$$\delta y' = ?$$

$$\delta y' = \frac{\delta x + \delta y}{2} - \frac{\delta x - \delta y}{2} \cos 2\theta - \tau xy \sin 2\theta$$

$$\delta y' = \frac{-22.60 + 0}{2} - \left(\frac{-22.60 - 0}{2}\right) \cos 2(40) : 2.544 - 40$$

$$\delta y' = -9.395 - 7.196 + 7822$$

$$\delta y' = -8.769 \ lb/inch^2$$

$$\tau x'y' = -\frac{\delta x - \delta y}{2} \sin 2\theta \ \tau xy \cos 2\theta$$

$$\tau x'y' = -\left(\frac{-22.60 - 0}{2}\right) \sin(40) + (2.544 \cos(-40))$$

$$= -(9.395)(-0.64) + 2.544 (0.766)$$

$$= -6.0128 - 09322$$

$$\boxed{\tau x'y' = -15.3348}$$

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## Principle Stress:

As:

We Know That principle Stress equation is given by

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_{1,2} = \frac{-22.60 + 0}{2} \pm \sqrt{\left(\frac{-22.60 - 0}{2}\right)^2 + (2.544)^2}$$

$$\sigma_{1,2} = -11.3 \pm \sqrt{17.7719}$$

$$\sigma_{1,2} = -11.3 \pm 4.215$$

$$\sigma_y = \sigma_1 = -11.3 + 4.215 = \boxed{-7.085}$$

$$\sigma_x = \sigma_2 = -11.3 - 4.215 = \boxed{-15.515}$$

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NOW!

We have to find $\theta p = ?$

$$\tan 2\theta p = \frac{2\tau_{xy}}{\left(\frac{\delta x - \delta y}{2}\right)} \quad \text{(formula)}$$

$$\tan 2\theta p = 2\frac{(\cancel{22.60}\ 2.544)}{\left(\frac{\cancel{22.60}\ 7.085yx}{2}\right)}$$

$$\theta p = \tan^{-1}\left(\frac{2\times 2.544}{9.395}\right) \Rightarrow \cancel{\theta p = \tan}$$

$$\theta p = \tan^{-1}\left(\frac{\cancel{2}.5088}{9.395}\right)$$

$$\theta p = \tan^{-1}(2.590)$$

$$\theta p = -68.89^\circ$$

$$\delta x'_{max} = \frac{-22.60+0}{2} + \frac{22.60-0}{2}\cos(68.89) + \cancel{2}2.544 \sin 2(-68.89)$$

$$\delta p_{max} = -9.395 - 1391 - 8.177$$

$$\boxed{\delta p_{max} = \cancel{0}\ 31.48}$$

NOW!

Maximum in plan shear stress

$$\tan 2\theta_s = -\frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\tau_{xy}}$$

$$\boxed{\tan\theta_s = \frac{+9.395}{50.88}}$$

~~$\theta_s = \tan^{-1}(0.38...)$~~

$\theta_s = \tan^{-1}(0.1847)$

$\boxed{\theta = 10.46^\circ}$ (Anticlock)

Putting this in general eq for $\sigma$ $\tau_{x'y'}$

$$\tau_{x'y'} = -\left[\frac{-22.60 - 0}{2}\right]\sin 2(10.46) + 2.544\cos 2(10.46)$$

$= (9.395)(0.6717) + (2.544)(0.7408)$

$= 6.310 + 9015$

$$\boxed{\tau_{x'y'} = 15.3256}$$

Coordinate of Centre (h, K)

$$= \left(\frac{\delta x - \delta y}{2}, 0\right)$$

$$(h, K) = \left(\left(\frac{-22.60 - 0}{2}\right), 0\right)$$

$$(h, K) = (-11.3, 0)$$

$\longleftrightarrow$

REDIUS OF Mohr's Circle:

$$r = \sqrt{\left(\frac{\delta x - \delta y}{2}\right)^2 - (\tau xy)^2}$$

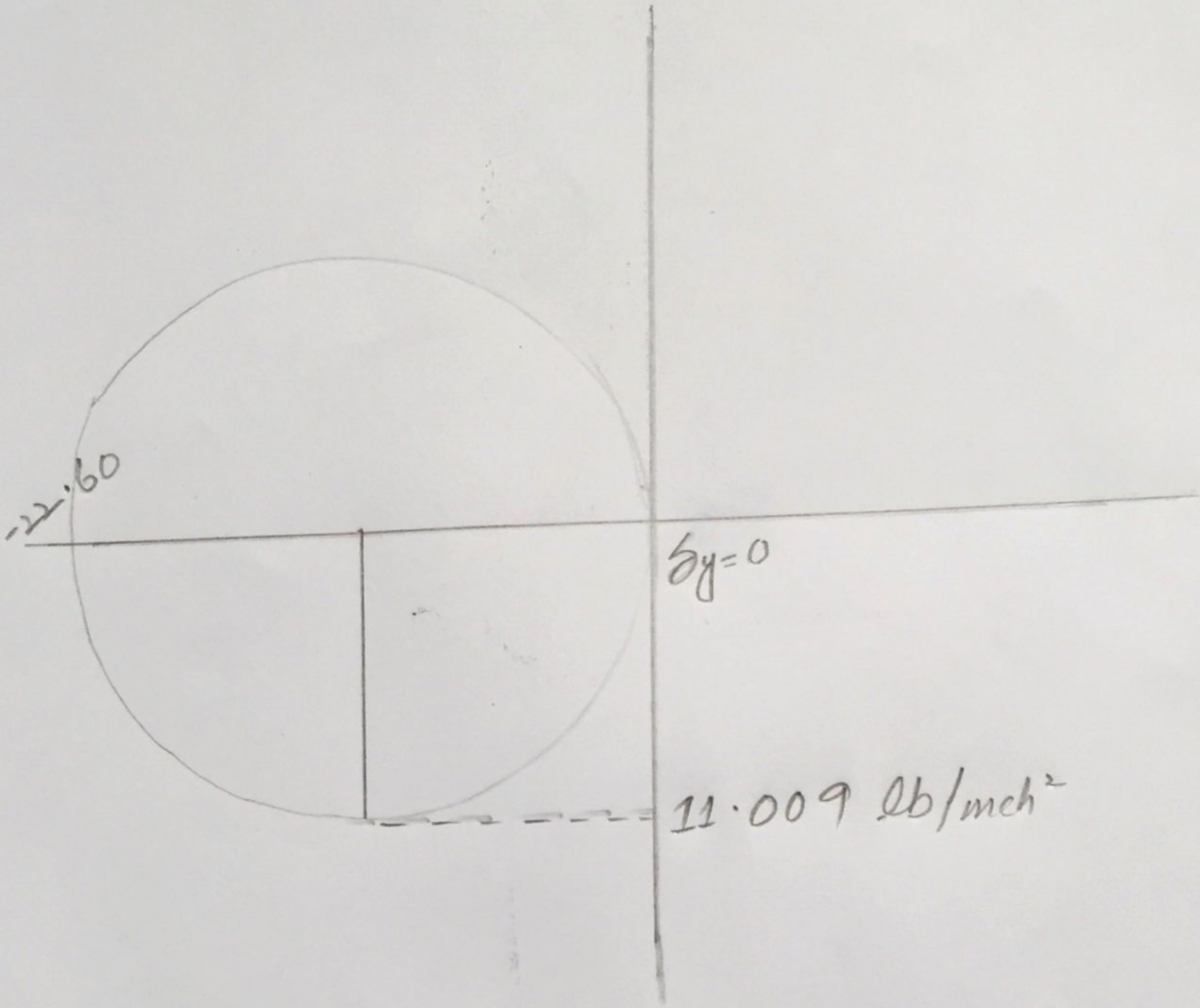
$$r = \sqrt{\left(\frac{-22.60 - 0}{2}\right)^2 + (2544)^2}$$

$$r = \sqrt{(-11.3)^2 + 6.471}$$

$$r = \sqrt{-127.69 + 6.471}$$

$$r = \sqrt{121.219}$$

$$\boxed{r = 11.009 \ lb/inch^2}$$




Maximum in plane shear Stress

Scale
3psi = 1cm