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Paper: Calculus

Department: Civil Engineering

Q.1)

Solution: $P=(4,1,3)$, $Q=(1,2,4)$

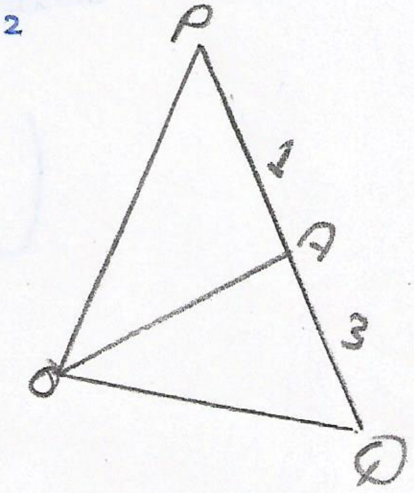
$$|PQ| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$= \sqrt{(1-4)^2+(2-1)^2+(4-3)^2}$$

$$= \sqrt{(-3)^2+(1)^2+(1)^2}$$

$$= \sqrt{9+1+1}$$

$$= \sqrt{11}$$



Position vector in the ratio 1:3

$$F(x,y,z) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}, \frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$$

$$= \left(\frac{(1)(1)+(3)(4)}{(1+3)}, \frac{(1)(2)+(3)(1)}{1+3}, \frac{(1)(4)+(3)(3)}{1+3}\right)$$

$$= \left(\frac{1+12}{4}, \frac{2+3}{4}, \frac{4+9}{4}\right)$$

$$= \left(13/4, 5/4, 13/4\right)$$

$$= \left(\frac{13}{4}i + \frac{5}{4}j + \frac{13}{4}k\right)$$

So the position vector dividing in the ratio 1 : 3 is

$$\left(\frac{13}{4} i + \frac{5}{4} j + \frac{13}{4} k \right)$$

Q.2) $\int \frac{4x^2+10x+4}{2x^2+x}$ ③

Solution:

It can be solve by integration by Partial fraction.

$$\frac{4x^3+10x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1} \quad — (A)$$

Xing to both sides by $x(2x+1)$

$$x(2x+1)\frac{4x^3+10x+4}{x(2x+1)} = \frac{A}{x}\cdot x(2x+1) + \frac{B}{2x+1}\cdot x(2x+1)$$

$$4x^3+10x+4 = A(2x+1) + Bx \quad — ①$$

Put $x=0$ in eq ①

$$4(0)^3+10(0)+4 = A(2(0)+1) + B(0)$$

$$\boxed{4 = A}$$

Now put $2x+1=0$

$$2x = -1$$

$$x = -\frac{1}{2} \quad \text{in eq ①}$$

$$4(-1/2)^3 + 10\left(-\frac{1}{2}\right) + 4 = A\left(2(-1/2)+1\right) + B\left(-\frac{1}{2}\right)$$

$$4(-1/8) + (-5) + 4 = A(-1+1) - \frac{1}{2}B$$

$$-1/2 - 5 + 4 = -1/2\,B$$

$$-1/2 - 1 = -\frac{1}{2}B$$

$$\frac{-1-2}{2} = \frac{-1}{2}B$$

$$\frac{-3}{2} = -1/2\,B$$

Xing to both side by 2.

$$2 \times \frac{-3}{2} = \frac{-1}{2} \times 2B$$

$$-3 = -B$$

$$\boxed{B = 3}$$

Now putting the value of A and B in eq (A)

$$\frac{4x^3+10x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Now taking integration

$$\int \left(4/x + 3/_{2x+1} \right) = \int 4/x + \int 3/_{2x+1}$$

$$= 4\int 1/x + 3\int \frac{1}{2x+1}$$

Apply integration

$$= 4\ln x + 3\ln |2x+1| + \ln c$$

$$= \ln(x)^4 + \ln(2x+1)^3 + \ln c$$

$$= \ln c(x)^4(2x+1)^3$$

Q.3(a) $\int_0^2 x^2 e^x dx$ ⑥

Solution:

$$= \int_0^2 x^2 e^x dx$$

$$= x^2 \int e^x dx - \int \left[\int e^x dx \cdot \frac{d}{dx} x^2 \right]$$

$$= x^2 e^x - \int e^x 2x dx$$

$$= x^2 e^x - 2\left[x \int e^x dx - \int \left[\int e^x \cdot 1 \right] dx \right]$$

$$= x^2 e^x - 2\left[xe^x - e^x + c \right]$$

$$= x^2 e^x - 2xe^x - 2e^x + 2c$$

⇒ Now taking limits

$$= x^2 e^x - 2xe^x - 2e^x \Big|_0^2$$

$$= 4e^2 - 4e^2 - 2e^2 - 0$$

$$\boxed{= -2e^2}$$ Answer

Q.3b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}$

Solution :

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}$$

By substitution

let $u = \sqrt{x}$

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$$du = \frac{1}{2\sqrt{x}}\, dx$$

$$2\,du = \frac{1}{\sqrt{x}}\, dx$$

if $x = 1 \Rightarrow u = 1$

if $x = 2 \Rightarrow u = \sqrt{2}$

$$= 2\int_1^{\sqrt{2}} \sin u\, du$$

$$= 2\left[-\cos u \Big|_1^{\sqrt{2}}\right]$$

$$= -2\left[\cos(\sqrt{2}) - \cos(1)\right]$$

$$= -2\,(0.999 - 0.9998)$$

$$= -2(-0.0008)$$

$$= 0.0016$$

Q.4) $u(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

Solution :

Three dimensional laplace equation is,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} + \frac{\partial^2 y}{\partial z^2} = 0 \quad \text{— (A)}$$

$$u = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}-1} \cdot 2x$$

$$\frac{\partial y}{\partial x} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2} \cdot 2x$$

again differentiate partially w.r.t x

$$\frac{\partial^2 u}{\partial x^2} = -\frac{1}{2}\left[(x^2+y^2+z^2)^{-3/2} 2 + 2x(-3/2)(x^2+y^2+z^2) \cdot 2x\right] \quad \text{— (1)}$$

$$u = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial y} = -1/2\,(x^2+y^2+z^2)^{-1/2-1} \cdot 2y$$

$$\frac{\partial u}{\partial y} = -1/2\,(x^2+y^2+z^2)^{-3/2} \cdot 2y$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{-1}{2}\left[(x^2+y^2+z^2)^{-3/2}\,2 + 2y(-3/2)(x^2+y^2+z^2)^{-5/2} \cdot 2y\right] - (2)$$

$$u = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial z} = -\frac{1}{2}\,(x^2+y^2+z^2)^{-1/2-1} \cdot 2z$$

$$\frac{\partial u}{\partial z} = -\frac{1}{2}\,(x^2+y^2+z^2)^{-3/2}\,2z$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{-1}{2}\left[(x^2+y^2+z^2)\,2 + 2z\,(x^2 y^2+z^2)^{-5/2} \cdot 2z\right]$$

(3)

Put ①, ② and eq ③ in A

$$\left(-\frac{1}{2}\left[(x^2+y^2+z^2)^{-3/2}\,2+2x\left(-\frac{3}{2}\right)(x^2+y^2+z^2)^{-5/2}\cdot 2x\right]\right)+$$

$$\left(-\frac{1}{2}\left[(x^2+y^2+z^2)^{-3/2}\,2+2y(-3/2)(x^2+y^2+z^2)^{-5/2}\cdot 2y\right]\right)+$$

$$\left(-\frac{1}{2}\left[(x^2+y^2+z^2)\cdot 2+2z(x^2+y^2+z^2)^{-5/2}\cdot 2z\right]\right)=0$$

$$0 = 0$$

Hence $u(x, y, z)$ satisfy 3 dimensional laplace equation.