NAME : FAWAD REHMAN
ID : S053
PAPER : DSP
Date : 27/06/2020
Instructor: Engr. Peer Mehar Ali shah

Question No: 01
(a).
Solution:

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

the characteristic equation is

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$ Hence,

$y_h(n) = C_1 2^n + C_2 n 2^n$

the particular solution is

$y_p(n) = K(-1)^n u(n).$

Substituting this solution into the difference equation, we obtain

$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2)$
$= (-1)^n u(n) - (-1)^{n-1} u(n-1)$

For $n = 2$

$K(1+4+4) = 2 \Rightarrow K = \frac{2}{9}$

the total solution is

$y(n) = \left[C_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n \right] u(n)$

Scanned with CamScanner

From the initial conditions, we obtain
$y(0)=1$, $y(1)=2$, then

$$C_1 + \frac{2}{9} = 1$$

$$\Rightarrow C_1 = \frac{7}{9}$$

$$2C_1 + 2C_2 - \frac{2}{9} = 2$$

$$\Rightarrow C_2 = \frac{1}{3}$$

Question No: 01
(b).
Solution:

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

the characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = \frac{1}{2}, \frac{1}{5}$ Hence

$$y_\lambda(n) = C_1 \frac{1}{2}^n + C_2 \frac{1}{5}^n$$

With $x(n) = \delta(n)$, we have

$$y(0) = 2$$

$$y(1) - 0.7y(0) = 0 \Rightarrow y(1) = 1.4$$

Hence

$$C_1 + C_2 = 2 \quad \text{And}$$

$$\frac{1}{2}c_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

$$\Rightarrow c_1 + \frac{2}{5}c_2 = \frac{14}{5}$$

these equation yield

$$c_1 = \frac{10}{3},\ c_2 = -\frac{4}{3}$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right] u(n)$$

the step response is

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left(\frac{1}{2}^n\left(2^{n+1} - 1\right)u(n) - \frac{1}{3}\left(\frac{1}{5}^n\left(5^{n+1} - 1\right)u(n)\right.\right.$$

Question No: 02

(a)

Solution:

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4, \; B = -3, \; C = -1$

Hence, $x(n) = \left[4(2)^n - 3 - n\right] u(n)$

Question No:02

(b).

Solution:

$$X(z) = \frac{1}{1-az^{-1}} \quad |z| > |a|$$

we know

$$x(n) = \frac{1}{2\pi j} \oint_c x(z)\, z^{n-1}\, dz$$

So

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1-az^{-1}}\, dz$$

$$= \frac{1}{2\pi j} \oint_c \frac{z^n\, dz}{z-a}$$

Where C is a circle at radius greater than $|a|$, we will evaluate this integral with $f(z) = z^n$.

we distinguish two cases.

1. If $n \geq 0$, $f(z)$ has only zeros and hence no poles inside C. the only pole inside C is $z=a$ hence

$$x(n) = f(z_0) = a^n \quad |a| \geq 0$$

2. If $n < 0$, $f(z) = z^n$ has an nth-order pole at $z=0$, which is also inside C. thus there are contributions from both poles.

For $n = -1$, we have

$$x(-1) = \frac{1}{2\pi j} \oint_c \frac{1}{z(z-a)} dz = \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=a} = 0$$

If $n = -2$, we have

$$x(-2) = \frac{1}{2\pi j} \oint_c \frac{1}{z^2(z-a)} dz = \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a} = 0$$

By continuing in the same way we can show that $x(n) = 0$ for $n < 0$.

Thus

$$x(n) = a^n u(n)$$

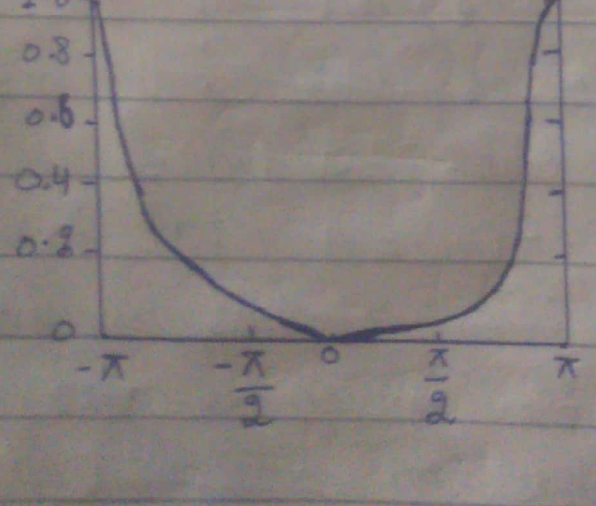
Question No: 03

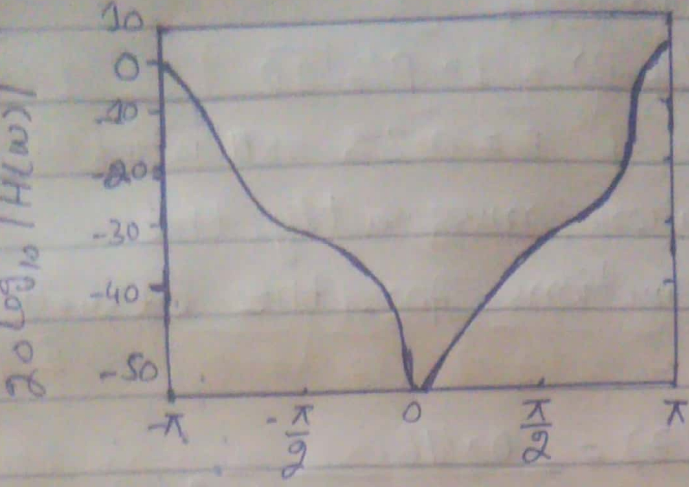
(a)

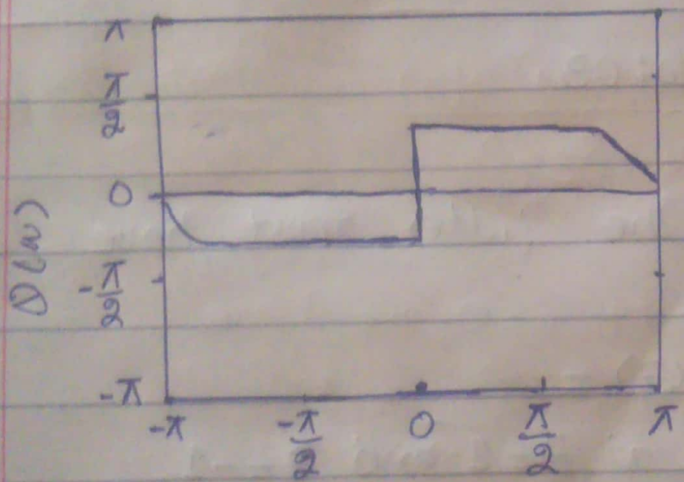
Solution: At $\omega = 0$, we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence $\quad b_0 = (1-p)^2$







At $w = \frac{\pi}{4}$,

$$H\left(\frac{\pi}{4}\right) = \frac{(1-P)^2}{(1-P \text{ [redacted] } e^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4)+jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2}+jP/\sqrt{2})^2}$$

Hence $\quad \dfrac{(1-P)^4}{[(1-P/\sqrt{2})^2+P^2/\sqrt{2}]^2} = \dfrac{1}{2}$

Or equivalently,

$$\sqrt{2}(1-p)^2 = 1 + p^2 - \sqrt{2}p$$

The value of $p = 0.32$ satisfies this equation consequently, the system functions for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

Question No: 03

(b)

Solution: The filter must have poles at

$$p(z) =$$

and zeros at $z=1$ and $z=-1$, Consequently the system function is

$$H(z) = \frac{G(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$

the gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$ thus we have

$$H\left(\frac{\pi}{2}\right) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$, thus we have

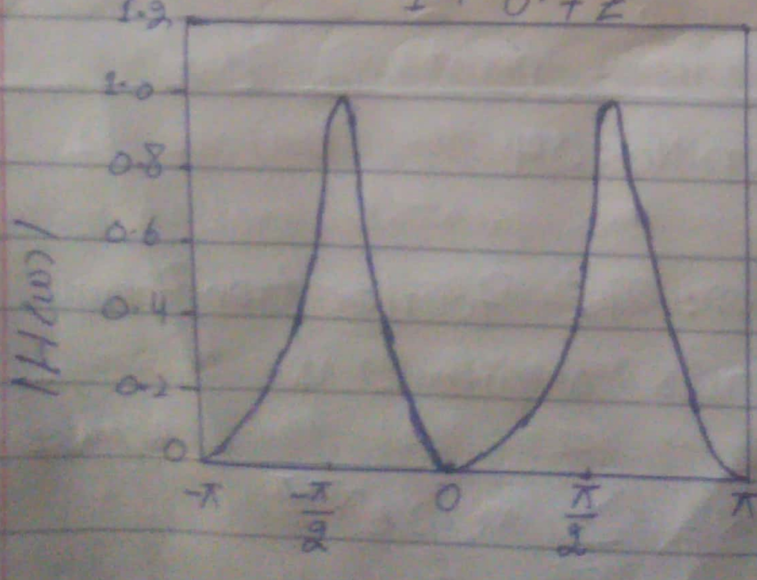
$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\,\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

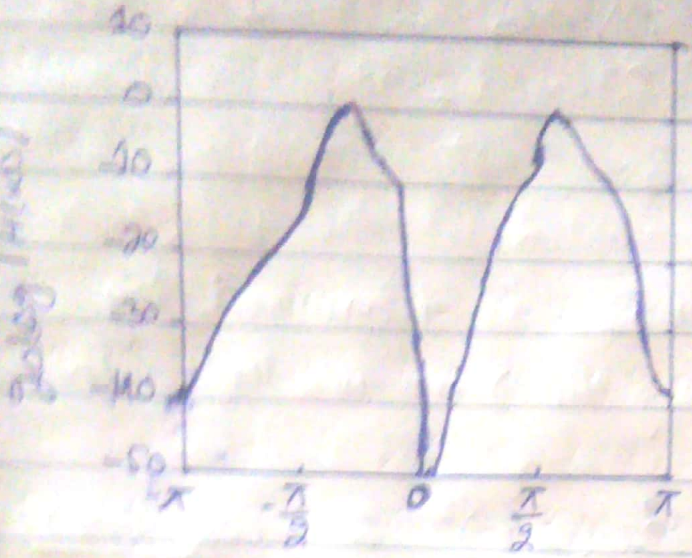
or equivalently

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfies this equation. therefore the system function for the desired filter is

$$H(z) = 0.15\,\frac{1-z^{-2}}{1+0.7z^{-2}}$$





Question No: 04

(a)

Solution: The Fourier transform of this sequence is

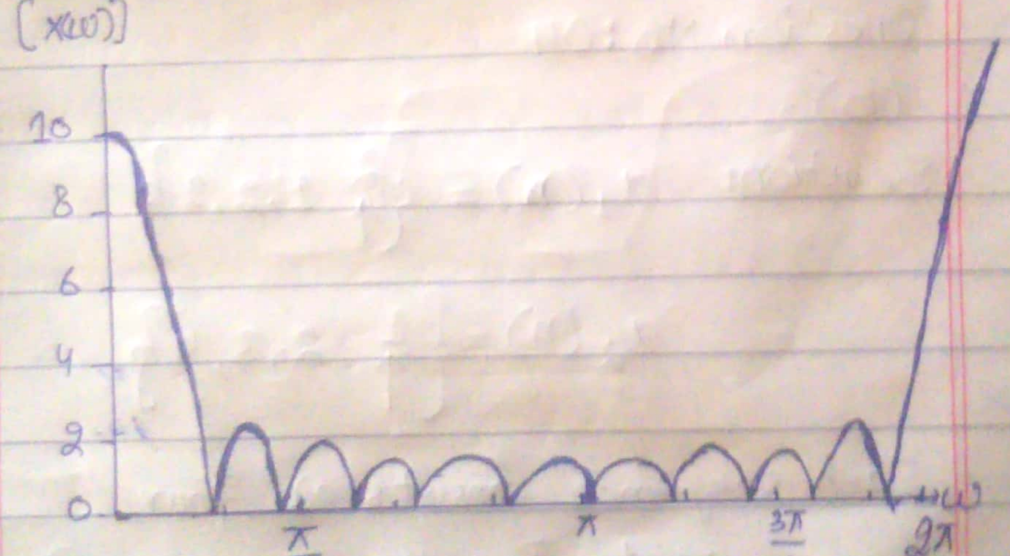
$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

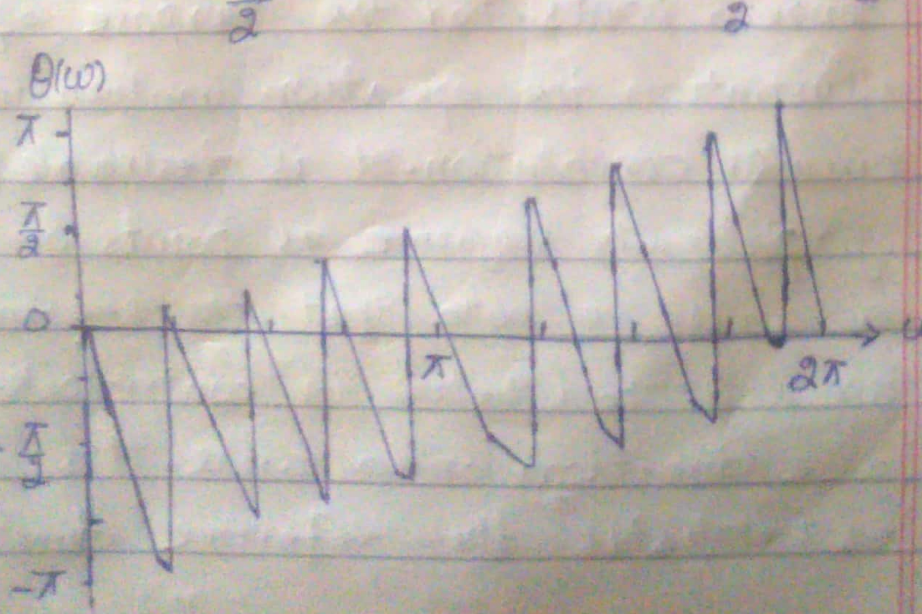
$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustrated below For $L = 10$ the N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equality spaced frequencies $\omega_K = 2\pi K/N$, $K = 0, 1 \ldots\ldots N-1$. Hence

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}} \quad K = 0, 1 \cdots N-1$$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





If N is selected such that N=L.
then the DFT becomes

$$X(K) = \begin{cases} L. & K=0 \\ 0. & K = 1,2 \cdots L-1 \end{cases}$$

Thus therefore there is only one nonzero value in the DFT. this is apparent from observation of $X(\omega)$. Since $X(\omega)=0$ at the frequencies $\omega_K = 2\pi K/L$, $K \neq 0$.

Question No: 04
(b).

Solution: $x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$

$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Each sequence consists of four nonzero points. For the purpose of illustration the operation involved in circular convolution. It is desirable to graph each sequence as points on a circle. We note that the sequences are graphed below in conterclock-wise direction on a circle. this establishes the reference direction

in rotating one of the sequences relative to each other.
Begining with $m = 0$, we have

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2((-n))_N$$

$x_2((-n))_4$ is simply the sequence $x_2(n)$ folded and graphed on a circle. In other words, the folded sequence is simply $x_2(n)$ graphed in clockwise direction.
The product sequence is obtained by multiplying $x_1(n)$ with $x_2((-n))_4$ point by point. Finally, we sum the values in the product sequence to obtain

$$x_3(0) = 14$$

For $m = 1$, we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

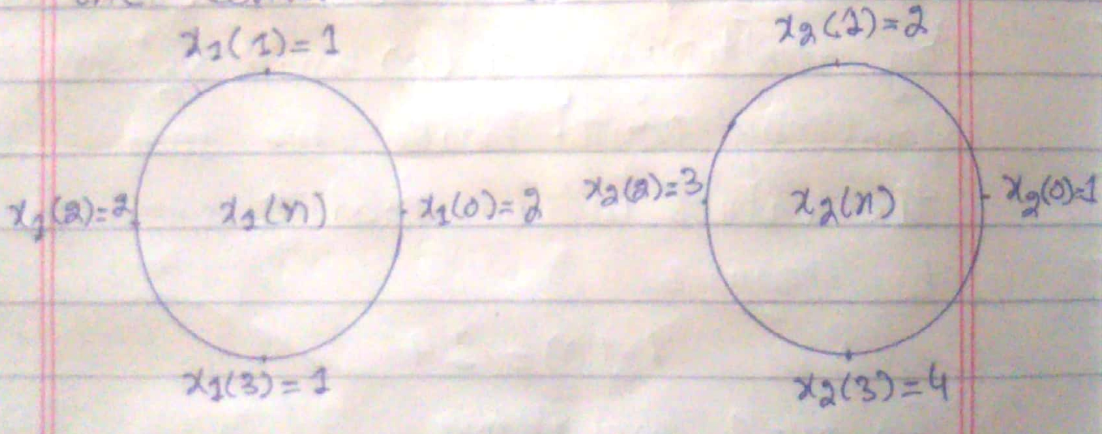
It is easily verified that $x_2((1-n))_4$ is simply the sequence $x_2((-n))_4$ rotated counterclockwise by one unit in time. this rotated sequence multiplies $x_1(n)$ to yield the product sequence. Finally, we sum the values in the product sequence to obtain $x_3(1)$. thus

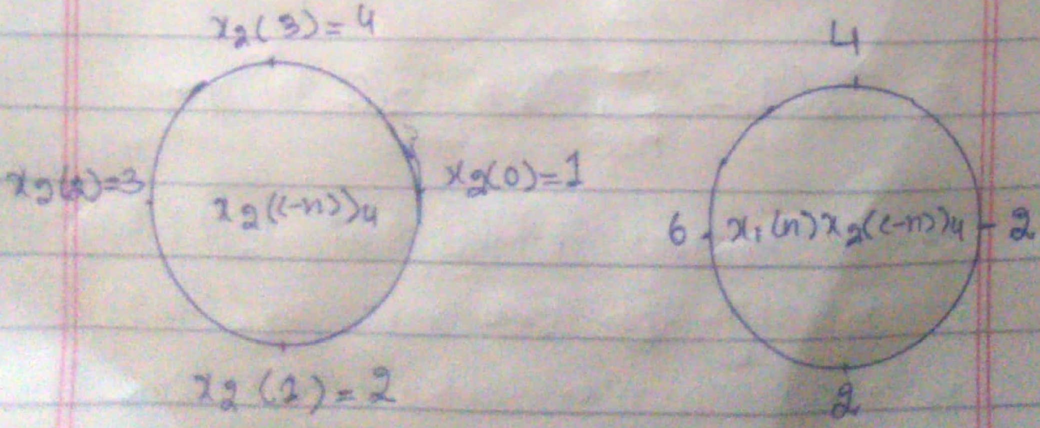
$$x_3(1) = 16$$

For $m=2$, we have

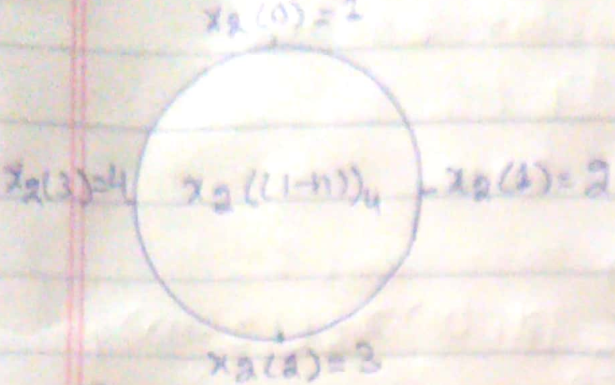
$$x_3(2) = \sum_{n=0}^{3} x_1(n)x_2((2-n))_4$$

Now $x_2((2-n))_4$ is folded sequence rotated two unit of time in the conterclockwise direction.

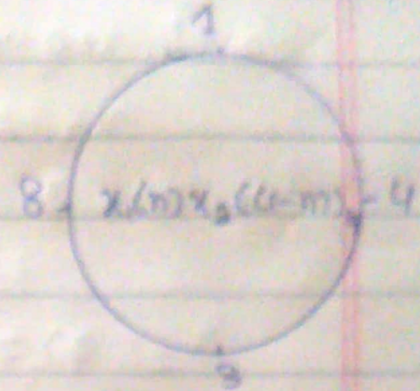




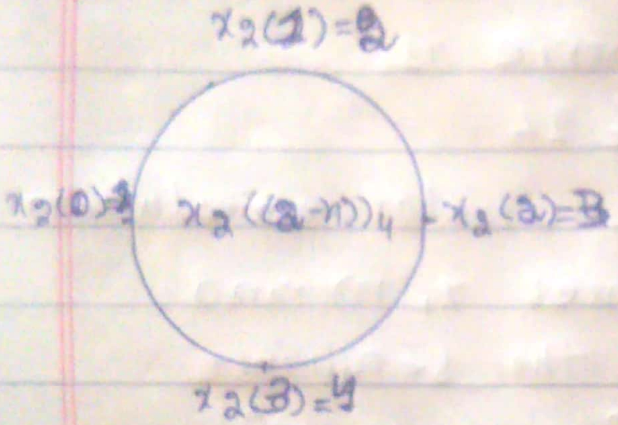
Folded sequence Product sequence



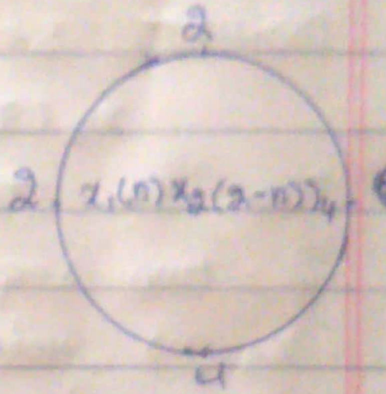
Folded sequence rotated by one unit in time



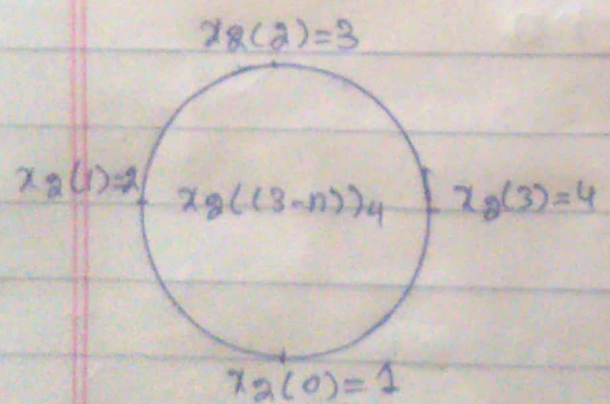
Product sequence



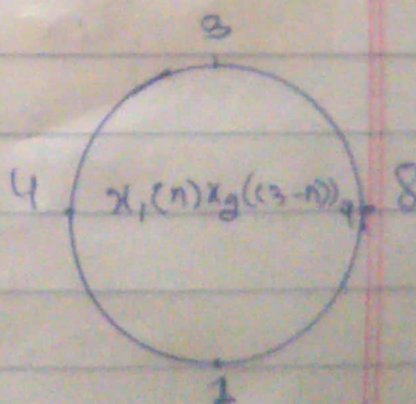
Product sequence



Folded sequence rotated by two unit in time



Folded sequence rotated by three units ~~by~~ in time



Product sequence

→ product sequence $x_1(n)\, x_2((2-n))_4$ by summing the four terms in the product sequence, we obtain.

$x_3(2) = 14$

For $m = 3$, we have

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2((3-n))_4$$

the sum of the values in product sequence is

$x_3(3) = 16$

therefore the circular convolution of two sequences $x_1(n)$ and $x_2(n)$ yields the sequence

$$x_3(n) = \{\underset{\uparrow}{14}, 16, 14, 16\}$$

"The End"