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Question no. 1

Find PQ where P is the paint in three dimentional Space with coordinates (4,1,3) and the point Q with co-ordinates (1,a,u). Find the distance between P and Q. Further, find the position vector of the point dividing PQ in the ratio 1:3.

Answer no. 1

Solution :-

Co-ordinate of $P = (4,1,3)$

$OP = 4i, 1j, +3k$

Or $OQ = \vec{OQ} - \vec{OP}$

$= (i + 2j + 4k) - (4i + 1j + 3k)$

$= -3i + 1j + 1k \longrightarrow (1)$

Now distance between P and Q $= |PQ|$

$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9 + 1 + 1}$

$= \sqrt{11} \longrightarrow (2)$

Let M be the point which divided PQ in ratio 1:3 then by the ratio theorum positive vector of M $= \vec{OM}$

$= \dfrac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1+3}$

$= \dfrac{12j + 3j + 9k + i + 2j + 4k}{4}$

$= \dfrac{13i + 5j + 13k}{4} \longrightarrow (3)$

Hence eq 1, 2 and 3 are required solution.

Question no. 2

Evaluate $\int \frac{4x^3 + 10x + 4}{2x^2 + x} dx$

Answer no. 2

Solution:-

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x} dx$$

$$
\begin{array}{r|l}
 & 2x - 1 \\
\hline
2x^2 + x & \cancel{4x^3} + 10x + 4 \\
 & \pm \cancel{4x^3} \quad \pm 2x^2 \\
\hline
 & -\cancel{2x^2} + 10x + 4 \\
 & \mp \cancel{2x^2} \mp x \\
\hline
 & 11x + 4
\end{array}
$$

So $2x - 1 + \frac{11x+4}{2x^2+x} = \frac{4x^3 + 10x + 4}{2x^2 + x}$

$$\Rightarrow \int \frac{4x^3 + 10x + 4}{2x^2 + x} = \int 2x - 1 + \int \frac{11x + 4}{2x^2 + x} \longrightarrow 1$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x} dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)} dx \longrightarrow 2$$

Now find

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1} \longrightarrow (A)$$

$$\frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$11x+4 = A(2x+1) + Bx \longrightarrow (3)$$

put $x=0$ in eq (3)

$$\boxed{4 = A}$$

Now put $x = -1/2$ in eq (3)

$$11(-1/2) + 4 = B(-1/2)$$

$$-11/2 + 4 = \frac{-B}{2}$$

$$\frac{-11+8}{\cancel{2}} = \frac{-B}{\cancel{2}}$$

$$-3 = -B \quad \Rightarrow \quad B = 3$$

Putting the value of A and B in eq (A)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integral on both sides

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4\,dx}{x} + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + 3/2 \ln|2x+1|$$

Putting these values in eq 2

$$= x^2 - x + 4\ln|x| + 3/2\ln(2x+1)$$

Now put the value in 1

$$\boxed{\int \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + 3/2\ln(|2x+1|) + C}$$

Question no. 3

Evaluate

a) $\int_0^2 x^2 e^x \, dx$

b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}} \, dx$

Answer no. 3

Solution:-

a) $\int_0^2 x^2 e^x dx$

Now first find integratim

$$= \int x^2 e^n dx$$

$$= x^2 \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x^2 \right) dx$$

$$= x^2 e^x - \int e^x (2x) dx$$

$$= x^2 e^x - 2 \int x e^x dx$$

$$= x^2 e^x - 2 \left(x \int e^x dx - \int \left(\left(\int e^x dx \frac{d}{dx} x \right) dx \right) \right)$$

$$= x^2 e^x - 2 \left(x e^x - \int e^x dx \right)$$

$$= x^2 e^x - 2x e^x + 2 e^x$$

Now put limits

$$= \left| x^2e^x - 2xe^x + 2e^x \right|_0^2$$

$$= \left(2^2e^2 - 2(2)e^2 + 2e^2 - (0 - 0 + 2e^0)\right)$$

$$= (4e^2 - 4e^2 + 2e^2 - 2)$$

$$\boxed{= 2e^2 - 2}$$

b) $\displaystyle\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

First find integration

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = ? \longrightarrow (1)$$

let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$2\,dy = \frac{1}{\sqrt{x}}\,dx \quad \text{putting in } (1)$$

$$\int \sin(y)(2dy) = 2\int \sin(y)\,dy$$

$$= 2(-\cos y)$$

$$= -2\cos y$$

Put $y = \sqrt{x}$

$$= -2\cos\sqrt{x}$$

Put limits

$$= -2\left|\cos\sqrt{x}\right|_{1}^{2} = -2\left(\cos\sqrt{2} - \cos 1\right)$$

$$\boxed{= -2\cos\sqrt{2} + 2\cos(1)}$$

Question no. 4

Verify that

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfices the three dimentional Laplace's equation.

Answer no. 4

Solution:-

The laplace equation in 3d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \longrightarrow (A)$$

So, $u(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

$$u(x,y,z) = (x^2+y^2+z)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x(-3/2)(x^2+y^2+z^2)^{-5/2}(\cancel{2}x)+(x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \longrightarrow (1)$$

Now

$$\frac{\partial u}{\partial y} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y(-3/2)(x^2+y^2+z^2)^{-5/2}(2y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \longrightarrow (2)$$

$$\frac{\partial u}{\partial z} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2) - (x^2+y^2+z^2)^{-3/2} \longrightarrow (3)$$

Putting (1), (2) and (3) in (A)

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$
$$+ 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2)+3z^2 - (x^2+y^2+z^2)\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2-x^2-\cancel{y^2}-\cancel{z^2}+3y^2-\cancel{x^2}-y^2-\cancel{z^2}+3z^2-\cancel{x^2}-\cancel{y^2}-z^2\right]$$

$$= (x^2+y^2+z^2)^{-5/2}(0) = 0$$

So the given solution $u(x,y,z)$ is Solution of leplace equation.