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Q Q1)

Find PQ where P is the point in three dimensional space.....
...... ?

S Sol)

co-ordinates of "P" = (4, 1, 3)

$OP = (4i + 1j + 3k)$

or $\overrightarrow{OQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (i + 2j + 4k) - (4i + j + 3k)$

$= -3i + j + k$

Now distance b/w P & Q = |PQ|

$= \sqrt{(-3)^2 + (1)^2 + (1)^2} = \sqrt{11}$ —— (2)

Let M be the point which divided PQ in ratio 1:3 then by ratio theorm position vector of M = $\overrightarrow{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1+3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4} \longrightarrow (3)$$

hence eq (1), (2), (3) are required Solutions.

Q No (2) ;

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx$$

Sol)

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x^2+x & \cancel{4x^3}+10x+4 \\ & \underline{\pm\cancel{4x^3} \qquad \pm 2x^3} \\ & \cancel{2x^3}+10x+4 \\ & \underline{\mp 2x^3 \mp x} \\ & 11x+4 \end{array}$$

So $2x-1+\dfrac{11x+4}{2x^2+x} = \dfrac{4x^3+10x+4}{2x^2+x}$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x} = \int 2x-1 + \int \frac{11x+4}{2x^2+x} \rightarrow 1$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \text{ —— } (2)$$

Now find

$$= \int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$= \frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \quad \text{——(A)}$$

$$= \frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$= 11x+4 = A(2x+1) + Bx \quad \text{——(3)}$$

Put $x = 0$ in (3)

$$4 = A$$

Now put $x = -1/2$ in (3)

$$11\left(-\tfrac{1}{2}\right) + 4 = B\left(-\tfrac{1}{2}\right)$$

$$\frac{-11+8}{\cancel{2}} = -\frac{B}{\cancel{2}}$$

$$-3 = -B \implies B = 3$$

Putting the value of A & B in (A)

$$= \frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

(A)

Taking integral on both side

$$= \int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4}{x} \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + 3/2 \ln|2x+1|$$

Putting these values in (2)

$$= x^2 - x + 4\ln|x| + 3/2 \ln(2x+1)$$

Now put these values in (1)

$$= \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + 3/2 \ln|2x+1| + C$$

Ans

Q3)

a) $$\int_0^2 x^2 e^x \, dx$$

Now finding integration

$$= \int x^2 e^x \, dx$$

$$= x^2 \int e^x dx - \int \left(\int e^x dx \, \frac{d x^2}{dx} \right) dx$$

$$= x^2 e^x - \int e^x (2x) \, dx$$

$$= x^2 e^x - 2 \left[x \int e^x dx - \int \left(\int e^x dx \, \frac{dx}{dx} \right) dx \right]$$

$$= x^2 e^x - 2 \left[x \int e^x dx - \int \left(\int e^x dx \, \frac{dx}{dx} \right) dx \right]$$

$$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$$

$$= x^2 e^x - 2x e^x + 2 e^x$$

Now applying limit

$= \left[x^2 e^x - 2x e^x + 2 e^x \right]_0^2$

$= \left(2^2 e^2 - 2(2) e^2 + 2e^2 - (0 - 0 + 2e^0) \right)$

$= \left(4e^2 - 4e^2 + 2e^2 - 2 \right)$

$= \underline{2e^2 - 2}$ Ans

b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}} \, dx$

Sol) First we do integration

$\int \frac{\sin\sqrt{x}}{\sqrt{x}} \, dx$ — (1)

let $y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$2dy = \frac{1}{2\sqrt{x}}$

$\boxed{2dy = \frac{1}{\sqrt{x}} dx}$ Puting in equation (1)

$\int \sin(y)(2dy) = 2 \int \sin(y) \, dy$

$= 2(-\cos y)$

$= -2\cos y$

puting $y = \sqrt{x}$

as per actual equation

$= -2\cos\sqrt{x}$

Now putting limit

$= -2 \left| \cos(\sqrt{x}) \right|_1^2 = -2\left(\cos\sqrt{2} - \cos 1\right)$

$= -2\cos\sqrt{2} + 2\cos(1)$ Ans

Q4)

Sol) The laplace equation in 3d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad - A$$

$\rightarrow$ So $u(x, y, z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

$\rightarrow$ $u(x, y, z) = (x^2+y^2+z^2)^{-1/2}$

$\rightarrow$ $\dfrac{\partial u}{\partial x} = \dfrac{-1}{\cancel{2}} (x^2+y^2+z^2)^{-1/2} (\cancel{2}x)$

$\rightarrow$ $\dfrac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$

$\rightarrow$ $\dfrac{\partial^2 u}{\partial x^2} = -\left[x(-3/x)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2} \right]$

$\rightarrow$ $\dfrac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2) - (x^2+y^2+z^2)^{-3/2} \quad -(1)$

Now

$$\Rightarrow \frac{\partial u}{\partial y} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$\Rightarrow \frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} = -\left[y\left(-\frac{3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} = -\left[y(-3/\cancel{2})(x^2+y^2+z^2)^{-5/2}(\cancel{2}y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\Rightarrow \boxed{\frac{\partial^2 u}{\partial y^2} = \left[3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}\right]} - (2)$$

$$\Rightarrow \frac{\partial u}{\partial z} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$\Rightarrow \frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$\Rightarrow \boxed{\frac{\partial^2 u}{\partial z^2} = \left[3z^2(x^2+y^2+z^2) - (x^2+y^2+z^2)^{-3/2}\right]}$$

Putting eq (1) (2) (3) in (A)

$$= 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2}$$
$$- (x^2+y^2+z^2)^{-3/2} +$$
$$3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2) + 3z^2-(x^2+y^2+z^2)\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[\cancel{3x^2}\cancel{-x^2}-\cancel{y^2}-z^2+3y^2-\cancel{x^2}-y^2\cancel{-z^2}+3z^2 -\cancel{x^2}-\cancel{y^2}-z^2\right]$$

$$= (x^2+y^2+z^2)^{-5/2}(0) = 0$$

So given $u(x,y,z)$ is solution of laplace equation.