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Dep. BE (E)

Subinited to Himayatullah

Assignment

Q1. $\int_0^{\frac{\pi}{4}} (1-\sin t)^{\frac{3}{2}} \cos 2t \, dt$ (1)

Solution:

Integration by part

$(1-\sin t)^{3/2}(\cos 2t) \cdot \int ((1-\sin t)^{3/2} \cos t) \, dt$

$= \frac{3}{2}\left((1-\sin t)^{1/2} \cos t - \frac{\sin 2t}{2}\right)$

$(1-\sin t)^{3/2}(\cos 2t)\Big|_0^{\pi/4} = \frac{6}{4}\int_0^{\pi/4} (1-\sin t) \cos t \sin t$

Now Let

$u = \sin t$

$\frac{d}{dt} u = \cos t$

$du = \cos t \, dt$

$(1-\sin t)^{3/2}(\cos t) = \frac{6}{4}\int_0^{\pi/4} (1-u)^{1/2} u^2 \, dt \, du$

Now Let

$t = 1-u$

②

$$\frac{dt}{du} = -1$$

$$dt = du$$

$$(1-\sin t)^{3/2} (\cos t) = \frac{6}{4} \int_0^{\pi/4} (1+x)^2 \; x^{1/2} = dx$$

$$= \frac{6}{4} \int_0^{\pi/4} (1-2t+t^2)(t^{1/2})\, dt$$

$$= \frac{6}{4} \int_0^{\pi/4} (t^{1/2} - 2t^{3/2} + t^{5/2})\, dt$$

Applying Integral

$$= -\frac{6}{4} \left| \frac{t^{5/2+1}}{5/2+1} - \frac{2t^{3/2+1}}{3/2+1} + \frac{t^{1/2+1}}{1/2+1} \right|_0^{\pi/4}$$

$$= \frac{-6}{4} \left(\left[\frac{2}{7}(t)^{2/3} - \frac{4}{5} t^{5/2} + \frac{2}{3} t^{3/2} \right] \right)_0^{\pi/4}$$

$$\boxed{\cancel{-\frac{6}{4} \left(\frac{2}{7} (\sin t) \Big|_0^{\pi/4} {}^{7/2} - \frac{4}{5} (\sin t \, 5)^{5/4} + \frac{2}{3} (\sin t) \right.}}$$

$$= -\frac{6}{4} \left[\frac{2}{7} (\sin t) \Big|_0^{\pi/4} {}^{7/2} - \frac{4}{5} (\sin t)^{5/4} \Big|_0^{\pi/4} + \frac{2}{3} (\sin t)^{3/2} \Big|_0^{\pi/4} \right.$$

$$= \frac{-6}{4} \left(\frac{2}{7} (\sin t 5)^{1/2} - \frac{4}{5} (\sin t 5)^{5/4} + \frac{2}{2} (\sin t 45)^{3/2} \right.$$

$$= \frac{6}{4} (0.28 + 0.085 - 0.33)$$

$$= \frac{6}{4} (0.035)$$

$$= 0.0525 \text{ Ans}$$

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Q2:- $\int_0^1 (4y - y^2 + 4y^3 + 1)^{\frac{-2}{3}} (12y^2 - 2y + 4)\,dy$

Solution:- $\int_0^1 (4y - y^2 + 4y^3 + 1)^{\frac{-2}{3}} (12y^2 - 2y + 4)\,dy$

$$\int_0^1 (4y - y^2 + 4y^3 + 1)^{\frac{-2}{3}} (12y^2 - 2y + 4)\,dy \quad (1)$$

By Using Subsitution method

let $x = 4y - y^2 + 4y^3 + 1$

$\frac{dx}{dy} = 4 - 2y + 12y^2$ - $dx = (4 - 2y + 12y^2)\,dy$

$$\int_0^1 x^{-2/3}\,dx$$

$$\frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1} + C$$

$$\frac{x^{1/3}}{1/3} + C$$

$3x^{1/3} + C$

$\Rightarrow (3\,(4y - y^2 + 4y^3 + 1) + 1$

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Putting the limits

$$\left(3\left(4(1)-(1)^2+4(1)^3+\cancel{(1)}\right)^{1/3}+C\right.$$

$$=3\left(4-\cancel{1}+4+\cancel{1}\right)^{1/3}+C$$

$$=3(8)^{1/3} \qquad = C \longrightarrow \text{orbitary Constant}$$

$$=3(2^3)^{1/3}$$

$$=3(2)=6$$ Ans