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Paper	Intro to Structural Dynamics & Earthquake Engg
Exam	Final term
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Question-1

Given data

$E = 29000 ksi$
$I = 150 in^4$
$L = 10 ft = 10 \times 12 = 120 inch$
$W = 7742 lb$

Sol:

The general E.O.M for SDOF system is

$$ku + cu' + m\ddot{u} = p(t)$$

In our case system is undamped $(c = 0)$ undergoing free vibration $p(t) = 0$

Hence general EOM $= ku + m\ddot{u} = 0$

$$k = \frac{3EI}{L^3}$$

$$k = \frac{3 \times 29000 \times 150}{(120)^3}$$

$$k = 7.5521 \ k/in$$

$$k = \frac{7.5521 \times 1000}{1/12}$$

$$k = 90625 \ lb/ft$$

$m = \frac{w}{g}$

$m = \frac{7742}{32.2}$

$m = 240.43$ slug

$\omega_n = \sqrt{\frac{k}{m}}$

$\omega_n = \sqrt{\frac{90625}{240.43}}$

$\omega_n = 19.41$ rad/sec

$T_n = \frac{2\pi}{\omega_n}$

$T_n = \frac{2 \times 3.14}{19.41}$

$T_n = 0.324$ sec

So

$ku + m\ddot{u} = 0$

$90625u + 240.43\ddot{u} = 0$

* where 'k' is in lb/ft and 'm' is in lb sec/ft²

General solution to EOM for undamped free vibration is

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$$u(0) = \frac{1}{2''} \Rightarrow \frac{1}{24}\ ft \quad \text{and } \dot{u}(0) = 0$$

$$u(t) = \frac{1}{24}\cos(19.41t) + \frac{0}{19.41}\sin(19.41t)$$

$$u(t) = \frac{1}{24}\cos(19.41t)$$

Equivalent static force at any time 't' is

$$f_s(t) = k \cdot u(t) = \frac{90625\cos(19.41t)}{24}$$

$$f_s(t) = 3776.04\cos(19.41t)$$

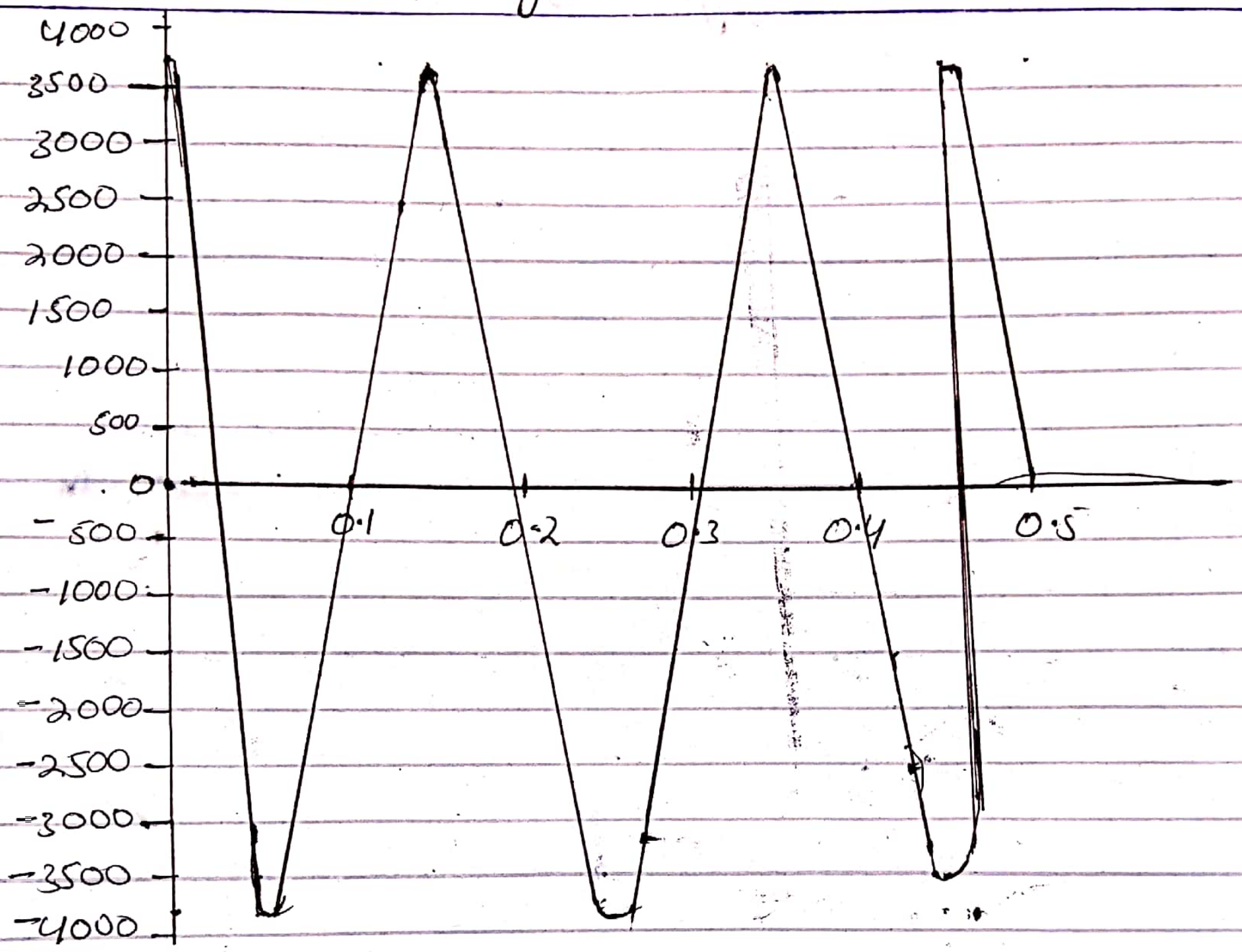
amplitude of dynamic displacement, u_0 for undamped free vibratio is

$$u_0 = \sqrt{(u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2}$$

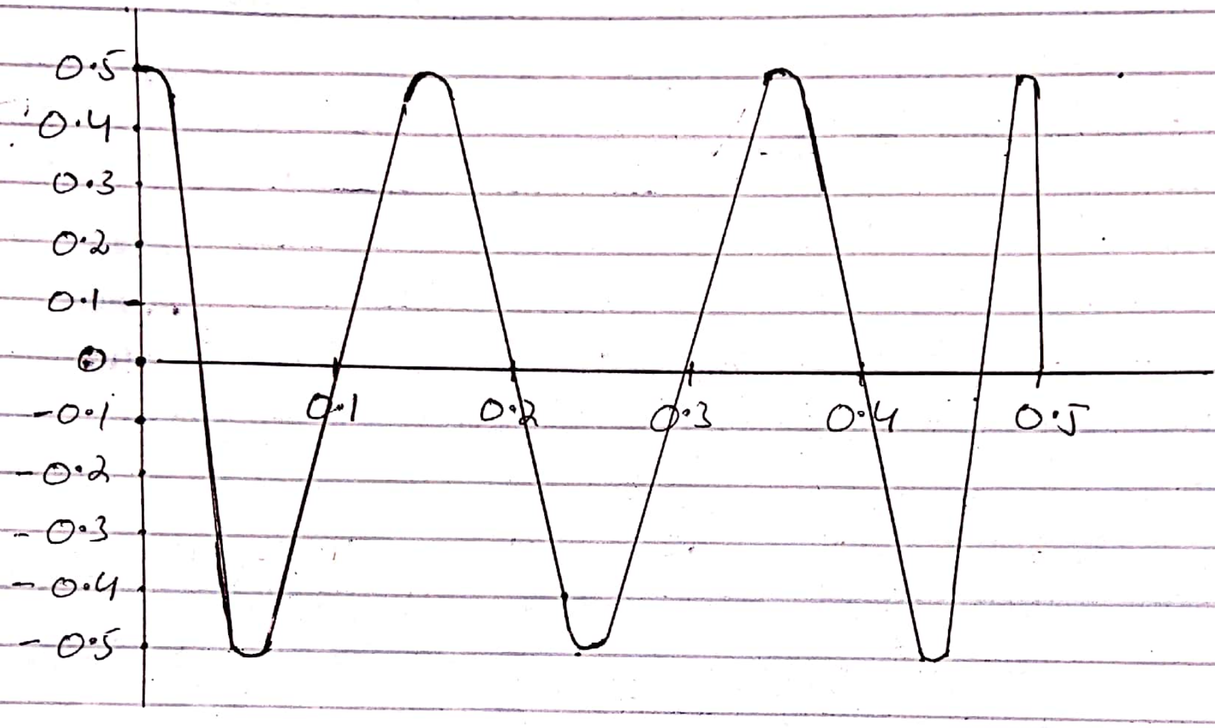
$$u_0 = \sqrt{\left(\frac{1}{24}\right)^2 + \left(\frac{0}{19.41}\right)^2}$$

$$u_0 = \frac{1}{24}\ ft$$

$$ku_0 = 90625 \times \frac{1}{24} \Rightarrow 3776.04\ lb$$



Variation of Equivalent Static Forces with time



variation of displacement with time

Question - 2:

Given:

→ consider Damping ratio $\zeta = 0.03$

→ $m = 240$ lb.sec²/ft

→ $k = 90625$ lb/ft

→ $\omega_n = 19.41$ rad/sec

SOl:

E.O.M for damped free vibration is

$ku + c\dot{u} + m\ddot{u} = 0$ — (1)

$c = \zeta \times 2m\omega_n$

$c = 0.03 \times 2(240)(19.41)$

$c = 279.504$ lb.sec/ft

put value in eq(1)

~~90625u + 85.26u~~

$90625 + 279.504\dot{u} + 240\ddot{u} = 0$

solution to E.O.M for damped free vibration is

$$u(t) = e^{-\zeta \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)\zeta\omega_n\right]\sin(\omega_D t)\right]$$

$\omega_D = 19.41$ rad/sec

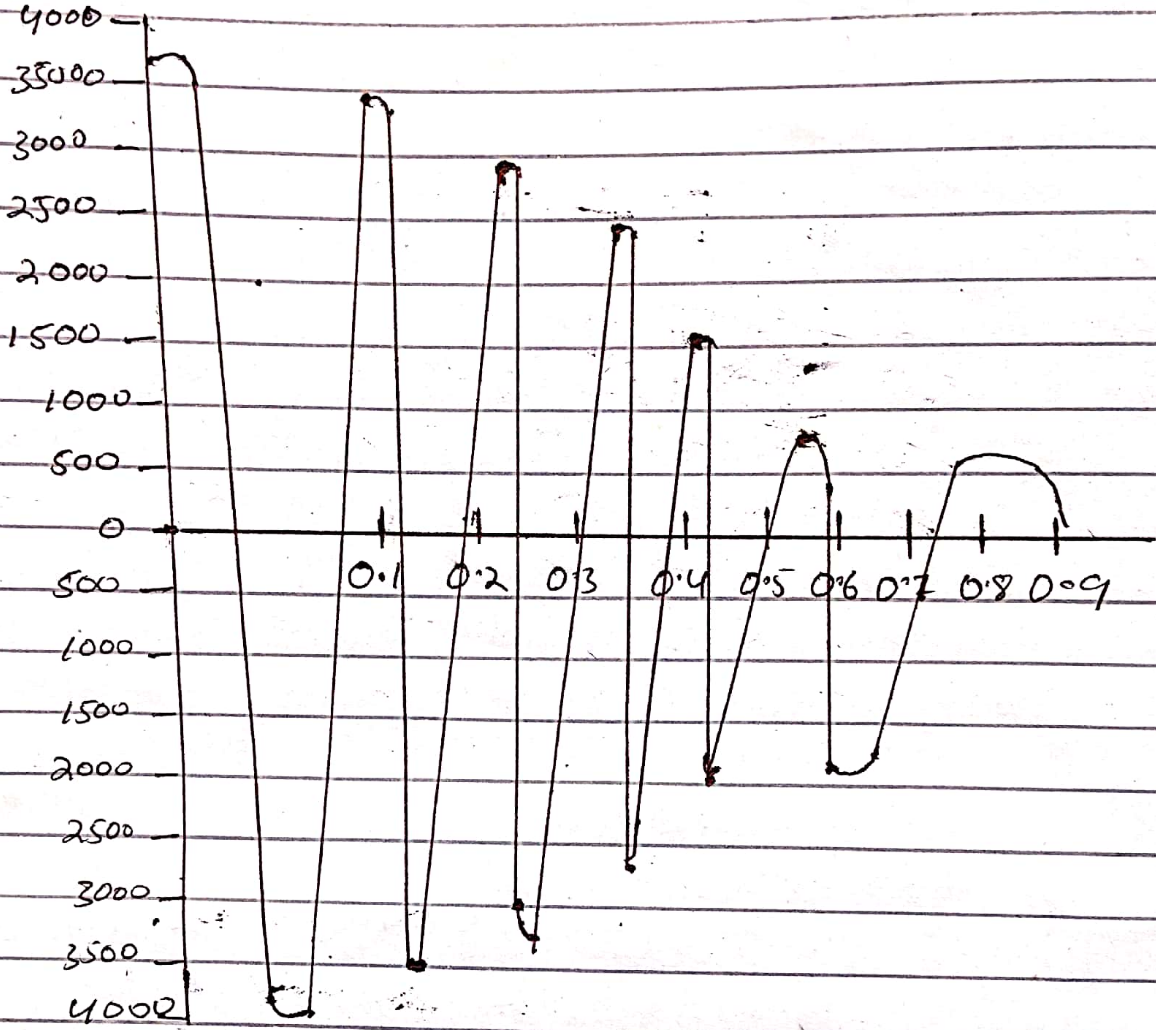
$$u(t) = e^{-0.03 \times 19.41}\left[\frac{1}{24}\cos(19.41t) + \frac{1}{19.41}\left[0 + \frac{1}{24} \times 0.03 \times 19.41 \sin(19.41t)\right]\right]$$

$$u(t) = e^{-0.5823}\left[0.4167\cos(19.41t) + 0.00125\sin(19.41t)\right]$$

$F_s(t) = k \cdot u(t)$

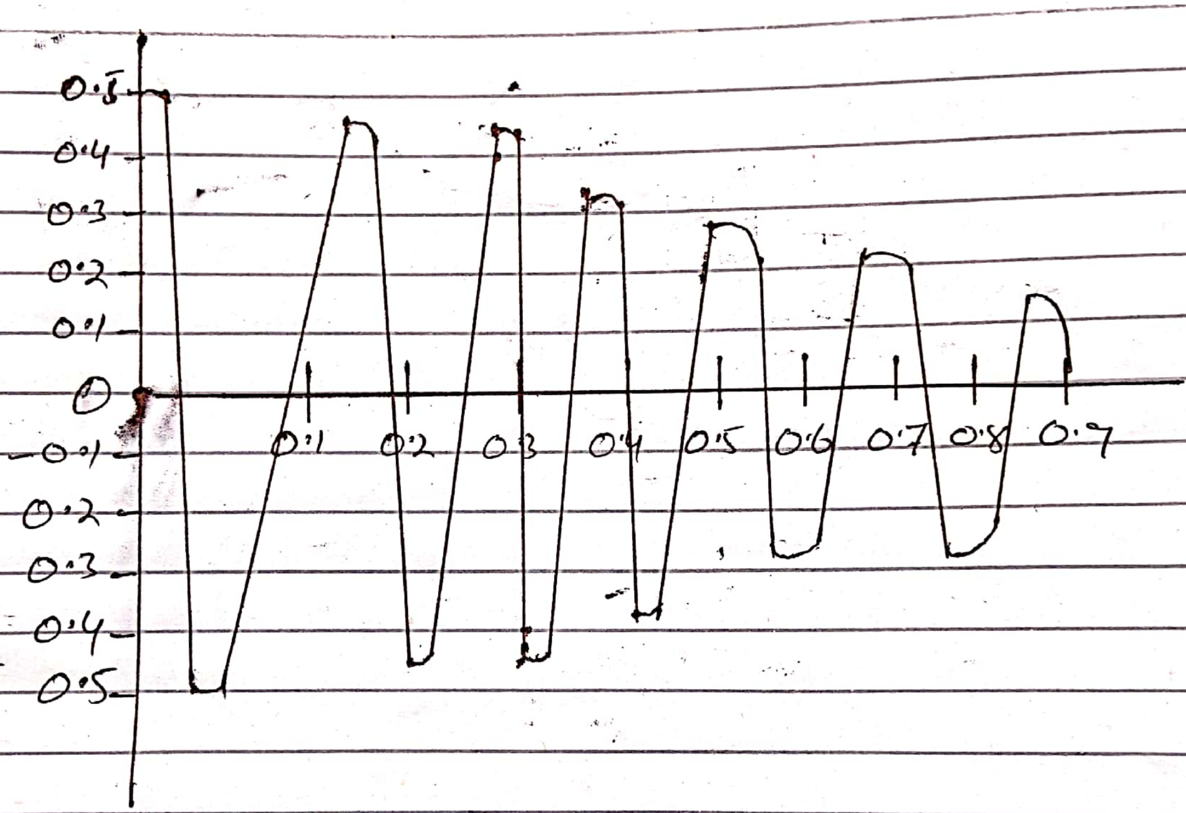
$$F_s(t) = 90625 \times e^{-0.5823}\left[0.4167\cos(19.41t) + 0.00125\sin(19.41t)\right]$$

$$F_s(t) = e^{-0.5823}\left[37763\cos(19.41t) + 113.28\sin(19.41t)\right]$$



~~Equal~~ Equivalent static forces

variation of ~~displacement~~ with time



variation of displacement with time

Question-3:

Given data:

→ Force = 60kips

→ Tank is displace = $\frac{7742}{1000} = 7.742''$

→ cycle = j = 7 cycles

→ Time = t = 3.57 sec

→ Amplitude of displacement = 2.286 cm = 0.9 inches

sol:

part a) $u_1 = 7.742''$

After j=7 , $u_{j+1} = u_{7+1} = u_8 = 0.9''$

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$7 = \frac{1}{2\times 3.14\zeta} \ln\left[\frac{7.742}{0.9}\right]$$

$$7 = 0.343/\zeta$$

$$\zeta = 0.049 = 4.9\%$$

part b: Natural period of un-damped vibration

7 cycles in 3.57 seconds

Time required to complete one cycle = $\frac{3.57}{7}$

$T_D = 0.51$ sec

NOW

$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$

$\frac{2\pi}{\omega_D} = 2\pi / \omega_n \sqrt{1-\zeta^2}$

$T_D = \frac{T_n}{\sqrt{1-\zeta^2}}$

$T_n = T_D \times \sqrt{1-\zeta^2}$

$T_n = 0.51 \times \sqrt{1-(0.049)^2}$

$T_n = 0.5093 = 0.51$ sec

Part C: Stiffness of structure (k):

$$k = \frac{60 \cos 60^\circ}{7.742}$$

$= 3.875$ k/in

$k = 46500$ lb/ft

Part D: weight of tank, w:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{\left(\frac{w}{g}\right)}} = \sqrt{\frac{k \times g}{w}}$$

$$\omega_n^2 = \frac{kg}{w}$$

Also $\omega_n = 2\pi / T_n$

$$w = kg / \frac{4\pi^2}{T_n^2}$$

$$w = \frac{kg\, T_n^2}{4\pi^2}$$

$$w = \frac{(46500)(32.2)(0.51)^2}{4(3.14)^2}$$

$= 9874.84$ lb $= 9.875$ k

Part E: Dumping coefficent (C):

we know that $\zeta = \frac{C}{2m\omega_n}$

$$C = \zeta \times 2m\omega_n$$

$$C = \zeta \times 2m(2\pi/T_n)$$

$$C = 0.049 \times 2\left(\frac{9874.84}{32.2}\right)\left(\frac{2 \times 3.14}{0.51}\right)$$

$$C = 370.075 \text{ lb.sec/ft}$$

part F: number of cycle to reduce the displacement amplitude to 0.5":

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2 \times 3.14 \times 0.049} \ln\left[\frac{7.742}{0.9}\right]$$

$$j = 6.99 \approx 7 \text{ cycles.}$$