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Degree — B.S (computer science)

Paper — Calculus

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QUESTION => 1

Part => 'A'

$$\frac{d}{dx}\left(\frac{3x^4-2x^3+5}{x^3+1}\right)$$

Sol:-

$$\Rightarrow \frac{(x^3+1)\frac{d}{dx}(3x^4-2x^3+5)-(3x^4-2x^3+5)\frac{d}{dx}(x^3+1)}{(x^3+1)^2}$$

$$\Rightarrow \frac{(x^3+1)(12x^3-6x^2)-(3x^4-2x^3+5)(3x^2)}{(x^3+1)^2}$$

$$\Rightarrow \frac{(12x^6-6x^5+12x^3-6x^2)-(9x^6-6x^5+15x^2)}{(x^3+1)^2}$$

$$\Rightarrow \frac{12x^6-6x^5+12x^3-6x^2-9x^6+6x^5-15x^2}{(x^3+1)^2}$$

$$\Rightarrow \frac{3x^6+12x^3-21x^2}{x^6+3x^3} \quad \text{Ans}$$

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PART :- 'B' :-

$$\frac{(x^3+1)^2}{x^3-1}$$

Sol :- $\frac{d}{dx}\left[\frac{(x^3+1)^2}{x^3-1}\right]$

diff w.r.t 'x'

$$\Rightarrow \frac{(x^3-1)\frac{d}{dx}(x^3+1)^2 - (x^3+1)^2\frac{d}{dx}(x^3-1)}{(x^3-1)}$$

$$\Rightarrow \frac{(x^3-1)\ 2(x^3+1)(3x) - (x^3+1)(3x)}{(x^3-1)^2}$$

$$\Rightarrow \frac{6x(x^3-1)(x^3+1) - (x^3+1)^2(3x)}{(x^3-1)^2}$$

$$\Rightarrow 3x(x^3+1)\left[\frac{2(x^3-1)-(x^3+1)}{(x^3-1)^2}\right]$$

$$\Rightarrow \frac{3x(x^3+1)\left[(2x^3-2)-(x^3+1)\right]}{(x^3-1)^2}$$

$$\Rightarrow \frac{3x(x^3+1)\left[(2x^3-2-x^3-1)\right]}{(x^3-1)^2}$$

$$\Rightarrow \frac{3x(x^3+1)(x^3-3)}{x^6+2x^3-1}$$ Ans

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QUESTION – 2 :-

PART 'A' :- $\int \frac{1}{\sqrt{x^5}} dx$

Sol :- $\int \frac{1}{x^{5/2}} dx$

$\int \frac{1}{x^{\frac{5}{2}}} dx$

$\int x^{-\frac{5}{2}} dx$

$= \int x^{-\frac{5}{2}} dx$

$\Rightarrow \frac{x^{-\frac{5}{2}+1}}{-\frac{5}{2}+1} + C$

$-\frac{2}{3} \cdot x^{-\frac{3}{2}} + C$ Ans

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QUESTION - 3

PART A :- $\int \frac{-x+9}{2x^2-3x+6}\,dx$

$\Rightarrow 2x^2 - 3x + 6$

$2x^2 - 6x - 2x + 6$

$2x(x-3) - 2(x-3)$

$(x-3)(2x-2)$

let $\frac{-x+9}{(x-3)(2x-2)} = \frac{A}{(x-3)} + \frac{B}{(2x-2)}$

x by $(x-3)(2x-2)$ on b.s

$-x+9 = A(2x-2) + B(x-3)$

put in $x = 3$

$-3+9 = A(2(3)-2) + B(3-3)$

$6 = A(6-2) + B(0)$

$A = -2$

$x = \frac{2}{2}$ in (x)

$\Rightarrow \frac{-2}{2}+9 = A\left(\cancel{2}\left(\frac{2}{\cancel{2}}\right)-2\right) + B\left(\frac{2}{2}-3\right)$

$\Rightarrow \frac{-2+9}{2} = A(0) + B\,\frac{-4}{2}$

$\Rightarrow \frac{7}{2} = B\,\frac{-4}{2}$

$B = 21$ Ans

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Thus $\frac{x+9}{(x-1)(2x-3)}\,dx = \int \left(\frac{3}{x-3} + \frac{-12}{2x-3}\right) dx$

$$3\int \frac{1}{x-3}\,dx - \frac{12}{2}\int \frac{2}{2x-3}$$

$$3\ln(x-2) - \frac{12}{2}\ln(2x-3) + C$$

Ans.

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PART 'B' :-

$$\int \frac{4x^2 + 8x}{(x^2+1)(x^2+2x+3)}\, dx$$

Let

$$\frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2x+3}$$

$$4x^2 + 8x = (Ax+B)(x^2+2x+3) + (Cx+D)(x^2+1)$$

$$4x^2 + 8x = Ax^3 + 2Ax^2 + 3Ax + Bx^2 + 2Bx + 3B + Cx^3 + Cx + Dx^2 + D$$

Equating coffient

$x^3, x^2, x, 1$

$x^3 = 0$, $A + C \Rightarrow C = -A$ — (1)

$x^2 = 4$, $2A + B + D$ → (2)

$x = 6$, $3A + 2B + C$ → (3)

$1 = 0$, $3B + D$ → (4)

Sub 2 & 4

$$\begin{array}{r} 4 = 2A + B + D \\ -\,0 = \quad -3B + B \\ \hline 4 = 2A - 2B \end{array}$$

$2 = A - B$ → (5)

Now putting $c = -A$ in (3)

$$y = 3A + 2B$$

$$2 = A + B \quad \rightarrow (6)$$

add (5) & (6)

$$2 = A - B$$
$$\underline{2 = A + B}$$
$$4 = 2A$$

$$A = 2$$

put in (5)

$$2 = A - B$$
$$2 = 2 - B$$
$$B = 0$$

put $B = 0$ in (4)

$$0 = 0 + D$$
$$D = 0$$

$c = -2$ Ans

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Q4 :-

PART 'A' :-

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$X + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 5-3 & 1-(-1) \\ -3-2 & 1-2 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix} \text{ Ans}$$

PART 'B'

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -6 \\ 2 & 0 \end{bmatrix}$$

$$X - \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} -4 & -6 \\ 2 & 0 \end{bmatrix}$$

$$X - \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 3 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 3 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & 5 \\ 3 & 5 \end{bmatrix} \text{ Ans}$$

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PART 'C'

$$X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$X + 2I - X = -109 - X$$

$$X + 2I - X = -109 - X$$

$$2I = -109 - X$$

$$\frac{2I}{2} = \frac{-109}{2} - \frac{X}{2}$$

$$I = \frac{-109 - X}{2}$$ Ans

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QUESTION - 5

(a) $A=\begin{bmatrix}1 & 4\\ 2 & 1\end{bmatrix}$, $B=\begin{bmatrix}-3 & 2\\ 4 & 0\end{bmatrix}$ $C=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$

Find A^2+BC

Sol:-

$$A=\begin{bmatrix}1 & 4\\ 2 & 1\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 4\\ 2 & 1\end{bmatrix}\times\begin{bmatrix}1 & 4\\ 2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}(1\times1)+(4\times2) & (1\times4)+(4\times1)\\ (2\times1)+(1\times2) & (2\times4)+(1\times1)\end{bmatrix}$$

$$A^2=\begin{bmatrix}1+8 & 4+4\\ 2+2 & 8+1\end{bmatrix}$$

$$A^2=\begin{bmatrix}9 & 8\\ 4 & 9\end{bmatrix}$$

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Now

$$B \times C$$

$$B = \begin{bmatrix} -3 & 2 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$B \times C = \begin{bmatrix} (-3 \times 1) + (2 \times 0) & (-3 \times 0) + (2 \times 2) \\ (4 \times 1) + (0 \times 0) & (4 \times 0) + (0 \times 2) \end{bmatrix}$$

$$B \times C = \begin{bmatrix} -3 + 0 & 4 \\ 4 & 0 \end{bmatrix}$$

$$B \times C = \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

Now

$$A^2 + BC$$

$$A^2 \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} 9 + (-3) & 3+4 \\ 4+4 & 9+0 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$$

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Hence

$$AD + BC = \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$$ Ans.

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