Assignment

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Subject: Discrete Structure

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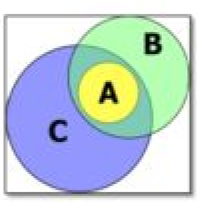
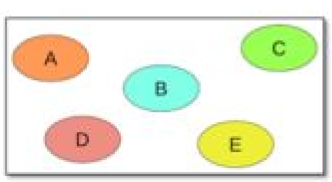
1) What is Venn diagram? Explain in detail the Application of Venn diagram.

ANSWER:

A Venn diagram is a diagrammatic representation of ALL the possible relationships between different sets of a finite number of elements. Venn diagrams were conceived around 1880 by John Venn, an English logician, and philosopher. They are extensively used to teach Set Theory. A Venn diagram is also known as **a Primary diagram, Set diagram or Logic diagram.**





Applications of VENN DIAGRAM:

- **Math:** Venn diagrams are commonly used in school to teach basic math concepts such as sets, unions and intersections. They're also used in advanced mathematics to solve complex problems and have been written about extensively in scholarly journals. Set theory is an entire branch of mathematics.
- **Statistics and probability:** Statistics experts use Venn diagrams to predict the likelihood of certain occurrences. This ties in with the field of predictive analytics. Different data sets can be compared to find degrees of commonality and differences.
- **Logic:** Venn diagrams are used to determine the validity of particular arguments and conclusions. In deductive reasoning, if the premises are true and the argument form is correct, then the conclusion must be true. For example, if all dogs are animals, and our pet Moro is a dog, then Moro has to be an animal. If we assign variables, then let's say dogs are C, animals are A, and Moro is B. In argument form, we say: All C is A. B is C. Therefore B is A. A related diagram in logic is called a Truth Table, which places the variables into columns to determine what is logically valid. Another related diagram is called the Randolph diagram, or R-Diagram, after mathematician John F. Randolph. It uses lines to define sets.
- **Linguistics:** Venn diagrams have been used to study the commonalities and differences among languages.
- **Teaching reading comprehension:** Teachers can use Venn diagrams to improve their students' reading comprehension. Students can draw diagrams to compare and contrast ideas they are reading about.
- **Computer science:** Programmers can use Venn diagrams to visualize computer languages and hierarchies.
- **Business:** Venn diagrams can be used to compare and contrast products, services, processes or pretty much anything that can

depicted in sets. And they're an effective communication tool to illustrate that comparison.

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2) What is Union? Draw Membership table for union using different examples.

ANSWER:

Union of Sets:

The **union** of two sets *A* and *B* is the set of elements, which are in *A* **or** in *B* **or** in both. It is denoted by $A \cup B$ and is read '*A* union *B*'
The following table gives some properties of Union of Sets: Commutative, Associative, Identity and Distributive. Scroll down the page for more examples.

Example :

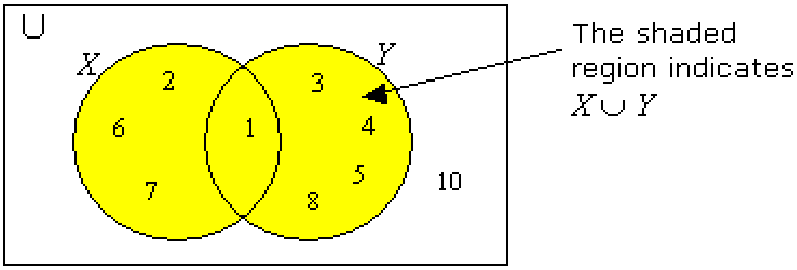
Given U = {1, 2, 3, 4, 5, 6, 7, 8, 10}

X = {1, 2, 6, 7} and *Y* = {1, 3, 4, 5, 8}

Find $X \cup Y$ and draw a Venn diagram to illustrate $X \cup Y$.

Solution:

$X \cup Y$ = {1, 2, 3, 4, 5, 6, 7, 8} ← 1 is written only once.



If *X* ⊂ *Y* then *X* ∪ *Y* = *Y*. We will illustrate this relationship in the following example.

Example:

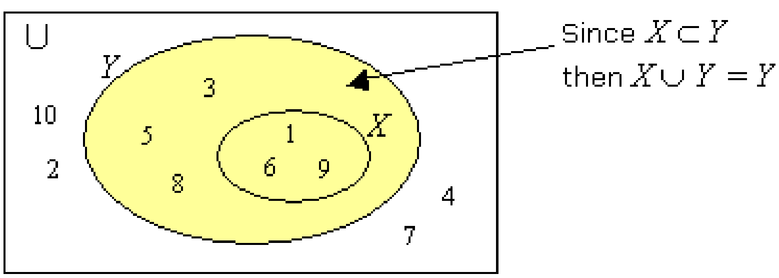
Given U = {1, 2, 3, 4, 5, 6, 7, 8, 10}

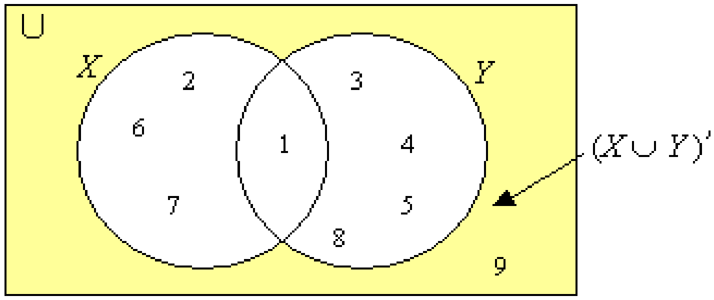
X = {1, 6, 9} and *Y* = {1, 3, 5, 6, 8, 9}

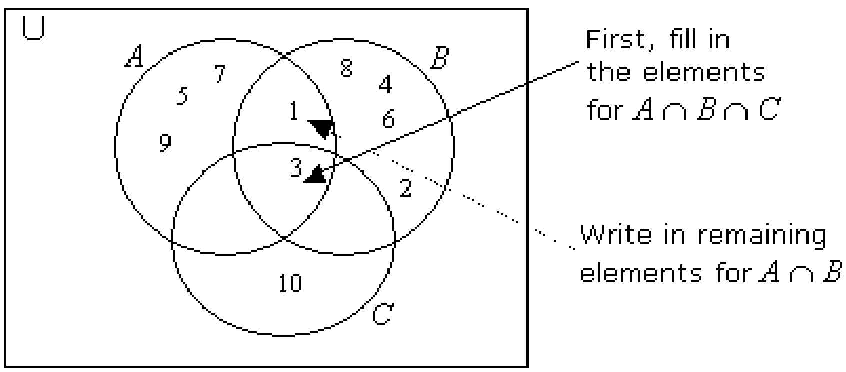
Find *X* ∪ *Y* and draw a Venn diagram to illustrate *X* ∪ *Y*.

Solution:

X ∪ *Y* = {1, 3, 5, 6, 8, 9}







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3) **What is Intersection? Draw Membership table for intersection using different examples.**

<u>ANSWER:</u>

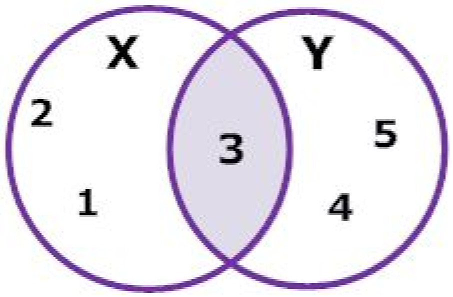
<u>Intersection:</u>

In previous lessons, we used Venn diagrams to represent relationships between sets. Let's look at the relationship of the sets described in example 1 below.

Example 1: Let *X* = {1, 2, 3} and Let *Y* = {3, 4, 5}. What elements do *X* and *Y* have in common?

Analysis: We will draw a Venn diagram of two overlapping circles. Elements that are common to both sets will be placed in the middle part, where the circles overlap.

Solution:



Explanation: The circle on the left represents set *X* and the circle on the right represents set *Y*. The shaded region in the middle is what they have in common. That is their intersection. The intersection of sets *X and Y* is 3.

The Venn Diagram in example 1 makes it easy to see that the number 3 is common to both sets. So the intersection of *X* and *Y* is 3, and this is written as:

***X* ∩ *Y* = {3}**

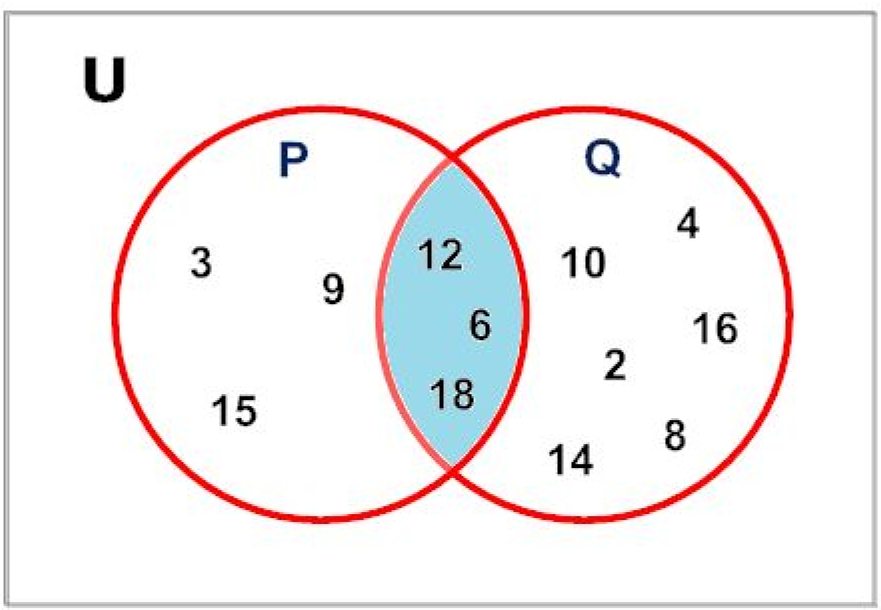
Definition:	The intersection of two sets, *X* and *Y,* is the set of elements that a common to both *X* and *Y*. It is denoted by ***X* ∩ *Y*,** and is read "*X i* *Y*".

So the intersection of two sets is the set of elements common to both sets. Let's look at some more examples of intersection.

Example 2: Let U = {counting numbers}, *P* = {multiples of 3 less than 20} and *Q* = {even numbers less than 20}. Draw and label a Venn diagram to show the intersection of *P* and *Q*.

Analysis: Start by filling in the elements in the intersection. Since *P* = {3, 6, 9, 12, 15, 18} and *Q* = {2, 4, 6, 8, 10, 12, 14, 16, 18}, we know that 6, 12 and 18 will be filled in first.

Solution:



Notation: ***P* ∩ *Q* = {6, 12, 18}**

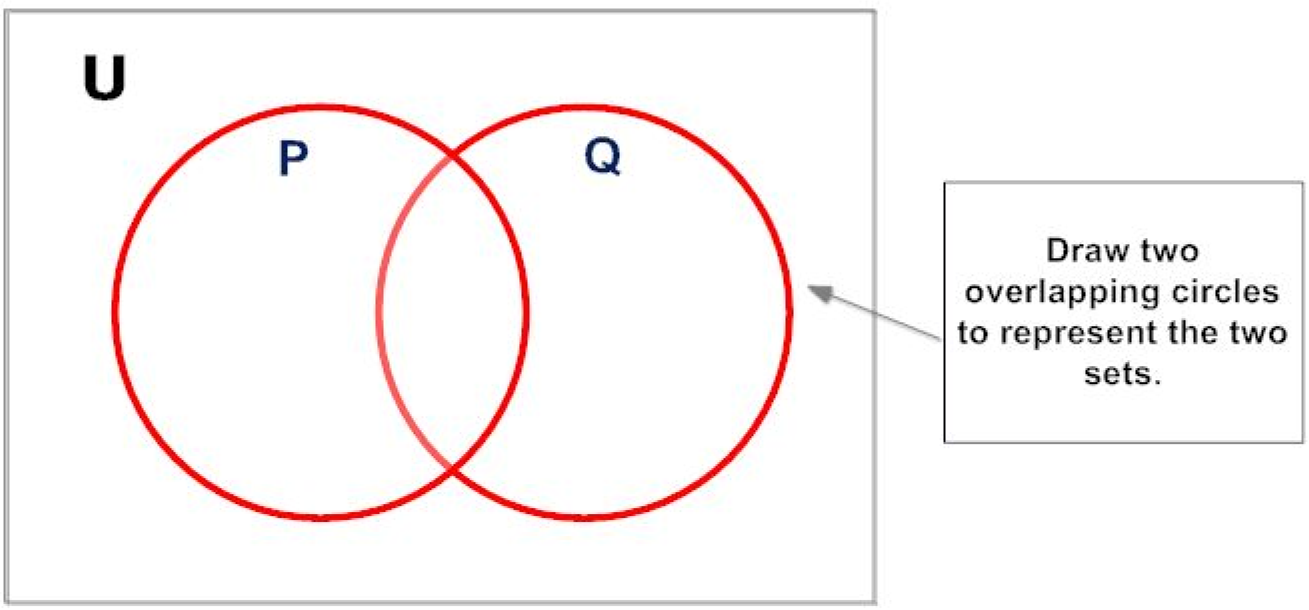
Another way to define the intersection of two sets is as follows:

A **∩** ***B*** **= { x | x ∈** ***A*** **and x ∈** ***B*** **}**

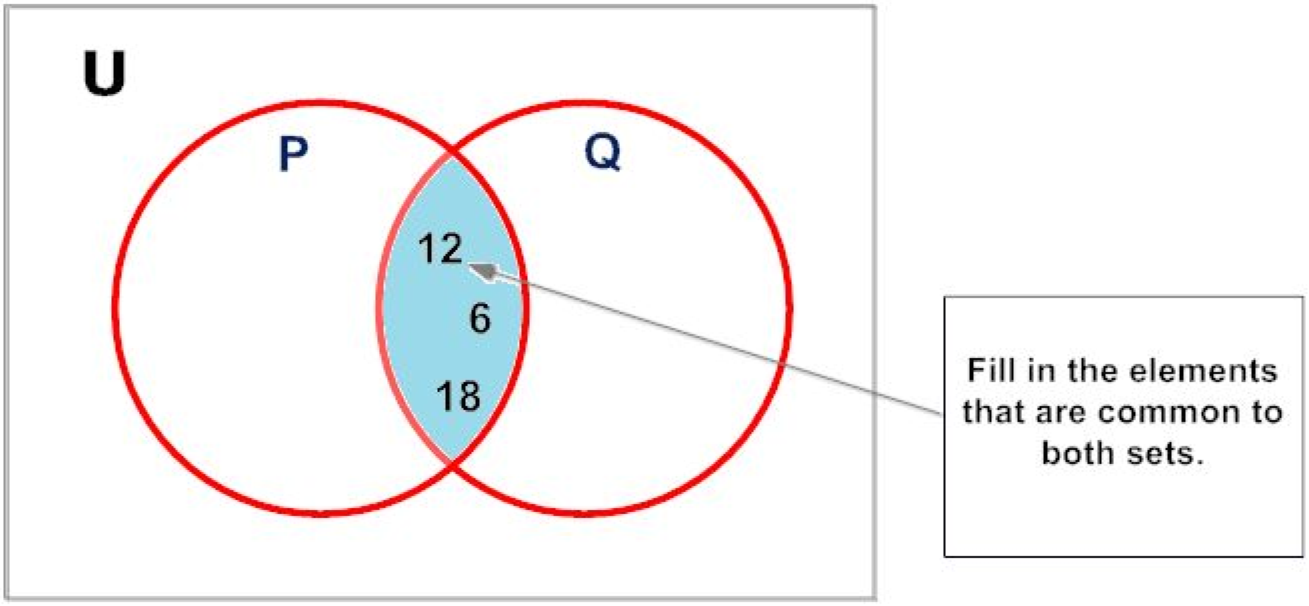
The procedure for drawing the intersection of two sets is shown below.

Procedure for Drawing the Intersection of Two Sets
Overlapping Sets

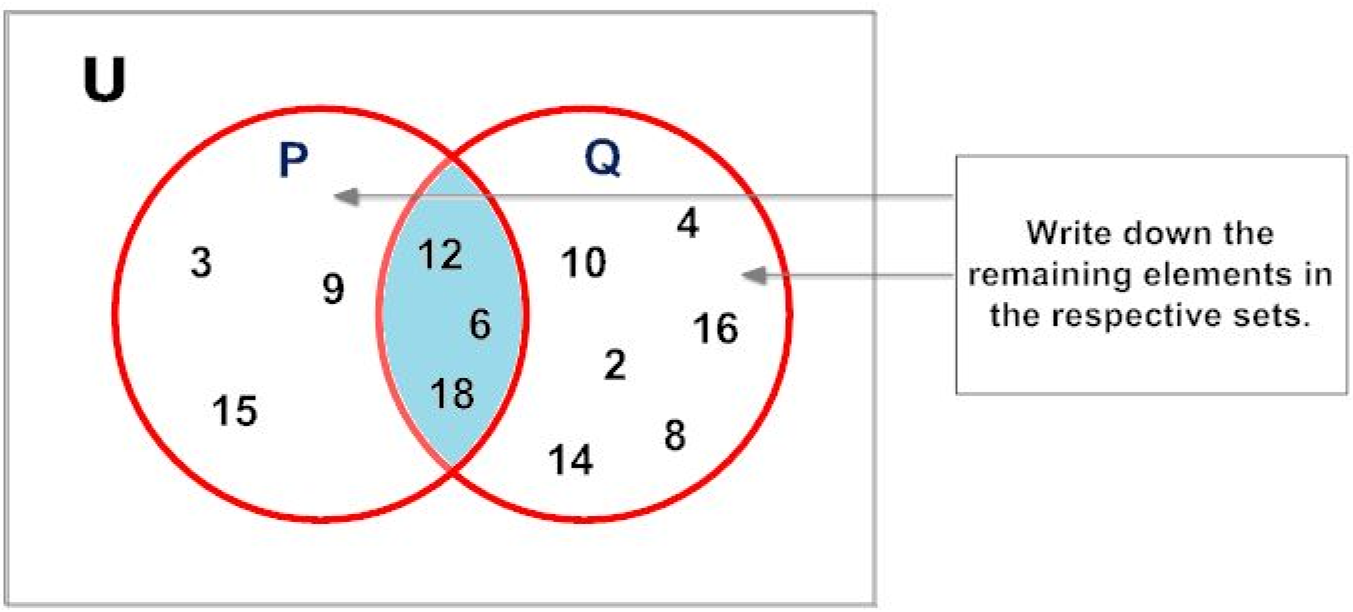
Step 1:



Step 2:



Step 3:

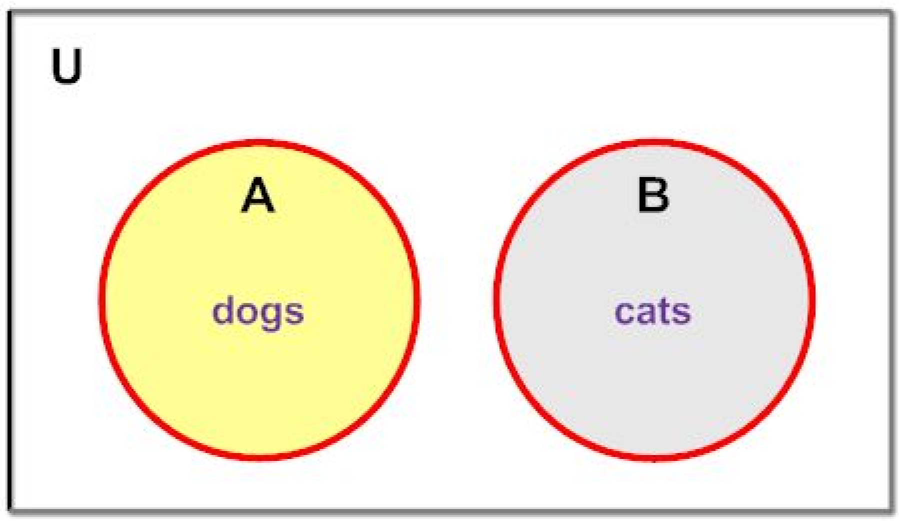


Let's look at the intersection of other types of sets. In example 3 below, **the given sets are not overlapping.**

Example 3: Let U = {animals}, *A* = {dogs} and *B* = {cats}. Draw and label a Venn diagram to show the intersection of *A* and *B*.

Analysis: Sets *A* and *B* do not overlap. These sets are disjoint, and have no elements in common.

Solution:



Notation: $\boldsymbol{A \cap B = \emptyset}$

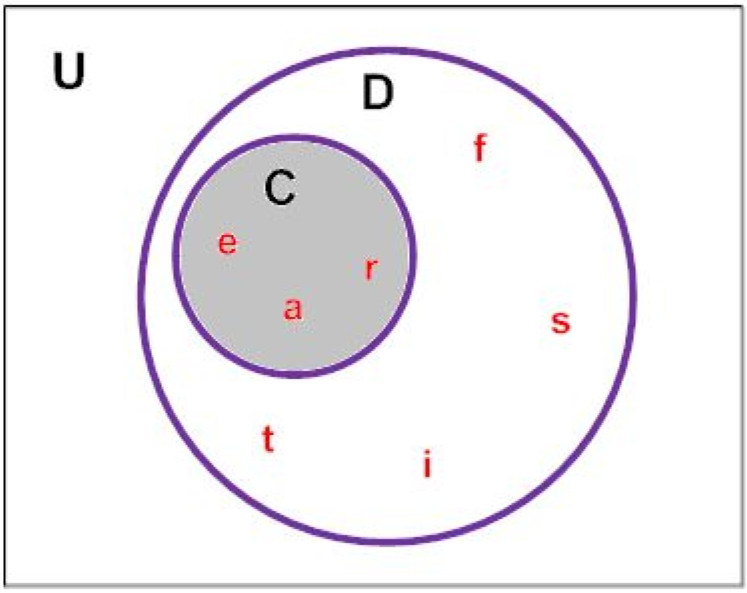
Two sets *A* and *B* are disjoint if their intersection is null. This is denoted by $\boldsymbol{A \cap B = \emptyset}$**, where Ø is the null or empty set.**

Recall that a **Universal Set** is the set of all elements under consideration, denoted by capital U. All other sets are subsets of the universal set. So in each example above, the circles are subsets of the Universal set. **We have examined the intersection of overlapping sets, and of disjoint sets. Let's look at the intersection of one set contained within another.**

Example 4: Let C = {a, r, e} and D = {f, a, i, r, e, s, t}. Draw and label a Venn diagram to show the intersection of sets C and D.

Analysis: C is a subset of D. Recall that this is denoted by C D.

Solution:



Explanation: It turns out that C ∩ D = {a, r, e}, which is equal to the set C.

In example 4, since *C* ⊂ *D , we get that C* ∩ *D = C. This relationship is defined below.*

If set A is a subset of set B, then the intersection of A and B is A:

If $A \subset B$, then $A \cap B = A$

The procedure for drawing the intersection of one set contained within another is shown below.

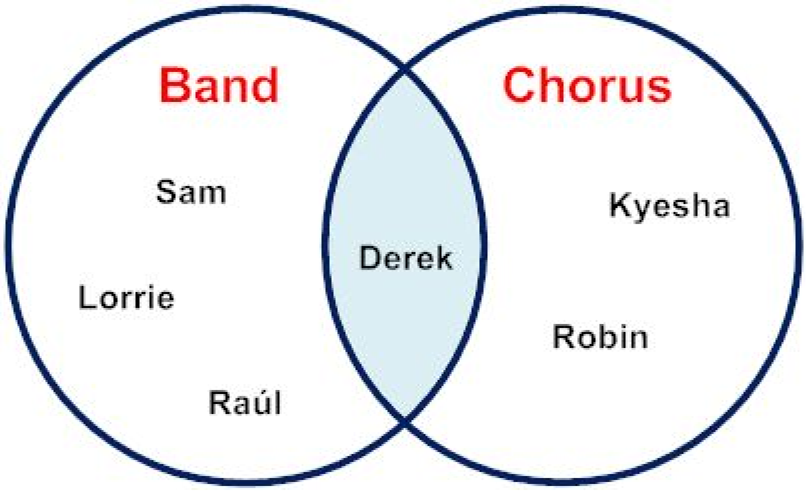
Procedure for Drawing the Intersection of One Set Contained Within An	
Step 1:	**Draw one circle within another circle.**
Step 2:	**Write down the elements in the inner circle.**
Step 3:	**Write down the remaining elements in the outer circle.**

Let's see if you can follow the challenge presented in Example 5.

Example 5: Given the Venn diagram below, name a member of Band that is not in both Band and Chorus.

Analysis: This problem is asking us to find a member of Band that is not in the intersection of Band and Chorus.

Solution:



Explanation: Sam, Lorrie and Raúl are each members of Band only. In addition, these students are not in Band and Chorus.

4) **What is Difference? Draw Membership table for Set difference using different examples.**

ANSWER:

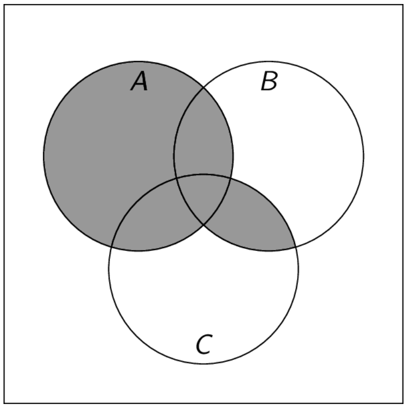
Membership Tables:

We combine sets in much the same way that we combined propositions. Asking if an element xx is in the resulting set is like asking if a proposition is true. Note that xx could be in any of the original sets.

What does the set A∪(B∩C)A∪(B∩C) look like? We use 11 to denote the presence of some element xx and 00 to denote its absence.

A11110000B11001100C10101010B∩C10001000A∪(B∩C)11111 000ABCB∩CA∪(B∩C)111111001101011000101111010000010 000000

This is a **membership table**. It can be used to draw the Venn diagram by shading in all regions that have a 11 in the final column. The regions are defined by the left-most columns.



We can also use membership tables to test if two sets are equal. Here are two methods of showing if A∩B‾‾‾‾‾‾‾‾‾=A‾‾‾∪B‾‾‾A∩B¯=A¯∪B¯:

- Showing each side is a subset of the other:

  x∈A∩B‾‾‾‾‾‾‾‾‾→→→→→→→x∉A∩B¬(x∈A∩B)¬(x∈A∧x∈B)¬(x∈A)∨¬(x∈B)x∉A∨x∉Bx∈A‾‾‾∨x∈B‾‾‾x∈A‾‾‾∪B‾‾‾x∈A∩B¯→x∉A∩B→¬(x∈A∩B)→¬(x∈A∧x∈B)→¬(x∈A)∨¬(x∈B)→x∉A∨x∉B→x∈A¯∨x∈B¯→x∈A¯∪B¯

  x∈A‾‾‾∪B‾‾‾→→→→→→x∉A∨x∉B¬(x∈A)∨¬(x∈B)¬(x∈A∧x∈B)¬(x∈A∩B)x∉A∩Bx∈A∩B‾‾‾‾‾‾‾‾‾x∈A¯∪B¯→x∉A∨x∉B→¬(x∈A)∨¬(x∈B)→¬(x∈A∧x∈B)→¬(x∈A∩B)→x∉A∩B→x∈A∩B¯

- Using membership tables:

  A11110000B11001100C10101010A∩B11000000A∩B‾‾‾‾‾‾‾‾‾00111111A‾‾‾00001111B‾‾‾00110011A‾‾‾∪B‾‾‾00111111ABCA∩BA∩B¯A¯B¯A¯∪B¯111100001101000010101011100010110110101001101001011110000 1111

Since the columns corresponding to the two sets match, they are equal.

It is **not sufficient** to simply draw the Venn diagrams for two sets to show that they are equal: you need to show why your Venn diagram is correct (typically with a membership table).

There is an additional way to prove two sets are equal, and that is to use **set identities**. In the following list, assume AA and BB are sets drawn from a universe UU.

- Identity Law: $A\cup\varnothing=AA\cup\varnothing=A$, $A\cap U=AA\cap U=A$
- Idempotent Law: $A\cup A=AA\cup A=A$, $A\cap A=AA\cap A=A$
- Domination Law: $A\cup U=UA\cup U=U$, $A\cap\varnothing=\varnothing A\cap\varnothing=\varnothing$
- Complementation Law: $\overline{\overline{A}}=A\bar{\bar{A}}=A$
- Commutative Law: $A\cup B=B\cup AA\cup B=B\cup A$, $A\cap B=B\cap AA\cap B=B\cap A$
- Associative Law: $A\cup(B\cup C)=(A\cup B)\cup CA\cup(B\cup C)=(A\cup B)\cup C$, $A\cap(B\cap C)=(A\cap B)\cap CA\cap(B\cap C)=(A\cap B)\cap C$
- Distributive Law: $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$, $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$
- Absorption Law: $A\cup(A\cap B)=AA\cup(A\cap B)=A$ and $A\cap(A\cup B)=AA\cap(A\cup B)=A$
- De Morgan's Law: $\overline{A\cap B}=\overline{A}\cup\overline{B}\bar{A\cap B}=\bar{A}\cup\bar{B}$, $\overline{A\cup B}=\overline{A}\cap\overline{B}\bar{A\cup B}=\bar{A}\cap\bar{B}$
- Complement Law: $A\cup\overline{A}=UA\cup\bar{A}=U$, $A\cap\overline{A}=\varnothing A\cap\bar{A}=\varnothing$
- Difference Equivalence: $A\setminus B=A\cap\overline{B}A\setminus B=A\cap\bar{B}$

Note the similarities to logical equivalences! Here are some examples of how to determine if two sets are equal:

- Is $(A \setminus C) \cap (B \setminus C)$(A∖C)∩(B∖C) equal to $(A \cap B) \cap \overline{C}$(A∩B)∩C¯? First, we can use a membership table:

  A11110000B11001100C10101010A∖C01000000B∖C01000100(A∖C)∩(B∖C)01000000A∩B11000000C‾01010101(A∩B)∩C‾01000000ABCA∖CB∖C(A∖C)∩(B∖C)A∩BC¯(A∩B)∩C¯11100010011011111110100000010000001001100000001001001000100000000000010

  Since the columns corresponding to the two sets match, they are equal. We can also use set identities:

  (A∖C)∩(B∖C)===(A∩C‾)∩(B∩C‾)(A∩B)∩(C‾∩C‾)(A∩B)∩C‾Difference EquivalenceAssociative LawIdempotent Law(A∖C)∩(B∖C)=(A∩C¯)∩(B∩C¯)Difference Equivalence=(A∩B)∩(C¯∩C¯)Associative Law=(A∩B)∩C¯Idempotent Law

- Is $(A \setminus C) \cap (C \setminus B)$(A∖C)∩(C∖B) equal to $A \setminus B$A∖B? Let's use some set identities:

  (A∖C)∩(C∖B)====(A∩C‾)∩(C∩B‾)(A∩B‾)∩(C∩C‾)(A∩B)∩∅∅Difference EquivalenceAssociative LawComplement LawDomination Law(A∖C)∩(C∖B)=(A∩C¯)∩(C∩B¯)Difference Equivalence=(A∩B¯)∩(C∩C¯)Associative Law=(A∩B)∩∅Complement Law=∅Domination Law

Note that, in general, $A \setminus B \neq \emptyset A \setminus B \neq \emptyset$ (\eg, let $A=\{1,2\},B=\{1\}A=\{1,2\},B=\{1\}$). Therefore, these sets are not equal. (Note the similarity to finding truth settings that invalidate an argument!)
