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SECTION :- A

SEMESTER :- 4TH BS(SE)

INTRUCTOR :- DAUD KHAN

PAPER : PROBABILITY AND STATISTICS

Q 1) GIVEN DATA:-

Sum = even
Sum > 8
Same outcome

SOLUTION:-

Two dices are thrown so outcomes :-

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (1, 7)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6) (2, 7)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6) (3, 7)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6) (4, 7)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6) (5, 7)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6) (6, 7)
(7, 1) (7, 2) (7, 3) (7, 4) (7, 5) (7, 6) (7, 7)

$n(S) = 36$

Probability of getting more than 8 = (3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4) (6,5), (6,6)

$$n(E) = 10$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{10}{36} = 5/18$$

$$\boxed{P(E) = 5/18}$$

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Q2) Sum of 2 has 1 way 1,1
Sum of 3 has 2 ways 1,2 and 2,1

Sum of 4 has 1,3, 2,2 ; 3,1
5 has 4 ways
6 has 5 ways
8 has 5 ways
9 has 4 ways
10 has 2 ways
⇒ Those or 15/36 for each side with a sum of 30/36
⇒ That leaves $6/36 = 1/6$ probability for a sum of 7.

Two dices are thrown so outcomes :-
(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)

(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

$$\boxed{n(S) = 36}$$

probability of getting sum more then 8 = (3,6), (4,5)

(4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)

$$n(E) = 10$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

$$\boxed{P(E) = 5/18}$$

Q3) a) $P(A \text{ wins} / A \text{ wins first}) = P(x=2) + P(x=3) + P(x=4)$

$$= \binom{4}{2} p^2 (1-p)^{4-2} + \binom{4}{3} p^3 (1-p)^{4-3} + \binom{4}{4} p^4 (1-p)^{4-4}$$

$$\boxed{= \sum_{i=2}^{4} \binom{4}{i} p^i (1-p)^{4-i}}$$

b) $P(\text{A wins first game} / \text{A wins}) = \dfrac{P(\text{A wins}/\text{A wins } 1^{st})\, P(\text{A win } 1^{st})}{P(\text{A win})}$

$$= \frac{\binom{4}{2}p^2(1-p)^{4-2} + \binom{4}{3}p^3(1-p)^{4-3} + \binom{4}{4}p^4(1-p)^{4-4}}{\binom{4}{i}p^3(1-p)^{4-3} + \binom{4}{4}p^4(1-p)^{4-4} + \binom{4}{5}p^5(1-p)^{4-5}}$$

$$= \frac{\sum \binom{4}{i} p^i (1-p)^{4-i}}{\sum \binom{5}{i} p^i (1-p)^{5-i}}$$

Q5) BINOMIAL DISTRIBUTION :-

DERIVATION :-

$$\mu = \sum_{k=0}^{n} k \binom{n}{k} p^{k} (1-p)^{n-k}$$

$$= np \sum_{k=0}^{n} k \frac{(n-1)!}{(n-k)!\,k!} p^{k-1} (1-p)^{(n-1)-(k-1)}$$

$$= np \sum_{k=1}^{n} \frac{(n-1)!}{(n-1)-(k-1)!\,(k-1)!} p^{k-1} (1-p)^{(n-1)-(k-1)}$$

$$= np \sum_{k=1}^{n} \binom{n-1}{k-1} p^{k-1} (1-p)^{(n-1)-(k-1)}$$

$$= np \sum_{l=0}^{n-1} \binom{n-1}{l} p^{l} (1-p)^{(n-1)-l}$$

$$= np \sum_{l=0}^{m} \binom{m}{l} p^{l} (1-p)^{m-l}$$

$$= np\,(p + (1-p)^{m}$$

$$= np$$

MEAN OF BINOMIAL DISTRIBUTION:-

$$= p \sum_{x=1}^{n} x \cdot \left[\frac{n}{x}\right] \left[\begin{matrix} n-1 \\ x-1 \end{matrix}\right] p^{x-1} q^{n-x}$$

$$= np(q+p)^{n-1} \quad (\text{since } p+q=1)$$

$$= np$$

$$E(x) = np$$

$\therefore$ The mean of binomial distribution is np.

VARIANCE OF BINOMIAL DISTRIBUTION:-

$$Var(x) = E(x^2) - E(x)^2$$

Here $E(x^2) = \sum_{x=0}^{n} x^2 \left[\begin{matrix} n \\ x \end{matrix}\right] p^x q^{n-1}$

$$\sum_{x=0}^{n} \{x(x-1)+x\} \left[\begin{matrix} n \\ x \end{matrix}\right] p^x q^{n-x}$$

$$\sum_{x=0}^{n} \{x(x-1)\} \left[\begin{matrix} n \\ x \end{matrix}\right] p^x q^{n-x} +$$

$$\sum x \left[\begin{matrix} n \\ x \end{matrix}\right] p^x q^{n-x}$$

MEAN OF BINOMIAL DISTRIBUTION:-

$$= p\sum_{x=1}^{n} x \cdot \left[\frac{n}{x}\right]\left[\begin{matrix} n-1 \\ x-1 \end{matrix}\right] p^{x-1} q^{n-x}$$

$$= np(q+p)^{n-1} \quad (\text{since } p+q=1)$$

$$= np$$

$$E(x) = np$$

$\therefore$ The mean of binomial distribution is np.

VARIANCE OF BINOMIAL DISTRIBUTION:-

$$Var(x) = E(x^2) - E(x)^2$$

Here $E(x^2) = \sum_{x=0}^{n} x^2 \left[\begin{matrix} n \\ x \end{matrix}\right] p^x q^{n-1}$

$$\sum_{x=0}^{n} \{x(x-1)+x\} \left[\begin{matrix} n \\ x \end{matrix}\right] p^x q^{n-x}$$

$$\sum_{x=0}^{n} \{x(x-1)\} \left[\begin{matrix} n \\ x \end{matrix}\right] p^x q^{n-x} +$$

$$\sum x \left[\begin{matrix} n \\ x \end{matrix}\right] p^x q^{n-x}$$

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$$\sum_{x=2}^{n} \{x(x-1)\}\left[\frac{n(n-1)}{x(x-1)}\right]\binom{n-2}{x-2} p^{x-2} q^{n-x}\}$$

$$+ \sum x \binom{n}{x} p^x q^{n-x}$$

$$= n(n-1)p^2 \{\sum \binom{n-2}{x-2} p^{x-2} q^{n-x}\}$$

$$+ np$$

$$= n(n-1)p^2 (q+p)(n-2) + np$$

$$= n(n-1)p^2 + np$$

Q6) <u>BINOMIAL DISTRIBUTION :-</u>

<u>DEFINITION :-</u>

Binomial distribution allows us to define the probability of observing a specific combinition of items which is derived from the fundemental formulas for the combinitions.

BINOMIAL DISTRIBUTION FORMULA:-

MATHEMATICALLY:-

$$P(x) = \binom{n}{x} p^x q^{n-1} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

where

n = the number of trials

x = the number of successes desired

p = probability of getting a success in one trial

q = $1-p$ = the probability of getting a failure in one trial

P = binomial parameter

c) p' = observed or sample binomial parameter

BINOMIAL FREQUENCY DISTRIBUTION:-

DEFINITION:-

If the binomial probability distribution is multiplied by N, the number of experiments or sets the resulting distribution is known as the binomial prequency distribution.

FORMULA :-

$$N\binom{n}{x}p^{x}q^{n-x}$$

EXAMPLE :-

Six dice are thrown 729 times. How many times do we expect at least three dice to show a 5 or a 6.

SOLUTION:-

The probality of getting a 5 or a 6 with one dice is $p = 2/6$

Since 6 dice are thrown and there are 729 sets so

$N = 729$

$p = 2/6$

$q = 1 - p$

$= 1 - 2/6$

$= 4/6$

$= 2/3$

So by using formula

$$729\left(\frac{2}{3} + \frac{1}{3}\right)^6$$

$$729\left[\sum_{x=3}^{6}\binom{6}{x}\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{6-x}\right]$$

$$= 729\left[\binom{6}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3 + \binom{6}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2 + \binom{6}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^1 + \binom{6}{6}\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^0\right]$$

$= 233$ Ans

7) COEFFICIANT OF VARIATION :-

Formula :-

$$CV = \frac{\text{standard deviation}}{\text{Mean}}$$

Set A :-

Mean = 45

SD = 3

Sample size = 1500

$$CV = \frac{3}{45} = 0.066$$

Set B :-

Mean = 60

SD = 11

Sample size = 3200

$$CV = \frac{11}{60} = 0.183$$

Set C:

Mean = 25

SD = 5

$$CV = \frac{5}{25} = 0.2$$

Set D:-

Mean = 25

SD = 15

$$CV = \frac{15}{25}$$

$$= 0.6$$