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Date: Umair

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ANSWER # 01:

$4y'' - 20y' + 25y = 0$

Solution:-

$4y'' - 20y' + 25y = 0$

$\Rightarrow \frac{4y''}{4} - \frac{20y'}{4} + \frac{25y}{4} = 0$

$\Rightarrow y'' - 5y' + \frac{25}{4} y = 0$

$\Rightarrow a = -5, \; b = 25/4$

$\Rightarrow \lambda^2 - 5\lambda + 25/4 = 0$

$\boxed{a^2 + b^2 - 2ab = 0}$: Using formula

$\Rightarrow \lambda^2 - 5\lambda + \left(\frac{5}{2}\right)^2 = 0$

$\Rightarrow \left(\lambda - \frac{5}{2}\right)^2 = 0$

$\Rightarrow \left(\lambda - \frac{5}{2}\right)\left(\lambda - \frac{5}{2}\right) = 0$

$\Rightarrow \lambda_1 = \frac{5}{4}, \quad \lambda_2 = \frac{5}{4}$

Same real roots

So

$y = (C_1 + C_2 x)\, e^{5/4\, x}.$

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ANSWER # 02 (a)

$y'' + 2y' + y = 0 \quad y(0) = 4, \; y'(0) = -6.$

Solution:-

$y'' + 2y' + y = 0$

$\lambda^2 + 2\lambda + 1 = 0$

$\lambda^2 + \lambda + \lambda + 1 = 0$

$\lambda(\lambda + 1) + 1(\lambda + 1) = 0$

$(\lambda + 1)(\lambda + 1) = 0$

$\lambda = -1, \quad \lambda = -1$

Roots are real & equal.

$y = c_2 e^{-x} + c_2 x e^{-x}$

$y' = c_1 e^{-x} + c_2 e^{-x} - x e^{-x}$

When $y = 4, x = 0$

$y = c_1 e^{-x} + c_2 x e^{-x}$

$y' = c_1 e^{-x} + c_2 e^{-x} - x e^{-x}$

when $y = 4, x = 0$

$y = c_1 e^{x} \pm c_2 x e^{-x}$

$4 = c_1 e^{-2} + c_2 e^{-2}$

$\boxed{4 = c_1} \rightarrow (1)$

when $x = 0, y = -6$

$-6 = c_2 e^{0} + c_2 e^{-e} - 0 e^{0}$

$\boxed{-6 = c_1 + c_2} \rightarrow (2)$

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Adding eq 1 & and eq 2

$$4 = \cancel{c_1}$$

$$\mp 6 = \pm \cancel{c_1} \pm c_2$$

$$10 = -c_2$$

$$\boxed{c_2 = -10}$$

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AWSWER #02 part (b)

$$x^2y'' + 3xy' + y = 0$$

Solution:-

$$x^2y'' + 3xy' + y = 0$$

$$m^2 + (a-1)m + b = 0$$

$$m^2 + (3-1)m + 1 = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$(m+1)(m+1) = 0$$

$$m = -1, \quad m = -1$$

The roots are real and equal

So

$$y = (C_1 + C_2 \ln x)x^m$$

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ANSWER #03

$y'' + y' - 6y = 6x^3 - 3x^2 + 12x$

$y'' + y' - 6y = 0$

Auxilary Eqn.

$\lambda^2 + \lambda + 6y = 0$

$\lambda^2 + \lambda - 6 = 0$

$\lambda^2 + 3\lambda - 2\lambda - 6 = 0$

$\lambda(\lambda + 3) - 2(\lambda + 3) = 0$

$\lambda + 3 = 0, \lambda - 2 = 0$

$\lambda = -3 \quad , \quad \lambda = 2$

Roots are real & distinct.

$y = c_1 e^{-3x} + c_2 e^{2x}$

chose y_0

$y_1 = k_3 x^3 + k_2 x^2 + k_1 x + k_0$

$y'_p = 3k_3 x^2 + 2k_2 x + k_1$

$y''_p = 6k_3 x - 2k_2$ put in eq (1)

$6k_3 x - 2k_2 + 3k_3 x^2 + 2k_2 x + k_1 - 6k_3 x^3 - 6k_2 x^3 - 6k_1 x - 6k_0 =$
$= 6x^3 - 3x^2 + 12x$

$-6k_3 = 6 \quad , \quad -6k_2 + 3k_1 = -3 \quad , \quad 6k_3 + 2k_2 + k = 12$

$k_3 = -1 \quad , \quad -6k_2 + 3(-1) = -3 \quad , \quad 6(-1) + 2(0) + k_3 = 12$

$, \quad -6k_2 - 3 = -3 \quad , \quad -6 + k_1 = 12$

$, \quad -6k = -3 + 3 \qquad k_1 = 18$

$-6k_2 = 0$

$\boxed{k_2 = 0}$

$-2k_2 + k_1 + k_0 = 0$

$-2(0) - 2 + k_0 = 0$

$\boxed{k_0 = 2}$

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$y'' - 4y + 4y' = x^2 e^{2x}$

Solution:-

$y'' - 4y + 4y' = x^2 e^2 x$

form eq

$y'' - 4y' + 4y = 0$

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$

$\lambda(\lambda - 2) - 2(\lambda - 2) = 0$

$(\lambda - 2)(\lambda - 2) = 0$

Roots are real & equal

$y = (c_1 + c_2 x) e^{2x}$

$y_1 = c_1 e^{2x} + c_2 x e^{2x}$

$y_1 = e^{2x}, \quad y_2 = x e^{2x}$

$y_1' = 2e^{2x}, \quad y' = e^{2x} + 2e x^{2x}$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$W = \begin{vmatrix} e^{2x} & x e^{2x} \\ 2e^{2x} & e^{2x} + 2e x^{2x} \end{vmatrix}$$

$w = e^{4x} + \cancel{2xe^{4x}} - \cancel{2xe^{4x}}$

$\boxed{w = e^{4x}}$

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$$y_p = -y_1 \int \frac{y_2 r(x)}{w} + y_2 \int \frac{y_1 r(x)}{w}$$

$$y_p = -e^{-3x} \int \frac{x e^{3x} \cdot x^2 e^{3x}}{e^{4x}} dx + x e^{2x} \int \frac{e^{2x} x^2 e^{2x}}{e^{2x}} dx$$

$$y_p = -e^{2x} \int \frac{x^3 \cancel{e^{4x}}}{\cancel{e^{4x}}} + x e^{2x} \int x^2 \frac{\cancel{e^{4x}}}{\cancel{e^{4x}}} dx$$

$$y_p = -e^{2x} \int x^2 dx + x e^{3x} \int x^2 dx$$

$$y_p = -e^{2x} \cdot \frac{x^4}{4} + x e^{2x} \cdot \frac{x^2}{3}$$

So

$$y = y_h + y_p$$

$$y = 4e^{2x} + 12xe^{2x} - e^{2x} \frac{x^4}{4} + x e^{4x} \frac{x^3}{3}$$

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ANSWER #05

Ordinary Differential Equation (ODEs)
Homogeneous Linear ODEs with constant co-efficients

$$y'' + ay' + by = 0$$

The characteristic equation

$$\lambda^2 + a\lambda + b = 0$$

It is a quadratic Equation
It has two roots λ_1 & λ_2

Case 1:- If the roots λ_1 & λ_2 are real and distinct
Then the solution is
$y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$.