

Date: 5/6/04
Umanir

(1)

ANSWER #01:

$$4y'' - 20y' + 25y = 0$$

Solution:-

$$4y'' - 20y' + 25y = 0$$

$$\Rightarrow \frac{4y''}{4} - \frac{20y'}{4} + \frac{25y}{4} = 0$$

$$\Rightarrow y'' - 5y' + \frac{25}{4}y = 0$$

$$\Rightarrow a = -5, b = \frac{25}{4}$$

$$\Rightarrow \lambda^2 - 5\lambda + \frac{25}{4} = 0$$

$a^2 + b^2 - 2ab = 0$ Using formula:

$$\Rightarrow \lambda^2 - 5\lambda + \left(\frac{5}{2}\right)^2 = 0$$

$$\Rightarrow \left(\lambda - \frac{5}{2}\right)^2 = 0$$

$$\Rightarrow \left(\lambda - \frac{5}{2}\right) \cdot \left(\lambda - \frac{5}{2}\right) = 0$$

$$\Rightarrow \lambda_1 = \frac{5}{2}, \lambda_2 = \frac{5}{2}$$

Same real roots

so

$$y = (C_1 + C_2 x) e^{5/4 x}$$

~~$y = C_1 e^{5/4 x} + C_2 x e^{5/4 x}$~~

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Umar

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ANSWER #02 (a)

$$y'' + 2y' + y = 0 \quad y(0) = 4, \quad y'(0) = -6$$

Solution:-

$$y'' + 2y' + y = 0$$

$$\lambda^2 + 2\lambda + 1 = 0$$

$$\lambda^2 + \lambda + \lambda + 1 = 0$$

$$\lambda(\lambda+1) + 1(\lambda+1) = 0$$

$$(\lambda+1)(\lambda+1) = 0$$

$$\lambda = -1, \quad \lambda = -1$$

Roots are real & equal:

$$y = C_1 e^{-x} + C_2 x e^{-x}$$

$$y' = C_1 e^{-x} + C_2 e^{-x} - x e^{-x}$$

When $y = 4, x = 0$

$$y = C_1 e^{-x} + C_2 x e^{-x}$$

$$y' = C_1 e^{-x} + C_2 e^{-x} - x e^{-x}$$

when $y = 4, x = 0$

$$y = C_1 e^{-x} + C_2 x e^{-x}$$

$$4 = C_1 e^{-2} + C_2 e^{-2}$$

$$\boxed{4 = C_1 + C_2} \rightarrow (1)$$

when $x = 0, y = -6$

$$-6 = C_1 e^0 + C_2 e^{-e} - 0 e^{0}$$

$$\boxed{-6 = C_1 + C_2} \rightarrow (2)$$

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Amount

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Date

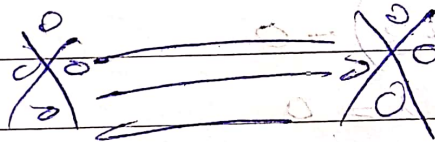
Adding eq (1) and eq (2)

$$4 = C_1$$

$$7b = +C_1 + C_2$$

$$10 = -C_2$$

$$\boxed{C_2 = -10}$$



ANSWER-Hod part (b)

$$x^2 y'' + 3xy' + y = 0$$

Solution:-

$$x^2 y'' + 3xy' + y = 0$$

$$m^2 + (a-1)m + b = 0$$

$$m^2 + (3-1)m + 1 = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

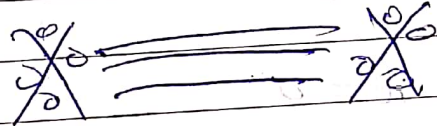
$$(m+1)(m+1) = 0$$

$$m = -1, \quad m = -1$$

The roots are real and equal

So

$$y = (C_1 + C_2 \ln x) x^{-1}$$





ANSWER #03

$$y'' + y' - 6y = 6x^3 - 3x^2 + 12x$$

$$y'' + y' - 6y = 0$$

Auxiliary Eqn.

$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda^2 + 3\lambda - 2\lambda - 6 = 0$$

$$\lambda(\lambda + 3) - 2(\lambda + 3) = 0$$

$$\lambda + 3 = 0, \lambda - 2 = 0$$

$$\lambda = -3, \lambda = 2$$

Roots are real & distinct.

$$y = c_1 e^{-3x} + c_2 e^{2x}$$

choose y_p

$$y_1 = k_3 x^3 + k_2 x^2 + k_1 x + k_0$$

$$y_1' = 3k_3 x^2 + 2k_2 x + k_1$$

$$y_1' = 6k_3 x - 2k_2$$

put in eq (1)

$$6k_3 x - 2k_2 + 3k_3 x^2 + 2k_2 x + k_1 - 6k_3 x^3 - 6k_2 x^2 - 6k_1 x - 6k_0 = 6x^3 - 3x^2 + 12x$$

$$-6k_3 = 6, \quad -6k_2 + 3k_1 = -3, \quad 6k_3 + 2k_2 + k_1 = 5$$

$$k_3 = -1, \quad -6k_1 + 3(-1) = -3, \quad 6(-1) + 2(0) + k_1 = 12$$

$$, \quad -6k_2 - 3 = -3, \quad -6 + k_1 = 12$$

$$, \quad -6k_2 = -3 + 3 \quad k_1 = 18$$

$$-6k_2 = 0$$

$$k_2 = 0$$

$$-2k_2 + k_1 + k_0 = 0$$

$$-2(0) - 2 + k_0 = 0$$

$$k_0 = 2$$

$$y'' - 4y + 4y' = x^2 e^{2x}$$

Solution:

$$y'' - 4y + 4y' = x^2 e^{2x}$$

Form eq

$$y'' - 4y' + 4y = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$$

$$\lambda(\lambda - 2) - 2(\lambda - 2) = 0$$

$$(\lambda - 2)(\lambda - 2) = 0$$

Roots are real & equal.

$$y = (C_1 + C_2 x) e^{2x}$$

$$y_1 = C_1 e^{2x} + C_2 x e^{2x}$$

$$y_1 = e^{2x}, \quad y_2 = x e^{2x}$$

$$y_1' = 2e^{2x}, \quad y_2' = e^{2x} + 2x e^{2x}$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$W = \begin{vmatrix} e^{2x} & x e^{2x} \\ 2e^{2x} & e^{2x} + 2x e^{2x} \end{vmatrix}$$

$$W = e^{4x} + 2x e^{-4x} - 2x e^{4x}$$

$$W = e^{4x}$$

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$$y_p = -y_1 \int \frac{y_2 x(x)}{w} dx + y_2 \int \frac{y_1 x(x)}{w} dx$$

$$y_p = -e^{-3x} \int x e^{3x} - x^2 e^{3x} dx + x e^{2x} \int \frac{e^{2x} x^2 e^{2x}}{e^{2x}} dx$$

$$y_p = -e^{-3x} \int \frac{x^3 e^{4x}}{e^{4x}} + x e^{2x} \int \frac{x^2 e^{4x}}{e^{4x}} dx$$

$$y_p = -e^{-3x} \int x^2 dx + x e^{2x} \int x^2 dx$$

$$y_p = -e^{-3x} \cdot \frac{x^3}{3} + x e^{2x} \cdot \frac{x^3}{3}$$

So

$$y = y_h + y_p$$

$$y = C_1 e^{2x} + C_2 x e^{2x} - e^{-3x} \frac{x^3}{3} + x e^{2x} \frac{x^3}{3}$$

~~$$y = C_1 e^{2x} + C_2 x e^{2x} - e^{-3x} \frac{x^3}{3} + x e^{2x} \frac{x^3}{3}$$~~

ANSWER #05

Ordinary Differential Equation (ODEs)
Homogeneous Linear ODEs with
constant co-efficients

$$y'' + ay' + by = 0$$

The characteristic equation

$$d^2 + ad + b = 0$$

It is a quadratic Equation
It has two roots λ_1 & λ_2

Case 1:- If the ^{roots} λ_1 & λ_2 are real
and distinct

Then the solution is
 $y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$

