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Section A

Samester Summer

Subject Calculas

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QUESTION #01

The function $g(t)$ is defined by

$g(t) = 0 \quad t < 0$

$t^2 \quad 0 \leq t \leq 3$

$2t + 3 \quad 3 < t \leq 4$

$12 \quad t > 4$

a) state any point of dis continuity

Solution:-

a) To check possibility of the discountinuity of the function is at $t = 0$ & 4

→ First at $t = 0$

$$g(t) = t^2$$

$g(0) = 0^2 = 0$

For R.H.L

$$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$$

$$= \lim_{h \to 0} 1 + h^2 + 2h$$

Apply limits

$$= 1 + 0^2 + 2(0)$$

$$= 1$$

For L.H.L

$$\lim_{h \to 0} g(1-h) = 2t + 3$$

$$= \lim_{h \to 0} 2(1-h) + 3$$

$$= \lim_{h \to 0} 2 - 2h + 3$$

Apply limit

$$= 2 - 2(0) + 3$$

$$= 5$$

$R \cdot H \cdot L \neq L \cdot H \cdot L = g(t) = 5$

Now at $t = 4$

$$g(4) = 2(4) + 3$$
$$= 8 + 3$$
$$= 11$$

For $R \cdot H \cdot L$

$$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h) + 3$$

$$\lim_{h\to 0} 2+2h+3$$

Apply limit

$= 2+2(0)+3 = 5$

For L.H.L

$$\lim_{h\to 0} g(1-h) = 12$$

$g(4) = R.H.L \neq L.H.L.$

point of ~~coun~~ discontinuity is at $t=4$

b) Find, if they exist

1) $\lim_{t \to 3} g$

SOLUTION:

For $g(t) = t^2$

R.H.L $\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$

$= \lim_{h \to 3} 1 + h^2 + 2h$

Apply limit

$= 1 + 3^2 + 2(3) \Rightarrow 16$

L.H.L

$\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t+3$

$$= \lim_{h \to 3} 2(1-h)+3$$

$$= \lim_{h \to 3} 2 - 2h + 3$$

Apply limit

$$= 2 - 2(3) + 3$$

$$= 2 - 6 + 3$$

$$= -1$$

R·H·L ≠ L·H·L (do not exist since L·H·L is −ve)

QUESTION #02

i) Find the Maclaurin's series

For

$y(x) = x^2 + \sin x$

SOLUTION:-

$y(x) = x^2 + \sin x$

Since we know that the Maclaurin's series is

$$y(x) = y(x_0) + y'(x - x_0) + \frac{y''(x_0)(x - x_0)^2}{2!} + \cdots$$

$+ \cdots$

Put $x_0 = 0$

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Now find

$y(0) = ?$

$y(x) = x^2 + \sin x$

$y(0) = 0 + \sin 0$

$y(0) = 0 + 0$

$y(0) = 0$

$y(x) = x^2 + \sin x$

$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$

$y'(x) = 2x + \cos x$

$y'(0) = 0 + 1$

$\boxed{y'(0) = 1}$

Since $y'(x) = 2x + \cos x$

$$\frac{d}{dx} y'(x) = 2\frac{d}{dx}x + \frac{d}{dx}\cos x$$

$$y''(x) = 2 - \sin x$$

$$y''(0) = 2 - \sin 0$$

$$y''(0) = 2 - \sin 0$$

$$= 2 - 0 = 2.$$

$$\boxed{y''(0) = 2}$$

$$y''(x) = 2 - \sin x$$

$$y''(x) = 0 - \cos x$$

$$y''(0) = -\cos 0$$

$$\boxed{y''(0) = -1}$$

Put in equation

$$y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \cdots$$

$$= x + \frac{2x^2}{2!} - \frac{x^3}{3!} + \cdots$$

$$= x + x^2 - \frac{x^3}{3!} + \cdots$$

So,

$$y(x) = x + x^2 - \frac{x^3}{3!} + \cdots$$

Ans

QUESTION #03

i) Find Y'' given

$$1 + xy = x^2 + y^2$$

SOLUTION:-

$$1 + xy = x^2 + y^2$$

Taking $\frac{d}{dx}$ on B.S

$$1 + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$1 + \left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) = 2x + 2y\frac{dy}{dx}$$

$$1 + \frac{xdy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$1+y+x\frac{dy}{dx} = 2x+2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x-y-1$$

$$(x-2y)\frac{dy}{dx} = 2x-y-1$$

$$\frac{dy}{dx} = \frac{2x-y-1}{x-2y} \longrightarrow (*)$$

$$\Rightarrow y' = \frac{2x-y-1}{x-2y} \longrightarrow (1)$$

Diff again

$$\frac{d}{dx}y' = \frac{d}{dx}\left(\frac{2x-y-1}{x-2y}\right)$$

Quotient rule

$$y'' = \frac{(x-2y)\frac{d}{dx}(2x-y-1) - (2x-y-1)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$y'' = \frac{(x-2y)(2-\frac{dy}{dx}) - (2x-y-1)(1-2\frac{dy}{dx})}{(x-2y)^2}$$

$$y'' = \frac{(x-2y)(2-y') - (2x-y-1)(1-2y')}{(x-2y)^2}$$

we know value of y' from eq (i)

$$\Rightarrow y'' = \frac{(x-2y)\left(2-\left(\frac{2x-y-1}{x-2y}\right)\right)-(2x-y-1)\left(1-2\left(\frac{2x-y-1}{x-2y}\right)\right)}{(x-2y)^2}$$

$$y'' = \frac{(x-2y)(2-2x-y-1)}{(x-2y)^2} - \frac{(2x-y-1)(1-2(2x-y-1))}{(x-2y)^3}$$

$$\Rightarrow y'' = \frac{-2x-y-3}{(x-2y)^2} - \frac{(2x-y-1)(1-2(2x-y-1))}{(x-2y)^3}$$

$2 \longrightarrow$

(15)

QUESTION 03(b)

$y = x^3(1+x^9)e^{6x}$

SOLUTION:

$x^3(1+x^9)e^{6x}$

$$\ln(y) = \ln(x^3(1+x)^9 e^{6x})$$

$$= \ln(x^3(1+x)^9) + \ln e^{(6x)}$$

$$= 3\ln x + 9\ln(1+x) + 6x$$

Now

$$\frac{\partial}{\partial x}\ln(y) = \frac{d}{dx}(3\ln x + 9\ln(1+x) + 6x)$$

$$= 3\frac{d}{dx}\ln x + 9\frac{d}{dx}(1+x) + 6\frac{dx}{dx}$$

$$= 3 \cdot \frac{1}{x} + 9 \cdot \frac{1}{1+x} + 6$$

$$\frac{d \ln(y)}{dx} = \frac{3}{x} + \frac{9}{x+1} - 6$$

Ans