Q1: $x^3y''' + 2x^2y'' + 2y = 10x + \frac{10}{x}$

Sol:-

$$y_p = 5e^{t} + \frac{10e^{-t}}{3(-1)^2 - 2(-1)}$$

$$= 5e^{t} + \frac{10e^{-t}}{3+2}$$

$$y_p = 5e^{t} + 5e^{-t}$$

General Solution :

$$y = y_p + y_c$$

$$y = C_1 e^{-x} + e^{-x}(C_1 \cos t + C_2 \sin t) + 5e^{t} + 5e^{-t}$$

Replace $e^{t} = x$ & $t = \ln x$

$$y = C_1 e^{-x} + e^{-x}(C_1 \cos \ln x + C_2 \sin \ln x) + 5x + 5x^{-1}$$

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Q2 :- $x^3y''' + 4x^2y'' - 5xy' - 15y = x^4$

Solution :- $x^3D^3y + 4x^2D^2y - 5xDy - 15y = x^4$

$$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4 \quad \text{---(1)}$$

Let $x = e^t \rightarrow t = \ln x$

$xD = \Delta$

$x^2D^2 = \Delta^2 - \Delta$

$\Delta^3D^3 = \Delta^3 - 3\Delta^2 + 2\Delta$

Substitute the values,

$$(\Delta^3 - 3\Delta^2 + 2\Delta + 4\Delta^2 - 4\Delta - 5\Delta - 15)y = x^4$$

$$(\Delta^3 + \Delta^2 - 7\Delta - 15)y = e^{4t}$$

For complementary Solution :- y_c :-

$$\Delta^3 + \Delta^2 - 7\Delta - 15 = 0$$

Synthetic Division :

3	1	1	-7	-15
		3	12	15
	1	4	+5	0

$$(\Delta - 3)(\Delta^2 + 4\Delta + 5) = 0$$

$$\Delta^2 + 4\Delta + 5 = 0$$

using Quadratic formula,

$a = 1, b = +4, c = 5$

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$$\Delta = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Delta = \frac{-4 \pm \sqrt{(4)^2-4(1)(5)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm 2i}{2} \Rightarrow \Delta = -2 \pm 1i$$

So $m_{1,2} = -2+i$, $m_{3,4} = -2-i$

$y_c = e^{-2x}(c_1\cos x + c_2 \sin x) + e^{-2x}(c_1\cos(-x) + c_2\sin(-x))$

$y_c = e^{-2x}(c_1\cos x + c_2 \sin x) + e^{-2x}(c_1\cos x - c_2\sin x)$

For $y_p = ?$

$$y_p = \frac{1}{\Delta^3+\Delta^2-7\Delta-15} e^{4t}$$

$$= \frac{1}{(4)^3+(4)^2-7(4)-15} e^{4t}$$

$$= \frac{1}{64+16-28-15} e^{4t}$$

$$y_p = \frac{e^{4t}}{37}$$

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$$y = y_c + y_p$$

$$y = e^{-2x}(c_1 \cos x + c_2 \sin x) + e^{-2x}(c_1 \cos x - c_2 \sin x) + \frac{e^{4t}}{37}$$

Again put $x = e^t$, $t = \ln x$

$$y = e^{-2x}(c_1 \cos x + c_2 \sin x) + e^{-2x}(c_1 \cos x - c_2 \sin x) + \frac{4x}{37}$$

Q3 :- $x^2y'' + 2xy' - 6y = 10x^2$; $y(1) = 1$, $y'(1) = -6$

Sol :- $x^2D^2y + 2xDy - 6y = 10x^2$

$$(x^2D^2 + 2xD - 6)y = 10x^2$$

let $x = e^t$, $t = \ln x$

$xD = \Delta$

$x^2D^2 = \Delta^2 - \Delta$

$$(\Delta^2 - \Delta + 2\Delta - 6)y = 10e^{2t}$$

$$(\Delta^2 + \Delta - 6)y = 10e^{2t}$$

For complementary Equation : y_c :-

$$\Delta^2 + \Delta - 6 = 0$$

$$\Delta^2 + 3\Delta - 2\Delta - 6 = 0$$

$$\Delta(\Delta + 3) - 2(\Delta + 3) = 0$$

$$(\Delta + 3)(\Delta - 2) = 0$$

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$\Delta + 3 = 0$, $\Delta - 2 = 0$

$\Delta = -3$, $\Delta = +2$

Since roots are real & distinct

$$y_c = c_1 e^{-3t} + c_2 e^{2t}$$

for $y_p = ?$

$$y_p = \frac{1}{\Delta^2 + \Delta - 6} e^{2t}$$

$$= \frac{1}{(2)^2 + (2) - 6} = \frac{1}{4 + 2 - 6} = 0$$

$$y_p = \frac{1}{2\Delta + \Delta} e^{2t} = \frac{1}{2(2) + 2} e^{2t}$$

$$y_p = \frac{1}{4 + 2} e^{2t} \Rightarrow \frac{1}{6} e^{2t}$$

$$y_p = \frac{e^{2t}}{6}$$

So General Solution :-

$$y = y_c + y_p$$

$$y = c_1 e^{-3t} + c_2 e^{2t} + \frac{e^{2t}}{6}$$

put $x = e^t$, & $t = \ln x$

$$y = c_1 x^{-3} + c_2 x^2 + \frac{x^2}{6} \quad (*) \qquad y(1) = 1$$

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(8)

$$y = c_1(1)^{-3} + c_2(1)^2 + \frac{(1)^2}{6}$$

$$y = c_1 + c_2 + \frac{1}{6}$$

$$(1) = c_1 + c_2 + \frac{1}{6}$$

$$c_1 + c_2 = 1 - \frac{1}{6}$$

$$= \frac{6-1}{6}$$

$$c_1 + c_2 = 5/6 \quad \text{---} (1)$$

Now Differentiate eq (*) w.r. to x

$$y' = -3c_1x^{-4} + 2c_2x + \frac{x}{3}$$

Now put $y'(1) = -6$

$$-6 = -3c_1(1)^{-4} + 2c_2(1) + \frac{(1)}{3}$$

$$-6 = -3c_1 + 2c_2 + \frac{1}{3}$$

$$-3c_1 + 2c_2 = -\frac{6}{1} - \frac{1}{3}$$

$$-3c_1 + 2c_2 = \frac{-18-1}{3} \Rightarrow -3c_1 + 2c_2 = \frac{-17}{3} \quad (2)$$

After solving eq (1) & (2)

$$\boxed{c_1 = 2} \qquad \boxed{c_2 = -1}$$

put in (*)

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Q#

$$y = 2x^{-3} - x^2 + \frac{x^2}{6}$$ → Ans

Q4:- $x^2y'' + 7xy' + 5y = x^5$; $y(0) = 2$ & $y(1) = 2$

Solution #

$$x^2D^2y + 7xDy + 5y = x^5$$

$$(x^2D^2 + 7xD + 5)y = x^5$$

Let $x = e^t$, $t = \ln x$

$$xD = \Delta$$

$$x^2D^2 = \Delta^2 - \Delta$$

$$(\Delta^2 - \Delta + 7\Delta + 5)y = x^5$$

$$(\Delta^2 + 6\Delta + 5)y = e^{5t}$$

for y_c :-

$$\Delta^2 + 6\Delta + 5 = 0$$

$$\Delta^2 + 5\Delta + \Delta + 5 = 0$$

$$\Delta(\Delta + 5) + 1(\Delta + 5) = 0$$

$$(\Delta + 5)(\Delta + 1) = 0$$

$$\Delta = -5 \quad , \quad \Delta = -1$$

Since roots are real & distinct,

$$y = c_1 e^{-5t} + c_2 e^{-t}$$

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For y_p :-

$$y_p = \frac{1}{D^2+6D+5} e^{5t}$$

$$= \frac{1}{(5)^2+6(5)+5} e^{5t}$$

$$= \frac{e^{5t}}{25+30+5} \Rightarrow y_p = \frac{e^{5t}}{60}$$

Now General Solution :-

$$y = y_c + y_p$$

$$y = c_1 e^{-5t} + c_2 e^{-t} + \frac{e^{5t}}{60} \quad \text{---(A)}$$

$$y = c_1 x^{-5} + c_2 x^{-1} + x^5/60 \quad \text{---(B)}$$

Now put $x=2$ & $y=2$ in eq (B)

~~$2 = c_1(0) + c_2(0) + (0)$~~

$$2 = c_1(2)^{-5} - c_2(2)^{-1} + \frac{1}{60}(2)^5$$

$$= -32c_1 + (-2)c_2 + \frac{1}{60}(32)^8$$

$$-32c_1 - 2c_2 = \frac{32}{15} \quad \text{---(C)}$$

Now Differentiate eq (B) w.r. to x

$$y' = -5c_1 x^{-6} - c_2 x^{-2} + 5x^4 \qquad y'(1) = 2$$

$$60$$

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$$2 = -5C_1(1)^{-6} - C_2(1)^{-2} + \frac{5(1)^4}{60}$$

$$2 = -5C_1 - C_2 + \frac{1}{12}$$

$$-5C_1 - C_2 = 2 - \frac{1}{12} \Rightarrow -5C_1 - C_2 = \frac{24-1}{12}$$

$$-5C_1 - C_2 = \frac{23}{12} \quad \text{(D)}$$

After solving (C) & (D) eqns:

$$C_1 = 580 \quad \& \quad C_2 = -9280$$

putting these values in eq (B)

$$y = 580x^{-5} - 9280x^{-1} + \frac{x^5}{60}$$ Ans

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Q5ᵗʰ-

$$(x+1)^2 y'' + 3(x+1)y' + 4y = x^2$$

Sol:

$$(x+1)^2 D^2 y + 3(x+1)Dy + 4y = x^2$$

$$\left((x+1)^2 D^2 + 3(x+1)D + 4\right) y = x^2$$

let $x+1 = e^t \Rightarrow t = \ln(x+1)$

$$(x+1)D = \Delta$$

$$(x+1)^2 D^2 = \Delta^2 - \Delta$$

$$\left(\Delta^2 - \Delta + 3\Delta + 4\right) y = e^{2t}$$

$$\left(\Delta^2 + 2\Delta + 4\right) y = e^{2t}$$

for y_c :-

$$\Delta^2 + 2\Delta + 4 = 0$$

$$\Delta^2 + 2\Delta + 2\Delta + 4 = 0$$

$$\Delta(\Delta+2) + 2(\Delta+2) = 0$$

$$(\Delta+2)(\Delta+2) = 0$$

$$\Delta = -2, \quad \Delta = -2$$

Since roots are real & repeated

$$y_c = (c_1 + c_2)e^{-2t}$$

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For y_p :-

$$y_p = \frac{1}{D^2 - 4D + 4} e^{2t}$$

$(2)^2 - 4(2) + 4$

$4 - 8 + 4 = 0$

$$y_p = \frac{1}{2D - 4} e^{2t}$$

$$y_p = \frac{1}{2} e^{2t}$$ After taking 2 times derivative

$$y = y_c + y_p$$

$$y = (c_1 + c_2) e^{2t} + \frac{1}{2} e^{2t}$$

replace $t = \ln(x+1)$ & $x+1 = e^t$

$$y = (c_1 + c_2)(x+1)^2 + \frac{1}{2}(x+1)^2$$

$$y = \left\{ (c_1 + c_2) + \frac{1}{2} \right\} (x+1)^2$$

General solution

The END

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