Date: / / 20 ID# 7549 Summer 2020

Qno 1 (a)

Drag:- A body which is wholy immersed in a homogenenous fluid may be subjected to two kind of force arising from relative motion between body and fluid these force are termed as drag and lift. if the force paralled to the motion then it is turmed as drag force.

there are two components

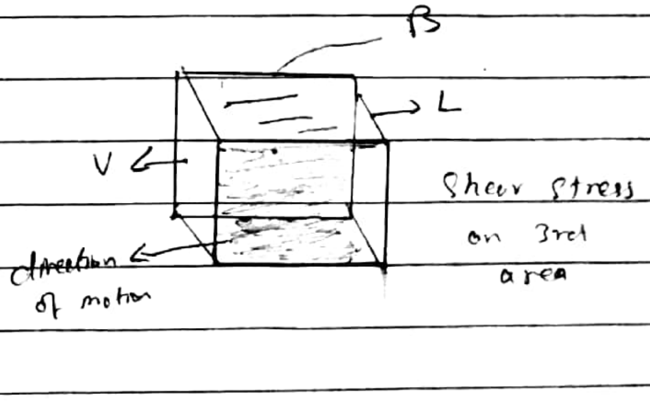
(1) **Pressure Drag:- (FP)** it is equal to integration of Component is direction of motion of all pressure force exerted on surface body

$$F_P = C_P \rho \frac{V^2}{2} A \quad \text{wher } C_P \text{ depend on shape}$$

(2) **Friction Drag:- (F_f)** it is equal to integration of Components of shear stress along surface of body in direction of motion

$$F_f = \underbrace{C_f \rho \frac{V^2}{2}}_{\text{Shear stress}} BL$$

Area = B × L



Signature:

Date: / / 20

Friction Drag of Boundry layer

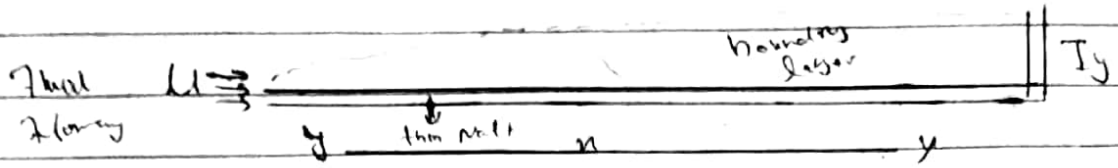
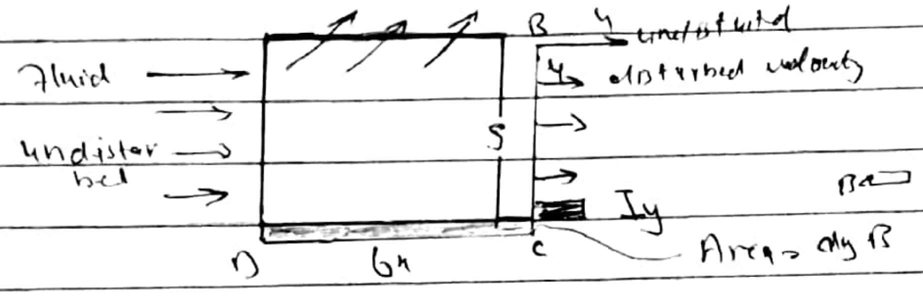


Fig Shows growth of boundry layer along one site of smooth plate inside the fluids

Now conside a control valume



Where δ is thickness of boundary layer and U is undisturbed velocity Thus $-F_x$ - drag = rate in momentum in x-direction leaving through BC + rate of momentum through AB) - rate of moment entering through DA)

$\Delta P = P_{out} - P_{in}$

Thus accourding to momentum

$$\Sigma F = \frac{d(P)}{dt} = \frac{d\,mov}{dt}$$

Where $\frac{dm}{dt} = \rho Q$ thus

$$F = \rho Q V$$

or

$$F = \rho A\, V \cdot V$$

$$F = \rho A V^2$$

DA → $\rho u\,(UB\delta)$

BC → $\rho B \int_0^{\delta} u^2\,dy$

Date: / / 20

$AB \rightarrow \rho u \left(UB\delta - B \int_0^{\delta} u \cdot dy \right)$

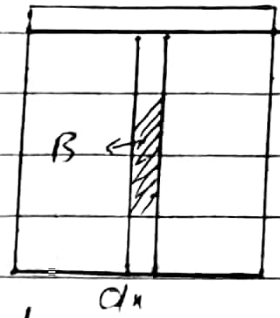
Putting value

$F_x = \rho B \int_0^{\delta} u (U - u)\, dy$

Solving it

Now to find local wall shear stress

$\tau_0 = \frac{dF_x}{B\,dx}$ - area



$F_x = \theta B U^2 \rho_x$

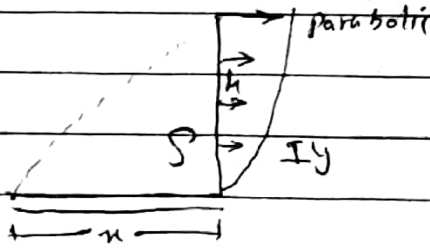
$\tau_0 = \theta B U^2 \rho_x$

$\tau_0 = \rho U^2 \propto \frac{d\theta}{dx}$ in general

Equation of shear stress

→ Laminar boundary layer

$\frac{u}{U} = F\left(\frac{y}{\delta}\right)$



Assume

$n = \frac{y}{\delta}$ or $y = n\delta$

thus $\frac{u}{U} = F(n)$ or $u = UF(n)$

In case of laminar flow

$\tau_0 = \mu \left(\frac{du}{dy}\right)$

Signature:

Date: / / 20

$$\tau' = \frac{\mu}{S}\frac{du}{dn} = \frac{\mu}{S}\left[\frac{d\tau(n)}{dn}\right]$$

Solving the equation

$$\tau_0 = \frac{\mu u \beta}{S} \rightarrow$$

As general equation is $\tau_0 = \rho u^2 \alpha \frac{ds}{dn}$

Equating $= \rho u^2 \frac{ds}{dn}$

or

$$S\,ds = \frac{\mu\beta}{\rho u \alpha}\,dx$$

Integrating the equation

$$\frac{S^2}{2} = \frac{\mu\beta}{\rho u \alpha} + C$$

Now at $x=0$ $S=0$ thus $C=0$

$$\frac{S^2}{2} = \frac{\mu\beta x}{\rho u \alpha}$$

Or $S = \sqrt{\frac{2\mu\beta x}{\rho u \alpha}}$ or $\sqrt{\frac{2\beta}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho u}}$

x-ing and ÷-ing by x

$$S = \sqrt{\frac{2\beta}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho u}} \cdot \frac{x}{\sqrt{x}\cdot\sqrt{x}}$$

Where $\alpha = 0.135$

$\beta = 1.03$

$R_n = \frac{\rho u x}{\mu}$

$S = \frac{4.91}{\sqrt{R_n}}\,x$, or $\frac{S}{x} = \frac{4.91}{\sqrt{R_n}}$

Date: / / 20

Now $\tau_0 = \frac{\mu u B}{\delta}$

thus putting value

$\tau_0 = 0.332 \frac{\mu u}{x} \sqrt{R_x}$

Where R_x is local Reynold number

Now

$F_D = B \int_0^l \tau_0 \, dx$

stress

Putting value

$F_D = 0.664 B \sqrt{\rho \mu u^3}$

As general equation is

$F_D = C_D \frac{\rho u^2}{2} B L$ → equation both equation

$C_D = 1.328 \sqrt{\frac{\mu}{\rho L V}} = \frac{1.328}{\sqrt{R}}$

Turbulant boundry layer

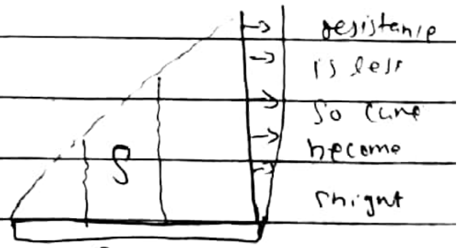


Figure show that velocity distribution in turbulant boundary layer char a much steeper gradient near wall and flatter through out reading layer the shear is greater in turbulant than in laminar layer As we have

$\tau_0 = \frac{f \rho V^2}{8}$ where V denotes average velocity of pipe

Signature:

Date: / / 20

Now we have obtained an approximate relation between v and U by using pipe factor f

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{f}}$$

Using friction factor of 0.02 from chart which is middle critical value

So $U = 1.235V$

Now we have

$$\tau_0 = f\rho\frac{V^2}{8}$$

As we have

$$f = \frac{0.316}{R^{0.25}}$$

thus $\tau_0 = 0.316 \frac{\rho V^2}{8}$

where $V = \frac{U}{1.235}$ thus

$$\tau_0 = \frac{0.316}{\left(\frac{D}{\nu}\left(\frac{U}{1.235}\right)\right)^{1/4}} \cdot \frac{\rho}{8}\left(\frac{U}{1.235}\right)^2$$

$$\delta D = 2r$$

thus $\tau_0 = \frac{0.023\rho U^2}{\left(\frac{\delta U}{\nu}\right)^{1/4}}$

As we have

Date: / / 20

$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

Equating both and integration For boundry Condition For boundry Condition of $x=0$ $\delta=0$

thus $\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{\nu}{Ux}\right)^{1/5} x$

For

$\alpha = 0.0972$

$\frac{\delta}{x} = \frac{0.377}{(R_x)^{1/5}}$

Putting value in equation

$\tau_0 = 0.0587 \frac{\rho U^2}{2} \left(\frac{\nu}{Ux}\right)^{1/5}$

Now $F_D = B \int_0^L \tau_0 dx$

$F_D = 0.0735 \rho \frac{U^2}{2} \left(\frac{\nu}{UL}\right) BL$

As $F_D = C_D \rho \frac{U^2}{2} BL$

equation both

$C_D = \frac{0.0735}{R^{1/5}}$

For $R > 10^7$

$C_D = \frac{0.455}{(\log R)^{2.58}}$

Signature:

Q no1 B

As

↳ Specific energy $E = y + \frac{V^2}{2g}$

The flow Q per unit width 'b'

Can be expressed as

↳ $q = \frac{Q}{b}$

Now average velocity will be

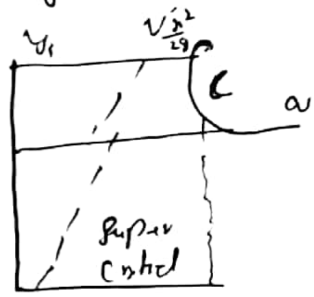
↳ $V = \frac{Q}{A} = \frac{qb}{by} = \frac{q}{y}$

Thus

$E = y + \frac{V^2}{2g} \Rightarrow y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$

$(E - y) = \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$ or $(E - y)y^2 = \frac{q^2}{2g}$

Thus point of E vs y will be parabola for particular q there will be two kind of possible values of y, for given E



The equation is

Cubic with three roots being negative points

C represent dividing points between two regions of flow thus for given q & value of E is minimum & flow of that point is critical the y_c critical flow depth of flow at that point is critical V_c and velocity at that point

Thus $E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$

For minimum specific energy $\frac{dE}{dy} = 0$

Thus

↳ $\frac{dE}{dy} = 1 - \frac{2}{2g}\left(\frac{q^2}{y^3}\right) = 0$

$$\frac{q^2}{gy^3} = 1 \Rightarrow q^2 = gy^3$$

$\hookrightarrow \dfrac{q^2}{gy^3} = 1 \Rightarrow q^2 = gy^3$

Now

$\hookrightarrow q^2 = gy^3$

& $q = vy \Rightarrow v^2y^2 = gy^3$

or

$v^2 = gy_{cr}$

or

$$\boxed{V_{cr} = \sqrt{gy_{cr}}}$$

Date: / / 20 ID # 7549 Summer 2020

Q no 2 Given

Water flows at rate, $Q = 3.5\ m^3/s$

Bed Slop, $S_o = 0.0008$

$n = 0.0219$

Width of bed in student ID = 7549 mm

Required

Depth of rectangular channel = ?

Critical Depth y_c = ?

Critical velocity V_c = ?

Flow is critical Sub-critical = ?

Sol

Manning Equation

$$Q = \left(\frac{1}{n} R_h^{2/3} S_o^{1/2}\right) A \rightarrow 0$$

Area $7549 * d$

perimeter $= d + 7549 + d$

Hydraulic Radical $R_h = \dfrac{\text{Area}}{\text{perameter}}$

$$\frac{7549d}{2d + 7549}$$

Put value in eq 1

$$Q = \left(\frac{1}{n} R_h^{2/3} S_o^{1/2}\right) A$$

$$3.5 = \left(\frac{1}{0.029} \times \frac{7549d}{2d + 7549}\right)^{2/3} \times (0.008)^{1/2} \times 7549d$$

$$\left(\frac{3.5 \times 0.0219}{(0.0008)^{1/2}}\right)^{2/3} = \left(\frac{7549}{2d + 7549}\right)^{2/3} \times 7549d$$

Signature:

$$\left(\frac{3.5 \times 0.0119}{\sqrt{0.008}}\right)^{3/2} = \frac{56.98d^2}{9d + 7.549}$$

$$4.461(9d + 7.549) = 56.98$$

$$8.922d + 33.67 = 56.98d^2$$

$$56.98 - 8.922d - 33.67 = 0$$

$$d = 0.719m$$

So the depth of channel is $\boxed{0.383m}$

Now

As q = discharge per unit width

$$q = \frac{Q}{b}$$

$$= \frac{3.5}{7.549} = 0.46$$

Date: / / 20

$q = 0.46$

→ Critical depth
using equation

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_c = \left(\frac{(0.46)^2}{9.81}\right)^{1/3}$$

$$y_c = 0.28 m$$

Critical velocity
using equation

$$V_c = g y_c$$

$$V_{cr} = \sqrt{(9.81)(0.28)}$$

$$V_{cr} = 1.73 m/s$$

$$V = \frac{Q}{A} \quad \frac{3.5}{7.549 \times 0.383}$$

$$V = 1.21 m$$

$y = 0.383m$ $y_{cr} = 0.28$ $V_{cr} = 1.73 m/s$

As $y > y_{cr}$
and
$V < V_{cr}$

Signature: So flow is sub critical flow

Q no 3 Given data

Width of Smooth plate, $B = 200mm$
$= 0.2m$

Length of Smooth plate, $L = 800 mm$
$= 0.8m$

Oil with Specific gravity, $S = 0.89$
Undisturbed velocity $U = 5m/s$
Kinematic Viscosity $\nu = 0.93 \times 10^{-4} m^2/s$

Required data

Friction drag on one side of a smooth plate $F_D = ?$

Sol Check the flow
As $\nu = 0.93 \times 10^{-4} m^2/s$

$$R = \frac{LU}{\nu} = \frac{(0.8)(5)}{0.93 \times 10^{-4}}$$

$R = 43010.75 < 500,000$
thus flow is laminar

Now
$$C_f = \frac{1.328}{\sqrt{R}} \Rightarrow \frac{1.328}{\sqrt{43010.75}}$$

$C_f = 6.403 \times 10^{-3}$
$C_D = 0.0064$

$$\Rightarrow F_D = C_D \rho \frac{U^2}{2} BL$$

$(0.0064)(S_{oil} \times \rho_{water}) \times (\frac{5^2}{2})(0.2)(0.8)$

$(0.0064)(0.89 \times 1000) \times \frac{5^2}{2} \times (0.2)(0.8)$

$F_D = 11.392 N$