Name: ZAHOOR Khan

I.D: 13835

Semster: 6th

Subject: linear Algebra

Teacher Name: Muhammad Shakeel

Page (1)

"Name: ZAHoor Khan I.D = 13835"

Q1 Compute adjoint of:

(i) $A = \begin{bmatrix} 1 & 2 & \text{2nd-ID} \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ ∵ 2nd-ID - 2nd number your ID.

Solution:

my I.D = 13835
2nd I.D Number is 3

$$Adj(A) = Adj \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} +\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} \\ +\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} +(3\times2-1\times1) & -(2\times2-1\times3) & +(2\times1-3\times3) \\ -(2\times2-3\times1) & +(1\times2-3\times3) & -(1\times1-2\times3) \\ +(2\times1-3\times3) & -(1\times1-3\times2) & +(1\times3-2\times2) \end{bmatrix}$$

$$= \begin{bmatrix} +(6-1) & -(4-3) & +(2-9) \\ -(4-3) & +(2-9) & -(1-6) \\ +(2-9) & -(1-6) & +(3-4) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}$$

Page(2) ZAHOOR Khan I,D: 13835.

Q1(ii) B:

$$\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$$

Solution.

As:

$$B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$$ $\therefore$ Adj B = (Co-f (B))^T

$\therefore$ finding minors and cofactors:-

$$B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$$

$$M.of\ B = \begin{bmatrix} 8 & -24 & 1 \\ 42 & -1 & -26 \\ 37 & 14 & -11 \end{bmatrix}$$

$$B = \begin{bmatrix} (+1)(8) & (-1)(-24) & (+1)(1) \\ (-1)(42) & (+1)(-1) & (-1)(-26) \\ (+1)(37) & (-1)(14) & (+1)(-11) \end{bmatrix}$$

$\therefore$ Co-f B = $\cancel{(+1)(8)}$

$$\begin{bmatrix} 8 & 24 & 1 \\ -42 & -1 & 26 \\ 37 & -14 & -11 \end{bmatrix}$$

Take Transpose of (ii) B.

$$\begin{bmatrix} 8 & 24 & 1 \\ -42 & -1 & 26 \\ 37 & -14 & -11 \end{bmatrix}^T$$

Q2 Find the cofactors of A_{21}, A_{31}, A_{33} if,

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

Solution:

As $A \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$

Finding minors and then co-factors:

$$\therefore m_{ij}(A) = \begin{bmatrix} 9 & -8 & -6 \\ -5 & -10 & 5 \\ -11 & 7 & -1 \end{bmatrix}$$

A_{21} A_{32} A_{33}

Now,

Co-factors of $A_{21} = ((-1)^{2+1} \cdot (5)) = (-1)^3 \cdot (5)$

$\Rightarrow A_{21} = -5$

$\therefore A_{31} = ((-1)^{3+1} \cdot (-11)) = (-1)^4 \cdot (-11)$

$\Rightarrow \boxed{A_{32} = -11}$

$\therefore A_{33} = ((-1)^{3+3} \cdot (-1)) = (-1)^6 \cdot (-1)$

$\Rightarrow \boxed{A_{33} = -1}$

Page (4)

Q 3 Find Eign Values and Eign Vector if,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} (2-\lambda) & 1 & 1 \\ 1 & (3-\lambda) & 2 \\ -1 & 1 & (2-\lambda) \end{vmatrix} = 0$$

$= (2-\lambda)((3-\lambda)\times(2-\lambda)-2\times 1)-1(1\times(2-\lambda)-2\times(-1))+1(1\times 1-(3-\lambda)\times(-1)) = 0$

$=(2-\lambda)((6-5\lambda+\lambda^2)-2)-1((2-\lambda)-(-2))+1(1-(-3+\lambda)) = 0$

$=(2-\lambda)(4-5\lambda+\lambda^2)-1(4-\lambda)+1(4-\lambda)=0$

$=(8-14\lambda+7\lambda^2-\lambda^3)-(4-\lambda)+(4-\lambda)=0$

$=(-\lambda^3+7\lambda^2-14\lambda+8)=0$

$=-(\lambda-1)(\lambda-2)(\lambda-4)=0$

$=(\lambda-1)=0$ or $(\lambda-2)=0$ or $(\lambda-4)=0$

= The eigenvalues of the matrix A are given by $\lambda = 1, 2, 4$

1. Eigenvectors for $\lambda = 1$

$$A-\lambda I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ -1 & 1 & 1 \end{bmatrix}$$

Now reduce this matrix

$R_2 \leftarrow R_2 - R_1$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$R_3 \leftarrow R_3 + R_1$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$

Interchanging row $R_2 \leftrightarrow R_3$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_2 \leftarrow R_2 \div 2$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_1 \leftarrow R_1 - R_2$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_3 \leftarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The system associated with the eigenvalue $\lambda = 1$

$$(A - I_3) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 = 0, x_2 + x_3 = 0$

$\Rightarrow x_1 = 0, x_2 = -x_3$

eigenvectors corresponding to the eigenvalue $\lambda = 1$ is

$$U = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & 0 \end{bmatrix}$$

Now, reduce this matrix

Interchanging row $R_1 \leftrightarrow R_2$

$$= \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$R_3 \leftarrow R_3 + R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$

Interchanging rows $R_2 \leftrightarrow R_3$

$$= \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$R1 \leftarrow R1 - R2$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$R3 \leftarrow R3 - R2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The system associated with eigenvalue $\lambda = 2$

$$(A - 2I) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 + x_3 = 0, \; x_2 + x_3 = 0$

$\Rightarrow x_1 = -x_3, \; x_2 = -x_3$

eigenvectors corresponding to the eigenvalue $\lambda = 2$ is

$$U = \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \end{bmatrix}$$

let $x_3 = 1$

$$U_2 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

* Find Matrix Eigenvalues.

(1)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

"Sollution"

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} (1-\lambda) & 0 & 0 \\ 0 & (1-\lambda) & 0 \\ 0 & 0 & (1-\lambda) \end{vmatrix} = 0$$

$= (1-\lambda)((1-\lambda)\times(1-\lambda) - 0\times 0) - 0(0\times(1-\lambda) - 0\times 0) + 0(0\times 0 - [1-\lambda)\times 0) = 0$

$= (1-\lambda)((1-2\lambda+\lambda^2) - 0) - 0(0-0) + 0(0-0) = 0$

$= (1-\lambda)(1-2\lambda+\lambda^2) - 0(0) + 0(0) = 0$

$= (1-3\lambda+3\lambda^2-\lambda^3) - 0 + 0 = 0$

$= (-\lambda^3+3\lambda^2-3\lambda+1) = 0$

$= -(\lambda-1)(\lambda-1)(\lambda-1) = 0$

$= (\lambda-1) = 0$ or $(\lambda-1) = 0$ or $(\lambda-1) = 0$

= The eigenvalues of the matrix A are given by $\lambda = 1$,

1. Eigenvectors for $\lambda = 1$

$$A-\lambda I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now reduce this matrix

The System associated with the eigenvalue $\lambda = 1$

$$(A-1I)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$

= eigenvectors corresponding to the eigenvalue $\lambda = 1$ is

$$V = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$