IQRA NATIONAL UNIVESITY

PESHAWAR

SUBMITTED BY SHARIQ

ID 13698

SUBMITTED TO SIR DAUD

SUBJECT DISCRTE STRUCTURE.

Q1 :-

Answers :-

Let a be the first term and d be the common difference of the arthmetic sequence.

Then

$$a_n = a + (n-1)d \quad n \geq 1$$

$$\Rightarrow a_3 = a + (3-1)d$$

and $a_8 = a + (8-1)d$

Given that $a_3 = 7$ and $a_8 = 17$, therefore

$$7 = a + 2d \quad \text{----} \quad (1)$$

$$17 = a + 7d \quad \text{----} \quad (2)$$

Subtracting (1) from (2) we get

$$10 = 5d$$

$$d = 2$$

Subsituating $d = 2$ in (1) we have

$$7 = a + 2(2)$$

Which gives $a = 3$

Thus $a_n = a + (n-1)d$

$a_n = 3 + (n-1)2$ using value of a & d

Hence the value of 36^{th} term is

$$a_{36} = 3 + (36-1)2$$

$$= 3 + 70$$

$$= 73.$$

Q 2 :-

Answer. Given:

$$f(x) = 2x+3$$

$$g(x) = -x^2+5$$

Required:-

$$fog\,(x) = ?$$

$$gof\,(x) = ?$$

Solution:

For $fog\,(x)$

$$(fof)(x) = f(f(x))$$

$$= f(2x+3)$$

$$= 2(\;) + 3$$ insert value

$$= 2(2x+3)+3$$

$$= 4x+6+3$$

$$= 4x+9$$

Now for $(g \circ g)(x)$

$$(g \circ g)(x) = g(g(x))$$

$$= -(\ )^2 + 5$$ insert value

$$= -(-x^2 + 5)^2 + 5$$

$$= -(-x^2 + 5)^2 + 5$$

$$= -(x^4 - 10x^2 + 25) + 5$$

$$= -x^4 + 10x^2 - 25 + 5$$

$$= -x^4 + 10x^2 - 20$$

Answer

$$f \circ g\,(x) = 4x + 9$$

$$g \circ g\,(x) = -x^2 + 10x^2 - 20$$

Q3:-

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \qquad n \geq 1$$

Solution:

P(1) is true

For $n = 1$

L.H.S of $P(1) = 1^2 = 1$

R.H.S of $P(1) = \frac{1(1+1)(2(1)+1)}{6}$

$$= \frac{(1)(2)(3)}{6} = \frac{6}{6} = 1$$

L.H.S = R.H.S R' P(1) is true

Inductive steps

Suppose P(K) is true for some integers

$$1^2 + 2^2 + 3^2 + \cdots + K^2 = \frac{K(K+1)(2K+1)}{6}$$

To prove P(K+1) is true i.e:

$$1^2 + 2^2 + 3^2 + \cdots + (K+1)^2 = \frac{(K+1)(K+1+1)(2(K+1)+1)}{6}$$

Consider L·H·S of above equation (2)

$$1^2+2^2+3^2+\dots+(k+1)^2 = 1^2+2^2+3^2+\dots+k^2+(k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} = (k+1)^2$$

$$= (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right]$$

$$= (k+1)\left[\frac{k(2k+1)+6(k+1)}{6}\right]$$

$$= (k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

Q4:-

Answer.

Definition:- It defines the relationship between two different sets of information. If two sets are considered the relation between them will be established if there is connection between the element of two or more non-empty sets.

Types:-

* Empty relation
* Universal relation
* Identity relation
* Inverse relation
* Reflexive relation
* Symetric relation
* Transitive relation
* Equivalance relation

Empty relation:

An empty relation (or void relation) is one in which there is no relation between any element of a set.

Example:

$$R = \varphi \subset A \times A$$

Universal relation:-

Universal relation is one in which every element of a set is related to each other.

Example:

$R = A \times A$

Identity relation:

In an identity relation every element of a set is related to itself only.

$I = \{(a,a), a \in A\}$

Inverse relation:-

Inverse relation is seen when a set has element which are inverse pairs of another set.

Example:

$R^{-1} = \{(b,a) : (a,b) \in R\}$

Reflexive Relation:-

Every relation maps to itself.

Example: $(a,a) \in R$

Symetric relation:

If $a = b$ is true then $b = a$ is also true

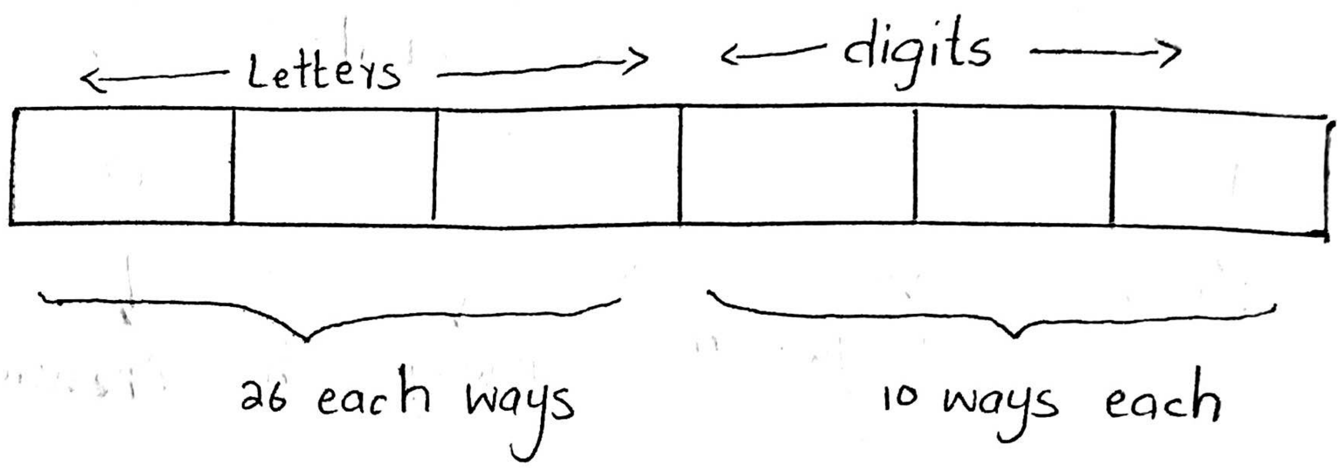
Example:

$aRb = bRa \;\forall\; a, b \in A$

Q5 :-

Answer :-

Each of the three letters can be written in 26 different ways and each of of the three digits can be written in 10 different ways.



Hence by the product rule there is total of

$$26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17,576,000$$

different licence plate possible.

b. The first and last place can be filled in one way only while each of the 2nd and 3rd places can be filled in 26 ways and each 4th and 5th can be filled in 10 ways.

← letters → ← digits →

A					0

one way ⏟ 26 ways each ⏟ 10 ways each One way

Number of licence plate that begin with A and end with zero are

$$1 \times 26 \times 26 \times 10 \times 10 \times 1 = 67600$$

3 Number of licence plates that begin with PQR are

$$1 \times 1 \times 1 \times 10 \times 10 \times 10 = 1000$$

← Letters → ← digits →

P	Q	R			

One way each ⏟ 10 ways each