QUESTION No: 1(a)

Identify:

$$\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$$

Solution:

$$\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$$

$$= \frac{\sqrt{2+0}-\sqrt{2}}{0} = \frac{\sqrt{2}-\sqrt{2}}{0}$$

$$= \frac{0}{0} \Rightarrow 0/0 \text{ From}$$

So then,

$$\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$$

Multiplying and dividing by $\sqrt{2+h}+\sqrt{2}$

$$= \lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h} \times \frac{\sqrt{2+h}+\sqrt{2}}{\sqrt{2+h}+\sqrt{2}}$$

$$= \lim_{h\to 0} \frac{(\sqrt{2+h}-\sqrt{2})(\sqrt{2+h}+\sqrt{2})}{(h)(\sqrt{2+h}+\sqrt{2})}$$

$$\lim_{h\to 0} \frac{\cancel{2}+h-\cancel{2}}{h(\sqrt{2+h}+\sqrt{2})}$$

$$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{2+h}+\sqrt{2})}$$

$$= \frac{1}{\sqrt{2+0}+\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}+\sqrt{2}}$$

$= \frac{1}{2\sqrt{2}}$ Ans

b) $y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$

Solution:-

$$y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$$

$$= (x + x^{-1})\frac{d}{dx}(x - x^{-1} + 1) + (x - x^{-1} + 1)(1 - x^{-2})$$

$$= x + x\frac{\cancel{1}}{x^2} + \frac{\cancel{1}}{x} + \frac{1}{x^2} + x - x\frac{1}{x^2} - \frac{\cancel{1}}{x} + \frac{1}{x^3} + 1 - \frac{1}{x^2}$$

$$= 2x + 1 - \frac{1}{x^2} + \frac{1}{x^3}$$ Ans

Q. Q2(a): $S = 160t - 16t^2 ft$

Given

a) $S = 160t - 16t^2$ ft

velocity is:

$$V = \frac{ds}{dt} = \frac{d}{dt}(160t - 16t^2)$$

$$= \frac{d}{dt}160t - \frac{d}{dt}16t^2$$

$$V = 160 - 32t$$

Maximum hight

$$V = 0$$

so

$$160 - 32t = 0$$

$$\frac{160}{32} = \frac{\cancel{32}t}{\cancel{32}}$$

$$t = 5 sec$$

$$S_{max} = S(5) = 160(5) - 16(5)^2$$

$$S_{max} = 400 ft$$

b) Given that

$$S = 256 ft$$

then

$$160t - 16t^2 = 256$$

$$16t^2 - 160t + 256 = 0$$

$$\frac{16}{16}(t^2 - 10t + 16) = \frac{0}{16}$$

$$\frac{\cancel{16}}{\cancel{16}}(t^2 - 10t + 16) = \frac{0}{16}$$

$$t^2 - 10t + 16 = 0$$

$t^2 - 8t - 2t + 16 = 0$

$(t-8)\ (t-2) = 0$

$t-8=0\ ,\ t-2=0$

$t_1 = 8sec \quad t_2 = 2sec$

Since

$V = 160 - 32t$

$t_1 = 2s$

$V_{(2)} = 160 - 32(2)$

$= 160 - 64$

$V_{(2)} = 96 m/s \Rightarrow$ velocity upward

$t_2 = 8s$

$V_{(8)} = 160 - 32(8)$

$= 160 - 256$

$= -96 m/s \Rightarrow$ velocity downward

c) Since

$V = 160 - 32t$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(160 - 32t)$

$a = 0 - 32 m/s^2$

$a = -32 m/s^2$

Q3 (a)

$$y = x^4 - 2x^2 + 2$$

Solution:

$$y = x^4 - 2x^2 + 2$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^2 + 2)$$

$$= \frac{d}{dx}x^4 - \frac{d}{dx}2x^2 + \frac{d}{dx}2$$

$$= 4x^3\frac{dx}{dx} - 2\times 2x\frac{dx}{dx} + 0$$

$$= 4x^3 - 4x + 0$$

$$\frac{dy}{dx} = 4x^2 - 4x$$

If the tangent is horizontal then $\frac{dy}{dx} = 0$

So,

$$4x^3 - 4x = 0$$

$$4x(x^2 - 1) = 0$$

$$4x = 0, \quad x^2 - 4 = 0$$

$$x^2 - 1 = 0$$

$$\frac{4x}{4} = \frac{0}{4}$$

$$x = 0 \qquad x^2 = 1$$

$$\sqrt{x^2} = \sqrt{1}$$

So,

$$x = 0, \ 1, -1$$

Their corresponding point is

$$y = x^4 - 2x^2 + 2$$

For $x = 0$

$$y = x^4 - 2x^2 + 2$$

$$= (0)^4 - (0)^2 + 2$$

$$= 0 - 0 + 2$$

$$y = 2$$

$x = 1$

$$y = x^4 - 2x^2 + 2$$
$$= (1)^4 - 2(1)^2 + 2$$
$$= 1 - 2 + 2$$
$$y = 1$$

$x = -1$

$$y = x^4 - 2x^2 + 1$$
$$= (-1)^4 - 2(-1)^2 + 2$$
$$= 1 - \cancel{2} + \cancel{2}$$
$$= 1$$

Hence, $(0,2)$ $(1,1)$, $(-1,1)$ //