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SECTION : A

SEMESTER : BS(SE) 4th

SUBJECT : CALCULUS AND ANALYTIC GEOMETRY

INSTRUCTOR : MUHAMMAD ABRAR KHAN

EXAMINATION : FINAL

Q 1) a) Differentiate $\frac{3x^4-2x^3+5}{x^3+1}$ w.r.to x.

SOLUTION:

$$\frac{3x^4-2x^3+5}{x^3+1}$$

formula : $\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$

$$=\frac{(x^3+1)(12x^3-6x^2+0)-(3x^4-2x^3+5)(3x^2+0)}{(x^3+1)^2}$$

$$=\frac{(x^3+1)(12x^3-6x^2)-(3x^4-2x^3+5)(3x^2)}{(x^3)^2+2(x^3)(1)+(1)^2}$$

$$=\frac{12x^6-6x^5-(9x^6-6x^5+15x^2)}{x^6+2x^3+1}$$

$$=\frac{12x^6-\cancel{6x^5}-9x^6+\cancel{6x^5}-15x^2}{x^6+2x^3+1}$$

$$=\boxed{\frac{3x^6-15x^2}{x^6+2x^3+1}}$$

1) b) Differentiate $\frac{(x^3+1)^2}{x^3\ \ 1}$ w.r.t x.

SOLUTION:

$$\frac{(x^3+1)^2}{x^3+1} = \frac{(x^3)^2 + 2(x^3)(1) + (1)^2}{x^3\ \ 1}$$

$$= \frac{x^6 + 2x^3 + 1}{x^3 - 1}$$

By using formula of quotient rule

$$= \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$= \frac{(x^3+1)(6x^5 + 6x^2 + 0) - (x^6+2x^3+1)(3x^2+0)}{(x^3-1)^2}$$

$$= \frac{6x^8 + 6x^5 + \cancel{6x^5} + 6x^2 - 3x^8 - \cancel{6x^5} - 3x^2}{x^6 - 2x^3 + 1}$$

$$= \frac{3x^8 + 6x^5 + 3x^2}{x^6 - 2x^3 + 1}$$

$$= \boxed{\frac{3(x^8 + 2x^5 + x^2)}{x^6 - 2x^3 + 1}}$$

Q 2) a) Find integration of $\int \frac{1}{\sqrt{x^5}} dx$.

SOLUTION:

Let $x^5 = u$ → putting in given equation

$\int \frac{1}{\sqrt{u}} dx$ → By substitution method.

$$= \int \frac{1}{u^{1/2}} dx$$

$$= \int u^{-1/2} dx$$

$$= \frac{u^{-1/2+1}}{-1/2+1}$$

$$= \frac{u^{1/2}}{+1/2} = 2u^{1/2}$$

putting $u = x^5$

$$= 2(x^5)^{1/2}$$

$$= \sqrt{2x^5}$$ Ans.

2) b) Find integration of $\int \frac{1}{(8x+7)^8} dx$

SOLUTION:

$\int \frac{1}{(8x+7)^8} dx \rightarrow ①$ By substitution method.

let $(8x+7) = u$ putting in ① we get

$= \int \frac{1}{u^8} dx$

$= \int u^{-8} dx$

$= \frac{u^{-7}}{-7}$ putting value of u.

$\frac{(8x+7)^{-7}}{-7}$

$= \frac{7}{(8x+7)^7} = \frac{7}{(8x+7)^7}$ Ans

Q 3) a) Find the integration of $\int \frac{-x+9}{2x^2-8x+6}dx$ by partial fractions.

SOLUTION:

$\int \frac{-x+9}{2x^2-8x+6}dx$ By partial fraction.

$$\int \frac{-x+9}{2(x^2-4x+3)} = \int \frac{-x+9}{2(x-3)(x-1)}$$

$$\frac{-x+9}{2(x-3)(x-1)} = \frac{A}{2(x-3)} + \frac{B}{(x-1)}$$

$$-x+9 = A(x-1) + 2B(x-3) \rightarrow (1)$$

putting $x = 1$ and $x = 3$ in (1)

$$-1+9 = A(+1-1) + 2B(+1-3)$$

$$8 = 0 + 2B(-2)$$

$$8 = -4B$$

$$B = -\frac{8}{4} = -\frac{2}{1}$$

$$\boxed{B = -2}$$

putting $x = 3$ in ①

$$-3 + 9 = A(3-1) + 2B(3-3)$$

$$6 = A(2) + 0$$

$$A = \frac{6}{2}$$

$$\boxed{A = 3}$$

$$\int \frac{-x+9}{2(x-3)(x-1)} = \int \frac{3}{2(x-3)}dx + \int \frac{-2}{(x-1)}dx$$

$$= \frac{3}{2}\int \frac{1}{(x-3)}dx - 2\int \frac{1}{(x-1)}dx$$

$$= \frac{3}{2}\int (x-3)^{-1}dx - 2\int (x-1)^{-1}dx$$

$$= \frac{3}{2}\frac{(x-3)^{0}}{0}dx - 2\frac{(x-1)^{0}}{0}dx$$

$$= \frac{3}{2} - 2$$

$$= \frac{3-4}{2}$$

$$= \frac{-1}{2} \text{ Ans.}$$

3) b) Find integration of $\int \frac{4x^2 + 8x}{(x^{2+1})(x^2+2x+3)} dx$ by partial fraction.

SOLUTION:

$$\int \frac{4x^2+8x}{(x^{2+1})(x^2+2x+3)} dx$$

$$\frac{4x^2+8x}{(x^3)(x-3)(x+1)} = \frac{A}{x^3} + \frac{B}{(x-3)} + \frac{C}{(x+1)} \rightarrow (1)$$

$$4x^2+8x = A(x-3)(x+1) + B(x^3)(x+1) + C(x^3)(x-3) \rightarrow (2)$$

putting $x = 3$ in (2) we get

$$4(3)^2 + 8(3) = A(3-3)(3+1) + B(3)^3(3+1) + C(3)^3(3-3)$$

$$60 = 0 + B(108) + 0$$

$$B = \frac{60}{108} = \frac{5}{9}$$

putting $x = -1$ in (2) we get

$$4(-1)^2 + 8(-1) = A(-1-3)(-1+1) + B(-1)^3(-1+1) + C(-1)^3(-1-3)$$

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$$-4 = 0 + 0 + C(-1)(-4)$$

$$C = \frac{-4}{4} = 1$$

$$\boxed{C = 1}$$

$$4 = A$$

$$\boxed{A = 4}$$

Now integrating:

$$= 4\int \frac{1}{x^3} + \frac{5}{9}\int \frac{1}{(x-3)} + 1\int \frac{1}{(x+1)}$$

$$= 4\int x^{-3} + \frac{5}{9}\int (x-3)^{-1} + \int (x+1)^{-1}$$

$$= 4\,\frac{x^{-2}}{-2} + \frac{5}{9}(1) + (1)$$

$$= \frac{2x^{-2}}{-1} + \frac{5}{9} + 1$$

$$= \frac{1}{2x^2} + \frac{14}{9}$$

$$= \frac{1}{2x^2} + \frac{14}{9} \quad \text{Ans}$$

Q4) a) $X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$

SOLUTION:

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

$$= X = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 5-3 & 1-1 \\ -3-2 & 1-2 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 0 \\ -5 & -1 \end{bmatrix} \text{ Ans}$$

b) $X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$

SOLUTION:

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 3 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & -2 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -2 \\ 3 & -1 \end{bmatrix} \text{ Ans}$$

c) $$X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

SOLUTION:

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2I$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \text{ Ans}$$

Q 5) $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

Find $A^2 + BC$

SOLUTION:

$$B \times C = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$BC = \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 0+0 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} -2 & 8 \\ 6 & 1 \end{bmatrix} \text{ Ans.}$$