Name = Farhan Akhtar

ID = 15554

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Q1 = A man throws two fair dice, what is the Conditional Probability that the Sum of two dice will be 7 :

1. The Sum is even
2. The Sum is greater than 8
3. The two dice had the Same Outcome.

Solution :—

Let $A = \{\text{The Sum is } 7\}$

$B = \{\text{The Sum is even}\}$

$C = \{\text{The Sum is greater than } 8\}$

$D = \{\text{The two dice lead the Same OutCome}\}$

$A = \{(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)\}$

$B = \{(1,3)(1,5)(2,2)(2,4)(2,6)(3,1)$
$(3,3)(3,5)(4,2) \ldots\ldots (6,6)\}$

$C = \{(3,6)(4,5)(4,6)(5,4)(5,5)(5,6)(6,3)$
$(6,4)(6,5)(6,6)\}$

$D = \{(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)\}$

$(A \cap B) = \{\ \}$

$(A \cap C) = \{\ \}$

$(A \cap D) = 9$

$P(A) = 6/36 \quad A(B) = \frac{17}{36}, \ P(C) = 10/36$

$P(D) = 6/36$

$P(A \cap B) = \frac{6}{36}, \ P(A \cap C) = \frac{6}{36}$ and

$P(A \cap D) = 0$

Hence

$$P(A/B) = P\left(\frac{A \cap B}{P(B)}\right) = \frac{6}{36} \times \frac{36}{18} = \frac{1}{3}$$

$$P(A/C) = P\left(\frac{A \cap C}{P(C)}\right) = \frac{\cancel{6}}{\cancel{36}} \times \frac{\cancel{36}}{\cancel{10}} = \frac{3}{5}$$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = 0 \times \frac{36}{6} = 0$$

① Q :- Show that in a Single throw of two dice, the Probability of throwing more than 7 is equal to that of showing less than 7, and hence find the probability of throwing exactly 7.

Solution :-

Sum of 2 has 1 way 1,1
Sum of 3 has 2 ways 1,2 & 2,1
Sum of 4 has 3 ways 1,3 ; 2,2 ; 3,1
5 has ways
6 has 5 ways
8 has 5 ways (Symmetry)
9 has 4 ways
10 has 3 ways
11 has 2 ways
12 has 1 way

Those are 15/36 for each Side with a Sum of 30/36.

That leaves a $6/36 = 1/6$ Probability for a Sum of 7.

Q3 :- A and B play a game in which A's probability of winning in 2/3. In Series of 8 games what is the probability that A will win.

1. Exactly 4 games
2. At least 4 games
3. From 3 to 6 games

Sol:- Given that :

$$P = 2/3 \quad n = 8$$

$$q = 1 - P$$

$$= 1 - 2/3$$

$$q = 1/3$$

Let "x" donates the number of games won by A then

(i) $P(x = 4)$

$$= \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4$$

$$= \frac{1120}{6561}$$

$$= 0.1707$$

(ii) $P(X \geq 4)$

$$1 - P(x < 4)$$

$$= 1 - \sum_{x=0}^{3} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= 1 - \left[\left(\frac{1}{3}\right)^{8} + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{7} + 28 \left(\frac{2}{3}\right)^{2}\left(\frac{1}{3}\right)^{6} + 56\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5}\right]$$

$$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561}$$

$$= 0.9121$$

3. $P(3 \leq x \leq 6)$

$$\sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3}\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} + \binom{8}{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{4} +$$
$$\binom{8}{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{3} \binom{8}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{2}$$

$$= \frac{8}{(3)^8}\left[56+140+224+224\right]$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561}$$

$$= 0.7852$$

Q4:- Let $C_1, C_2 \cdots CMC_1, C_2 \cdots, CM$ be a partition of the Sample Space SS, and AA and BB be two events, Suppose we know that

- A and B are Conditionaly independent given C_i, for all $i \in \{1, 2 .. M\}$

* B is Independent of all C_is.

=> Prove that A and B are Independent.

Ans :- Proof :-

Since the C_is from a partition of the Sample Space we can apply the law of total probability for $A \cap B$.

$$P(A \cap B) = \sum_{i=1}^{m} P(A \cap B / C_i) P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^{m} P(A/C_i) P(B/C_i) P(C_i)$$

:- (A and B are Conditionaly Independent)

$$P(A \cap B) = \sum_{i=1}^{m} P(A/C_i)\, P(B)\, P(C_i)$$

$\therefore$ (B is Independent of all C_i)

$$P(A \cap B) = P(B) \sum_{i=1}^{m} P(A/C_i)\, P(C_i)$$

$$P(A \cap B) = P(B)\, P(A)$$

law of total Probability.

Hence A and B are Independent.

Q5 :-

Solution :-

⇒ Binomial distribution :-

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

for $x = 0, 1, 2 \ldots\ldots n$

$\mu = np$ // mean

$\sigma = np(1-p)$ // variance

A binomial random variable can be thought of as the sum of n independent Bernoulli random variables each with mean p and variable $p(1-p)$.

Let $U_1 \ldots U_n$ be independent Bernaulli random variables.

$\Sigma(U_i) = p$ and $Var(U_i) = p(1-p)$

$X = U_1 + \ldots\ldots U_n$

$Var(X) = Var(U_1) + \ldots\ldots Var(U_n)$

The binomial theorem :-

$$(a+b)^m = \sum_{y=0}^{m} \binom{m}{y} a^y b^{m-y}$$

$$E(x) = \sum_{x} x p(x)$$

$$= \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-1)-(x-1)!} \cdot p^{x-1} (1-p)^{(n-1)-(x)}$$

$$m = (n-1),\ y = (x-1)$$

$$= np \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} \binom{m}{y} p^y (1-p)^{m-y}$$

Now,

$$Var(X) = E[(x-\mu)^2]$$

$$= \sum_x (x-\mu)^2 \, p(x)$$

$$E[(x-\mu)^2] = E(x^2) - [E(x)]^2$$

$$E[x(x-1)] = \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} \cdot p^x (1-p)^{n-x}$$

$$E[x(x-1)] = n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{(n-2)-(x-2)}$$

By binomial theorem.

$$E[x(x-1)] = n(n-1)p^2$$

$$E(x^2 - x) = n(n-1)p^2$$

$$E(x^2) - E(x) = n(n-1)p^2$$

Since :-

$\Rightarrow$ $E(x) = np$, which is mean of binomial.

$\Rightarrow$ $E(x^2) = n(n-1)p^2$

$$E(x^2) = n(n-1)p^2 + np$$

$$Var(X) = E(x^2) - [E(x)]^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$Var(x) = np[(n-1)p + 1 - np]$$

$\Rightarrow$ This is variance of binomial distribution.

Q6 :- Differentiate between bi-nominal frequency distribution and Bi-nominal ~~dis~~ distribution with the help of formulas?

Ans : **Binominal distribution :-**

A binominal distribution Can be though of as simply the probability of a success or failure outcome in an experiment or Servey that is repeated multiple times.

$$P(x-x)\ f(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

Binominal frequency distribution :-

If the binominal probability distribution is multiplied by N the number of experiment or sets, the delection distribution is known as binominal frequency distribution.

$$N\binom{n}{x}\left(p^{x}q^{n-x}\right)$$

Q 7 :-

Solution :-

Measure	Data Set of A	B	C	D
Co-efficient of Variation	$CV = \frac{3}{45} \times 100$	$CV = \frac{11}{60} \times 100$	$CV = \frac{5}{50} \times 100$	$CV = \frac{15}{25} \times 100$
	$CV = 6.7$	$CV = 18.3$	$CV = 10$	$CV = 60$

Ans