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SECTION	"B"
Deptt:	BE Civil

Q.No 1

Solve the following objective is type question

1: The order of matrix A is $m \times p$ and the order of B is $p \times n$.

The order of Matrix A is $m \times p$ and the order of matrix B is $p \times n$ then order of Matrix AB is?

The order of Matrix AB $= m \times n$

2: The number of non zero rows is Echelon form?

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \end{bmatrix}$

The number of non zero rows less then or equal to the disnit of matrin depend on the row operation of matrix.

iii - If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = ?$

Sol:-

Singular matrix then $a = ?$

$|B| = 0$ $\quad \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$

$|B| = 1 \times a - 4 \times 2 = 0$

$\Rightarrow a - 8 = 0$

So,

$a = 8$ Ans:-

IV:- If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Sol:-

$\begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$= -2i^2 - i^2$ $\quad \because i^2 = -1$

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$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$$= 3 \text{ Ans:-}$$

VI) Solution of $\frac{dy}{dx} + 2xy = y = ?$

Solution:-

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1 - 2x)$$

$$\frac{dy}{dx} = (1 - 2x)dx$$

$$\int \frac{dy}{y} = \int (1 - 2x)dx$$

$$\ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\ln y = x - x^2 + c$$

$$e^{\ln y} = e^{x^2 - x + c}$$

$$y = e^{x - x^2 + c}$$ Ans:-

(V) The Matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Solution:-

$\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Scalar Matrix is $\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ Ans:

(Vii) The order and degree of differential equation?

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

Solution:-

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Squar on b/s

$$\left(\frac{dy}{dx}\right)^{3\times 2} = 1 + \left(\frac{dy}{dx}\right)^2$$

$$\left(\frac{dy}{dx}\right)^6 = 1 + \left(\frac{dy}{dx}\right)^2$$

order = 1

Degree = 6

viii) The order and degree differential equation

$$\frac{d^2Y}{dx^2} - 4xY = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is } = ?$$

Solution:-

$$\left(\frac{d^2y}{dx^2}\right)^1 - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$$

Sol:- order = 2

Degree = 1

(ix)- The differential equation $2\frac{dy}{dx} + x^2Y = 2x + 3,\ Y(0) = 5$ is ?

$$2\frac{dy}{dx} + x^3y = 2x + 3$$

$$\int 2\,dy = \int (2x + 3 - x^2y)\,dx$$

$$2y = \frac{2x^2}{2} + 3x - y\frac{x^3}{3} + c$$

$$2y = \frac{2x^3}{2} + 3x - y\frac{x^3}{3} + c$$

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{6} + c$$

Put $x = 0$, $y = 5$

$$5 = 0 + 0 - 0 + c$$

$$5 = c$$

{So it is homgeneous Equation}

$$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$-\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0 \right]$$

$$\Rightarrow -\left[-(-2+\lambda-1) + 1(6-3\lambda-2\lambda+\lambda^2-1)\right.$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \longrightarrow C$$

Put a, b and c in eq B

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2$$
$$+6\lambda-8$$

(X) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Sol:-

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1-1 & b-a & b^2-a^2 \\ 1-1 & c-a & c^2-a^2 \end{vmatrix} \begin{matrix} \\ R_2 - R_1 \\ R_3 - R_1 \end{matrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$

Expand by C_1

$$= 1\begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix} - 0 + 0$$

$$= \{(b-a)(c^2-a^2)\} - \{(b^2-a^2)(c-a)\}$$

$$= (b-a)(c-a)(c-\cancel{a}-b+\cancel{a})$$

$$= (b-a)(c-a)(c-b)$$ Ans:.

Q#2:- a

Express the Determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c_3 \end{vmatrix}$$

So the Product of factors which are linear in a, b, c.

Sol:-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3 - c^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$$

a.b, c common

$\ast$ $abc\,(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$

$\ast$ $abc\,[bc\,(c - b) - ac\,(c + a) + ab\,(b - a)]$ Ans:-

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Q # 2:-

Part, B:-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution:-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic equtian $\rightarrow |A - \lambda I| = 0 \rightarrow$ A

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Take Determinent

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} = 0 \longrightarrow (B)$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expand by R_1

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -1 - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$(3-\lambda)\Big[\big((3-\lambda)(2-\lambda)-(-1)(-1)\big)+1\big((-1)(2-\lambda)-(-1)$$

$$(-1)\big)-1\big((-1)(-1)-(-1)(3-\lambda)\big)\Big]$$

$$=(3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)(+1+3-\lambda)$$

$$=\ (3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$$

$$=\ 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$=\ \boxed{-\lambda^3-8\lambda^2-18\lambda\ \ +8}\rightarrow \text{(A)}$$

$$\Rightarrow +1\begin{vmatrix}-1 & -1 & -1\\ -1 & 3-\lambda & -1\\ 0 & -1 & 2-\lambda\end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1\begin{vmatrix}3-\lambda & -1\\ -1 & 2-\lambda\end{vmatrix}-(-1)\begin{vmatrix}-1 & -1\\ -1 & 2-\lambda\end{vmatrix}+0$$

$$\Rightarrow\ -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$\Rightarrow\ -\lambda^2-5\lambda-5-3+\lambda$$

$$=\ \boxed{-\lambda^2+6\lambda-8}\ \text{—}\ (b)$$

$$= -2\lambda^3 + 16\lambda^2 - 36\lambda + 16 + \lambda^4 - 8\lambda^3 + 18\lambda^2 - 8\lambda$$

$$-\lambda^2 + 6\lambda - 8 - \lambda^2 + 16\lambda - 8$$

$$\Rightarrow \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 16\lambda^2 - \lambda^2 - \lambda^2$$

$$-36\lambda - 8\lambda + 6\lambda + 6\lambda + 16 - 16$$

$$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$$

Now:-

Solve upper equation By synthetic Equation

	1	-10	32	-32
2			-16	32
	1	-8	16	0

We get:-

$$(\lambda - 2)(\lambda^3 - 8\lambda + 16\lambda) = 0$$

$$\Rightarrow \lambda(\lambda - 2)(\lambda^2 - 8\lambda + 16) = 0$$

$\lambda = 0$ | $\lambda - 2 = 0$, $\lambda = 2$ | $\lambda^2 - 8\lambda + 16 = 0$

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda-4) - 4(\lambda-4) = 0$

$(\lambda - 4) = 0$ | $(\lambda - 4) = 0$

$\lambda = 4$ | $\lambda = 4$

So,

$\lambda_1 = 0$

$\lambda_2 = 2$

$\lambda_3 = 4$

$\lambda_4 = 4$ Ans:-

(1)

Q # 3: The rate of change in the form of differential equation is given by?

$$(x^2 + 3y^2)\,dx - 2xy\,dy = 0$$

$x = 2$, $y = 6$

Sol:

$$(x^2 + 3y^2)\,dx - 2xy\,dy = 0$$

$$\Rightarrow (x^2 + 3y^2)\,dx = 2xy\,dy$$

$2xydx$ divide on B.S

know we get.

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow (*)$$

Let $y = vx$

(4)

Diff

$$dy = Vdx + xdv$$

Dividing by dx

$$\frac{dy}{dx} = v + \frac{xdv}{dx} \rightarrow (a)$$

Put (a) in *

$$v + \frac{xdv}{dx} = \frac{1}{2}\left[\frac{x}{xv} + 3\frac{vx}{x}\right]$$

$$v + \frac{xdv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying B.S by 2.

$$2v + \frac{2xdv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiplying B.S by $\frac{dx}{dv}$

We get

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Multiplying bothsides by $\frac{v}{x(1+v^2)}$

We get

$$\frac{2v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on both sides

$$\int\frac{2v}{1+v^2}\,dv = \int\frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Taking "e" on bothsides

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

$1+v^2 = xc$

Put $v = y/x$

$1+(y/x)^2 = xc$

$x^2+y^2 = x^3 c \rightarrow (**)$

Put $x=2$, $y=6$ in

$(4)+(36) = 8c$

$c = \frac{40}{8}$

$\underline{c = 5 \quad \text{Put in } (**)}$

So $x^2+y^2 = 5x^3$

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x-1)$

Taking "$\sqrt{\ }$" on B.S

$\boxed{y = +x\sqrt{5x-1} \quad , y = -x\sqrt{5x-1}}$ OR.