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QNo 1 :-

$$g(t) = \begin{cases} 0, & t<0 \\ t^2 & 0 \le t \le 3 \\ 2t+3, & 3 < t \le 4 \\ 12, & t>4 \end{cases}$$

(a) first we cheak continuity at $t=0$

Condition (i) $g(0) = (0)^2 = 0$, which is defined

Condition (ii) $L.H.S = \lim_{t \to 0^-} g(t)$

$= \lim_{t \to 0^-} (0) = 0$

$$R.H.S = \lim_{t \to 0^+} g(t)$$

$$= \lim_{t \to 0^+} t^2 = (0)^2 = 0$$

$$\boxed{\text{So } \lim_{t \to 0} g(t) = 0}$$

Thus $g(t)$ is continus at $t = 0$

Condition (iii) $\lim_{t \to 0} g(t) = g(0)$

b at $t = 3$

Condition (i) $g(t) = t^2$

$g(3) = (3)^2 = 9$

which is defined.

Condition (ii) $L.H.S = \lim_{t \to 3^-} t^2 = 9$

$R.H.S = \lim_{t \to 3^+} (2t+3) = 2(3) + 3 = 9$

So $\lim_{t \to 3} g(t) = 9$

Condition (iii) $\lim_{t \to 3} g(t) = g(3)$

Now we check at $t=4$.

Condition (i) $g(t) = 2t+3$

$$g(4) = 2(4)+3 = 11$$

Condition (ii) $L.H.S = \lim_{t \to 4} (2t+3)$

$$= 2(4)+3 = 11$$

$R.H.S = \lim_{t \to 4^+} (12) = 12$

So $\lim_{t \to 4} g(t)$ does not exist

So $g(t)$ is only discontinous at $t=4$.

Q2 Find the Maclaurin Series for $Y(x) = x^2 + \sin x$

Sol #

As we Know that.

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots + \frac{x^n f^n(0)}{4!} \quad \text{---(1)}$$

$$f(x) = x^2 + \sin x$$

$$Y(x) = x^2 + \sin x$$

$$Y(x) = x^2 + \sin x \Rightarrow Y(0) = 0 + 0 = 0$$

$$Y'(x) = 2x + \cos x \Rightarrow Y'(0) = 2(0) + \cos\cos 1$$

$$Y''(x) = 2 - \sin x \Rightarrow Y''(0) = 2 - \sin\cos = 2$$

$$Y'''(x) = \cos x \Rightarrow Y'''(0) = -\cos(0) - 1$$

$$Y(x) = \sin x \Rightarrow Y^{IV}(0) = 0$$

$$Y(x) = \cos x \Rightarrow Y^{V}(0) = 1$$

$$Y(x) = -\sin x \Rightarrow Y^{VI}(0) = 0$$

Putting these value in Equ # (1)

$$= 0 + x\,1 + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(-1) + \frac{x^4}{4!}(0) + \frac{x^5}{5!} + \frac{x^6}{6!}(0) \quad \text{—} (1)$$

$$= x + \frac{2x^2}{2!} - \frac{x^3}{3!} + 0 + \frac{x^5}{5!} + 0 + \cdots\cdots$$

$$= x + \frac{2x^2}{2} - \frac{x^3}{3!} + \frac{x^2}{5!} + 0 \cdots\cdots$$

$$= x + x^2 - \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} \cdots\cdots$$

is the required Moclousis Expossion.

QNo 3 (i)

$$1 + xy = x^2 + y^2$$

$$\frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(x \cdot y) = \frac{d}{dx}(x^2) + \frac{d}{dx}(y^2)$$

$$0 + x \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}(x) = 2x + 2y \cdot \frac{dy}{dx}$$

$$\Rightarrow x \cdot \frac{dy}{dx} + y \cdot (1) = 2x + 2y \cdot \frac{dy}{dx}$$

$$\Rightarrow x \cdot \frac{dy}{dx} + y = 2x + 2y \cdot \frac{dy}{dx}$$

$$\Rightarrow x \cdot \frac{dy}{dx} - 2y \frac{dy}{dx} = 2x - y$$

$$\Rightarrow (x - 2y) \frac{dy}{dx} = 2x - y$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{2x-y}{x-2y}}$$

$$\frac{dy}{dx} = \frac{2x-y}{x-2y}$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\cdot\frac{d}{dx}(2x-y)-(2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$= \frac{(x-2y)\cdot\left(2-\frac{dy}{dx}\right)-(2x-y)\left(-2\frac{dy}{dx}\right)}{(x-2y)^2}$$

$$= \frac{(x-2y)\left(2-\frac{dy}{dx}\right)-(2x-y)\left(-2\frac{dy}{dx}\right)}{(x-2y)^2}$$

$$= 2x-4y-x\frac{dy}{dx}+xy\frac{dy}{dx}-\left(-4x\cdot\frac{dy}{dx}+2y\frac{dy}{dx}\right.$$

$$= \frac{2x-4y-x\frac{dy}{dx}+2y\frac{dy}{dx}+4x\cdot\frac{dy}{dx}-2y\frac{dy}{dx}}{(x-2y)^2}$$

$$= \frac{2x-4y+3x\frac{dy}{dx}}{(x-2y)^2}$$

$$= \frac{2x-4y+3x\left(\frac{2x-y}{x-2y}\right)}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{2x-4y+3x\left(\frac{2x-y}{x-2y}\right)}{(x-2y)^2}$$

$$= \frac{2x-4y+6x^2-3yx/(x-2y)}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{2x-4y+\frac{6x^2-3xy}{(x-xy)}}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{(2x-4y)(x-2y)+6x^2-3xy}{(x-xy)}}{}$$

$$= \frac{\frac{2x^2-8xy+8y^2+6x^2-3xy}{(x-2y)}}{(x-2y)^2}$$

$$= \frac{2x^2-8xy+8y^2+6x^2-3xy}{(x-2y)^3}$$

$$\frac{d^2y}{dx^2} = \frac{8x^2+8y^2-11xy}{(x-2y)^3}$$ Note $x-2y \neq 0$

QNo 3 b Part:-

$$Y = x^3 (1+x)^9 . e^{6x}$$

$$\ln y = \ln (x^3 (1+x)^9 . e^{6x})$$

$$= \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$$

$$= 3 \ln x + 9 \ln (1+x) + 6x \cdot \ln e$$

$$\ln y = 3 \ln x + 9 \cdot \ln (1+x) + 6x(1)$$

Take derivative both side.

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(3 \cdot \ln x) + \frac{d}{dx}(9 \cdot \ln(1+x)) + \frac{d}{dx}(6x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 3 \cdot \frac{d}{dx}(\ln x) + 9 \cdot \frac{d}{dx}(\ln(1+x)) + \frac{6 \cdot d}{dx}(x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 3 \cdot \left(\frac{1}{x}\right) + 9 \cdot \left(\frac{1}{1+x}\right) + 6(1)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+x} + 6$$

$$\frac{dy}{dx} = y\left(\frac{3}{x} + \frac{9}{1+x} + 6\right)$$

$$\frac{dy}{dx} = \left(x^3(1+x)^9 e^{6x}\right)\left(\frac{3}{x}+\frac{9}{1+x}+6\right)$$

Ans:-