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program BSSE

Subject differential equation

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Q1 (a) 2nd order linear homogenous (non-homogenous) differential equation

A differential equation of only order is homogenous if once all the terms involving the unknown function are collected together on one side of the equation the other side is identically zero.

i.e:-
$y'' - 2y' + y = 0$ is a 2nd order homogenous DE

A ~~diff~~ differential equation of any order is non-homogenous if once all the terms involving the unknown functions are collected together on one side the other side is not identically zero.

i.e $y'' - 2y' + y = 4$.

Q 1

(i) $4y'' - 6y' + 7y = 0$

~~proof~~ Let's find the root of the charateristic equation

$4r^2 - 6r + 7 = 0 \Rightarrow r = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(7)}}{2(4)}$

$r = \frac{6 \pm \sqrt{36 - 112}}{8}$

$r = \frac{6 \pm \sqrt{-76}}{8}$

$r = 6/8 + 2\sqrt{19}i$

$r = 3/4 \pm \frac{\sqrt{19}i}{4}$

$r_1 = 3/4 + \frac{\sqrt{19}i}{4}, \quad r_2 = 3/4 - \frac{\sqrt{19}i}{4}$

So it has the complex conjugate roots $\varphi_1(x) = e^{r_1(x)} \cos r_1(x)$
$\varphi_2(x) = e^{r_2 x} \sin r_2(x)$

$y = C_1 e^{3/4 x} \cos i\sqrt{19}/4 (x) + e^{3x/4} \sin i\sqrt{19}/4 (x) \, C_2.$

Q1 (ii) $y'' - 4y - 12y = 3e^{5x}$

Solution

The characteristic equation and it's roots:

$$y^2 - 4y - 12 = (y-6)(y+2) = 0$$

$$y' = -2, \quad y_2 = 6$$

The Complementary Solution is then,

$$y_1(t) = C_1 e^{-2t} + C_2 e^{rt}.$$

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Q2

(i) $16y'' - 40y' + 25y = 0$ $y(0) = 3$ $y'(0) = -9/4$

Solution

Below are the characteristics and it's roots:

$16y^2 - 40y + 25 = (4y^{(2)} - 5)^2 = 0$ $y_1 = 5/4 =$ $y^2 = 5/4$

General solution and it's derivatives are

$y(t) = C_1 e^{5t/4} + C_2 e^{5t/4}$

$y(t) = 5/4\, C_1 e^{5t/4} + C_2 e^{5t/4} +$ $5/4\, C_2\, t e^{5t/4}$

By putting in the initial condition.

$3 = y(0) = C_1$

$-9/4 = y'(0) = 5/4\, C_1 + C_2$

The Solution for IVP is than $y^t = 3e^{5t/4} - 6te^{5t/4}$.

Q2 (ii)

$y'' + 14y' + 49y = 0 \quad y(-4) = -1 \; y'(-4) = 5$

Solution

The characteristic equation and it's root are:

$y^2 + 14y + 49 = (y+7)^2 = 0 \quad y_1 = 7$

$y_2 = -7$

The general solution and it's derivatives are $y(t) = C_1 e^{-7t} + C_2 t e^{-7t}$

$y't = -7 C_1 e^{-7t} + C_2 e^{-7t} - 7 C_2 t e^{-7}$

putting in the initial condition.

$-1 - y(-4) = C_1 e^{28} - 4 C_2 e^{28}$

$5 - y(-x) = -7 C_1 e + C_1 e^{28} + 28 C_2 e^{28}$

$-7 C_1 e^{28} + 29 C_2 e$

it gives the following constant by solving

$C_1 = -9e^{-28}$

$C_1 = -2e^{-28}$

The Solution for IVP is

$y(t) = -9e^{-28} e^{-7t} - 2te^{-28} e^{-7t}$

$y(t) = -9e^{-7}e^{t(-4)} - 2te^{-7(t+4)}$

Q2 (iii)

$y'' - 4y' + 9y = 0 \quad y(0) = 0, \; y'(0) = -8$

The characteristic equation for this DE is

$$y^2 - 4y + 9 = 0$$

The roots of the equation for this are

$y_1 = 2 + \sqrt{5}i$
$y_2 = 2 - \sqrt{5}i$

The general solution to the differential equation is then

$y(t) = C_1 e^{2t} \cos(\sqrt{5}t) + C_2 e^{2t} \sin(\sqrt{5}t)$

Applying initial condition doing with derivation

$y(t) = C_2 e \sin(\sqrt{5}t)$
$y(t) = 2C_2 e^{2t} \sin(\sqrt{5}t) +$

$\sqrt{5} C_1 e^{2t} \cos(\sqrt{5}t)$

$-8 = y'(0) = \sqrt{5}(2) = C_2 = -8/\sqrt{5}$

Q2

(iv) $y'' - 8y' + 17y = 0 \quad y(0) = 4 \; y'(0) = -1$

The charactirist equation roots are

$y^2 - 8y + 17 = 0$
$y_1 = 4 + i$
$y_2 = 4 - i$

The general solution as well as derivative is,

$y(t) = C_1 e^{4t} \cos(t) + C_2 e^{4t} \sin(t)$

$y'(t) = 4C_1 e^{4t} \cos(t) - C_1 e^{4t} \cos(t) + 4C_2 e^{4t} \sin(t) + C_2 e^{4t} \cos(t)$

By applying the initial condition gives the following

$-4 = y(0) = C_1$
$-1 \; y'(0) = 4C_1 + C_2$

The solution is then
$y(t) = -4e^{4t} \cos(t) + 15e^{4t} \sin(t)$.

Solution is then

$y(t) = -8/5\, e^{2t} \sin.$

Q3 Define Laplace transform along with example find the laplace of the transform of the given functions.

$f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$

part (i) $F(s)\ 6 \frac{1}{s-(-5)} + \frac{1}{s-3} + 5 \frac{3}{s^{3+1}} - 9/s$

$= 6/s+5 + 1/s-3 + 30/s^4 - 9/s$

Q3

part (ii) $g(t) = 4 \cos(4t) - 9 \sin(4t) + 2 \cos(10t)$

$g(s) = 4 \frac{s}{s^2+(4)^2} - 9 \frac{4}{s^2+(4)^2} + 2 \frac{s}{s^2+(10)^2}$

$= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}$

Q3 part 3

$H(t) = e^{3t} + \cos(6t) - e^{3t}\cos(6t)$

Solution

$H(s) = \frac{1}{s-3} + \frac{s}{s^2+(6)^2} - \frac{s-3}{(s-3)^2+(6)^2}$

$= \frac{1}{s-3} + \frac{s}{s^2+36} - \frac{s-3}{(s-3)^2+36}$

Q 4 Solve the following ivp using laplace forms

part 1 $y'' - 10y' + 9y = 5t \quad y(0) = -1 \quad y'(0) = 2$

Solution

Taking transform of every term

$\mathcal{L}\{y''\} - 10\,\mathcal{L}\{y'\} + 9\,\mathcal{L}(y) = \mathcal{L}(5t)$

By formula, we get

$s^2 Y(s) - s y(0) - y'(0) - 10(sY(s) - y(0)) + 9Y(s) = 5/s^2$

putting in the initial conditions

$(s^2 - 10s + 9)\,Y(s) + s - 12 = 5/s^2$

Solve for $Y(s)$

$$Y(s) = \frac{5}{s^2(s-9)(s-1)} + \frac{12-s}{(s-9)(s-1)}$$

$$Y(s) = \frac{5 + 12s^2 - s^3}{s^2(s-9)(s-1)}$$

The practical function of transform will be

$$Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-9} + \frac{D}{s-1}$$

$$5 + 12s^2 - s^3 = A \cdot s(s-9)(s-1) + B(s-9)(s-1) + Cs^2(s-1) + Ds^2(s-9)$$

Solving for constants

$s = 0$ $5 = 9B \Rightarrow B = 5/9$
$s = 9$ $16 = -8D \Rightarrow D = -2$
$s = 2$ $248 = 648C \Rightarrow C = 31/8$
$45 = -14A + \frac{4345}{81} \Rightarrow A = 50/81$

plugging in the constant gives

$$Y(s) = \frac{50/81}{s} + \frac{5/9}{s^2} + \frac{31/81}{s-9} - \frac{2}{s-1}$$

By taking the inverse transform the solution is

$$y(t) = 50/81 + 5/9\, t + 31/81\, e^{9t} - 2e^{t}$$

Q4

(ii) $y'' - 6y' + 15y = 2\sin(3t)$ $y(0) = -1$
$y'(0) = -4$

Solution.

Taking the laplace transform of everything and plug in initial conditions

$$s^2Y(s) - sy(0) - y'(0) - 6(sY(s) - y(0)) + 15Y(s) = 2\frac{3}{s^2+9}$$

$$(s^2 - 6s + 15)Y(s) + s - 2 = \frac{6}{s^2+9}$$

$$Y(s) = \frac{-s^3 + 2s^2 - 9s + 24}{(s^2+9)(s^2-6s+15)}$$

we get the partial fraction decomposition.

$$Y(s) = \frac{As+B}{s^2+9} + \frac{Cs+D}{s^2-6s+15}$$

now, setting numerators equal give

$$-s^3 + 2s^2 - 9s + 24 = (As+B)(s^2-6s+15) + (Cs+D)(s^2+9)$$

$$= (A+C)s^3 + (-6A+B+D)s^2 + (15A-6B$$

$+ 15B + 9D$

Solving for constants :

$$\left.\begin{aligned} s^3 &= A + C = -1 \\ s^2 &= -6A + B + D = 2 \\ s^1 &= 15A - 6B + 9C = -9 \\ s^0 &= 15B + 9D = 24 \end{aligned}\right\} \quad A = \frac{1}{10} \quad B = \frac{1}{10} \quad C = \frac{-11}{10} \quad D = 5/g$$

applying in the constant gives

$$Y(s) = 1/_{10}\left(\frac{s+1}{s^2+9} + \frac{-11s+25}{s^2-6s+15}\right)$$

$$= \frac{1}{10}\left(\frac{s}{s^2+9} + \frac{13/3}{3s^2+9} - \frac{11(s-3)}{(s-3)^2+6} - 8\frac{\frac{\sqrt{r}}{\sqrt{r}}}{(s-3)^2+r}\right)$$

Finally take the inverse transform on a Solution will be then

$$y(t) = \frac{1}{10}\left(\cos(3t) + \frac{1}{3}\sin(3t) - 11e^{3t}\cos(\sqrt{r}t) - \frac{8}{\sqrt{6}}e^{3t}\sin\sqrt{6}t\right)$$

Q3 Define laplace transform along with example.

part (a)

Laplace Transform.

laplace transform is integral transform that converts a function of real variable (t) to the function of complex variable s.

i.e

The laplace transform of a function f(t) for t>0 is defining by the following integral over 0-∞

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

general example is

$$F(s) = \int_0^\infty f(t) e^{-st} dt.$$