"Assigment # 2"

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Subject * Differential Equation.
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1. Use the method of seperation of variables to find the General solution.

(a) $x' = \sqrt{x}$

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$x = \frac{t^2}{4} + \frac{C_1 t}{2} + C_2 \frac{1}{4}$

(b) $y' = e^{-2x}$

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$y = -\frac{1}{2} e^{-2x} + C_1$

(c) $y' = 1 + y^2$

$y' = 1+y^2$

$y = \tan(t + C_1)$

(d) $u' = \frac{1}{5-2u}$

$u' = \frac{1}{5-2u}$

$u = -\frac{1}{} \ln(5-2u) + C_1$

(e) $x' = au + b$, $a, b > 0$

(f) $Q' = \frac{Q}{4+Q^2}$

$Q' = \frac{Q}{4+Q^2}$

$\therefore (Q') = 0$

Derivative of constant $\frac{d}{dx}(Q) = 0$

(g) $x' = e^{x^2}$

$x' = e^{x^2}$

$\frac{\sqrt{\pi}}{2} \operatorname{erf}(x) = t + c_1$

(h) $y' = r(a-y)$

$y' = r(a-y)$

$y = -e^{-rt-c_1} + a$

2 solve $y' = r(a-y)$ where r and a are constant

* solution:-

$$y' = r(a-y)$$

$\therefore$ $r \Rightarrow$ constant

$a \Rightarrow$ constant

$$y = -e^{-rt-c_1} + a$$

Q (Find the General Solution)

(a) $x' = \dfrac{2x}{t+1}$

$$x'(t) = \frac{2\,x(t)}{t+1}$$

$$x(t) = \frac{1}{2} t x' + \frac{x'(t)}{2}$$

$$2\,x(t) = (t+1)\,x'(t)$$

$$x(t) = c_1(t+1)^2$$

(b) $\theta = t\sqrt{t^2+1}\sec\theta$

Given, $\theta = t\sqrt{t^2+1}\,\sec\theta$

$$\cancel{t}, \theta = \cancel{t}\sqrt{t^2+1}\,\sec\theta$$

$$t = \sqrt{\frac{-\sec\theta + \sqrt{4\theta^2 + \sec^2(\theta)}}{2\sec(\theta)}}$$

$$t = -\sqrt{\frac{(-\sec\theta) \pm \sqrt{(\sqrt{4\theta^2} + \sec^2(\theta}}{2\sec\theta}}$$

(c) $= (2u+1)u' - (t+1) = 0$

$(2u(t)+1)u'(t) - (t+1) = 0$

$u'(t)(1+2u(t)) = 1+t$

$u(t) = \frac{1}{2}\left(-\sqrt{c_1 + 2t^2 + 4t + 1} - 1\right)$

$u(t) = \frac{1}{2}\left(\sqrt{c_1 + 2t^2 + 4t + 1} - 1\right)$

(d) $R' = (t+1)(R^2+1)$

$R'(t) = (t+1)(R(t)^2 + 1)$

$\frac{R'(t)}{1+R(t)^2} = 1+t$

$R'(t) = (1+t)R(t^2) + 1 + t$

$R(t) = \tan\left[\frac{1}{2}(c_1 + t(t+2))\right]$

(e) $y' + y + \frac{1}{y} = 0$

$$y'(x) + y(x) + \frac{1}{y(x)} = 0$$

$$\frac{y'(x)}{-\frac{1}{y(x)} - y(x)} = 1$$

$$y'(x) = -y(x) - \frac{1}{y(x)}$$

$$y(x) = -\sqrt{e^{c_1-2x} - 1}$$

$$y(x) = \sqrt{e^{c_1-2x} - 1}$$

(f) $(t+1)x' + x^2 = 0$

$$(t+1)x'(t) + (x)(t)^2 = 0$$

$$\frac{x'(t)}{x(t)^2} = \frac{1}{-1-t}$$

$$x(t) = \frac{1}{c_1 + \log(t+1)}$$