Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:		**Total Marks:**	50

Student Details

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Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$ To the input $x(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	Marks 7 CLO 2
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$	Marks 7 CLO 2
Q2.	(a)	Determine the causal signal x(n) having the z-transform $x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6 CLO 2
	(b)	Evaluate the inverse z- transform using the complex inversion integral $X(z) = \frac{1}{1-az^{-1}} \quad \lvert z\rvert > \lvert a\rvert$	Marks 6 CLO 2
Q.3	(a)	A two- pole low pass filter has the system response $H(z) = \frac{b_0}{(1-pz^{-1})^2}$ Determine the values of b_0 and p such that the frequency response H(ω) satisfies the condition H(0) = 1 and $\lvert H(\frac{\pi}{4})\rvert^2 = \frac{1}{2}$.	Marks 6 CLO 3

	(b)	Design a two-pole bandpass filter that has the center of its passband at ω = π/2, zero in its frequency response characteristics at ω = 0 and ω = π and its magnitude response in $\frac{1}{\sqrt{2}}$ at ω = 4π/9.	Marks 6 CLO 3
Q 4	(a)	A finite duration sequence of Length L is given as $x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & otherwise \end{cases}$ Determine the N- point DFT of this sequence for $N \geq L$	Marks 6 CLO 2
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step $x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$	Marks 6 CLO 2

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Q 1

(a)

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

$x(n) = (-1)^n u(n)$

$y(-1) = 0$

$y(-2) = 0$

The characteristic equation is

$$\lambda^n - 4\lambda^{n-1} + 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 4\lambda + 4) = 0$$

roots are $\lambda = 2, 2$

Hence

$$y_h(n) = c_1 \lambda_1^n + c_2 \lambda_2^n$$

$$= c_1 (2)^n + c_2 (2)^n$$

The particular solution is

$$y_p(n) = K(-1)^n u(n)$$

substituting this solution into the differencial equation, we obtain

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2)$$
$$= (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$, $K(1+4+4) = 2 \Rightarrow K = \frac{2}{9}$

The total solution is

$$y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right]u(n)$$

For the initial conditions, we obtain

$y(-1) = 0$ and $y(-2) = 0$ then

$$c_1 + \frac{2}{9} = 0$$

$$\Rightarrow \boxed{c_1 = -\frac{2}{9}}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 0$$

$$\boxed{c_2 = \frac{1}{3}}$$

$$y(n) = \left[-\frac{2}{9}2^n + \frac{1}{3}2^n + \frac{2}{9}(-1)^n\right]u(n)$$

Q1 (b)

sol:- $y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$

The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = \frac{1}{2}, \frac{1}{5}$ Hence, $y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n$

with $x(n) = \delta(n)$, we have

$$y(0) = 2$$

$$y(1) - 0.7y(0) = 0 \Rightarrow y(1) = 1.4$$

Hence $c_1 + c_2 = 2$ and

$$\frac{1}{2}c_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

$$\Rightarrow c_1 + \frac{2}{5}c_2 = \frac{14}{5}$$

These equation yield

$$c_1 = \frac{10}{3}, \; c_2 = \frac{-4}{3}$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n)$$

The step response is

$$s(n) = \sum_{k=0}^{n} h(n-k),$$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left(\frac{1}{2}^n(2^{n+1}-1)u(n) - \frac{1}{3}\left(\frac{1}{5}^n(5^{n+1}-1)u(n)\right.\right.$$

Q2

(a)

sol:- first we express $x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$ in terms of positive powers of z, in the form

$$\frac{x(z)}{z} = \frac{z^2}{(2z-1)(z-1)^2}$$

$$\frac{X(z)}{z} = \frac{A_1}{2z-1} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$$

Find A, B and C

A = 4

B = -3

&

C = -1

Hence

$$x(n) = \left[4(2)^n - 3 - n\right] u(n)$$

Q 2
(b)

Sol:-

we have

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1-az^{-1}} dz$$

$$= \frac{1}{2\pi j} \oint_c \frac{z^n \, dz}{z-a}$$

where c is a circle at radius greater than $|a|$. we shall evaluate this integral using $\frac{1}{2\pi j} \oint_c \frac{f(z)}{z-z_0} dz$ with $f(z) = z^n$.

we distinguish two cases.

① If $n \geq 0$, $f(z) = z^n$ has only zeros and hence no poles ~~inside~~ inside c. The only pole inside c is $z = a$. Hence.

$$x(n) = f(z_0) = a^n \quad n \geq 0$$

② If $n < 0$, $f(z) = z^n$ has an n^{th}-order pole at $z = 0$ which is also inside c. Thus there are contributions from both poles. For $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j} \oint_c \frac{1}{z(z-a)} dz = \left.\frac{1}{z-a}\right|_{z=0} + \left.\frac{1}{z}\right|_{z=a}$$

$$x(-1) = 0$$

If $n = -2$, we have

$$x(-2) = \frac{1}{2\pi j} \oint_c \frac{1}{z^2(z-a)} dz$$

$$= \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a} = 0$$

By continuing in the same way we can show that $x(n) = 0$ for $n < 0$. Thus

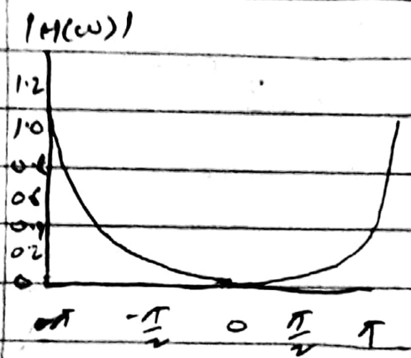
$$x(n) = a^n u(n)$$

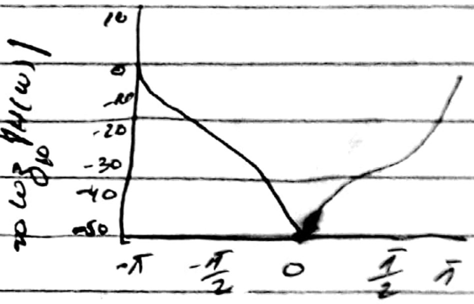
P3
(a)
sol^n

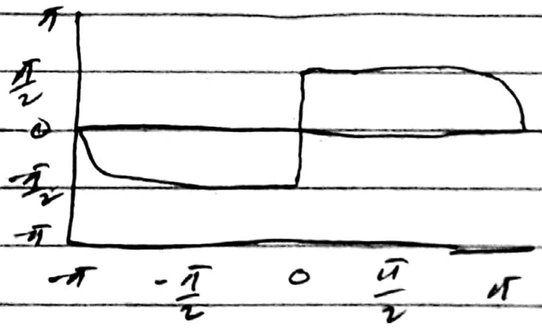
At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

$$b_0 = (1-p)^2$$







At $\omega = \pi/4$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4) + jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2} + jp/\sqrt{2})^2}$$

Hence

$$\frac{(1-p)^4}{[(1-p/\sqrt{2})^2 + p^2/2]^2} = \frac{1}{2}$$

or, equivalently

$$\sqrt{2}(1-p)^2 = 1+p^2-\sqrt{2}p$$

The value of $p = 0.32$ satisfies this equation. Consequently, the system function for desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

Q3
(A)
sol:-

Clearly, the filter must have pole at

$$P_{1,2} = re^{j/\pi/2}$$

and zeros at $z=1$ and $z=-1$. consequently, the system function is

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\left(\frac{z^2-1}{z^2+r^2}\right)$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$. Thus we have

$$H\left(\frac{\pi}{2}\right) = G\frac{2}{1-r^2} = 1$$

$$= G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

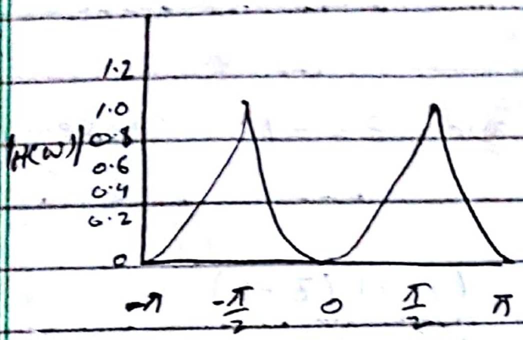
or, equivalently,

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfies this equation. Therefore, the system function for the desired filter is

$$H(z) = 0.15\,\frac{1-z^{-2}}{1+0.7z^{-2}}$$

Its frequency response is illustrated in fig



Q4
(a)
Sol:-

The fourier transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustrated in fig for $L=10$. The N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally spaced frequencies $\omega_K = 2\pi K/N$, $K = 0, 1, \ldots, N-1$. Hence

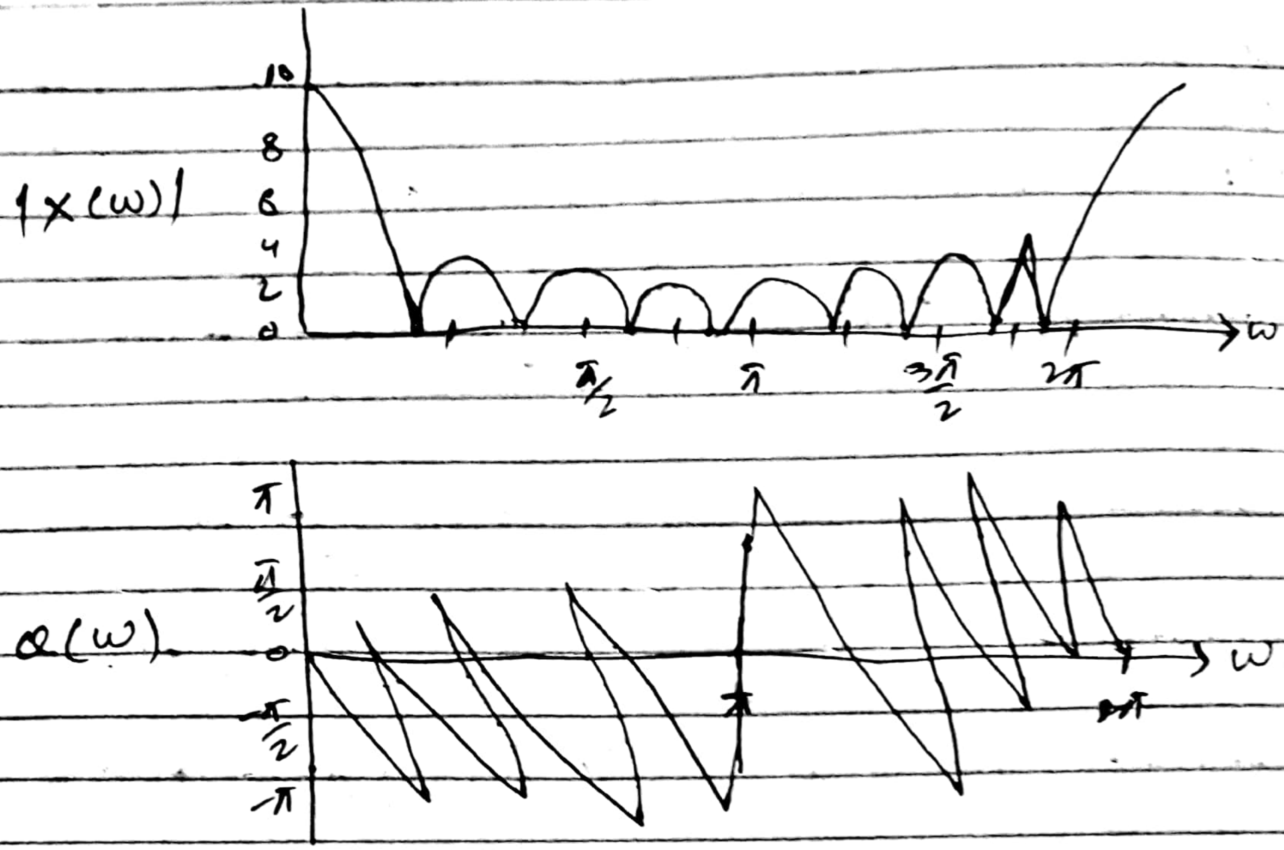
$$X(K) = \frac{1-e^{-j2\pi KL/N}}{1-e^{-j2\pi K/N}}$$

$K = 0, 1, 2, \ldots, N-1$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$

If N is selected such that $N=L$ then the DFT becomes

$$X(K) = \begin{cases} L, & K=0 \\ 0, & K = 1, 2, \ldots L-1 \end{cases}$$



magnitude & phase characteristic

Q4 (b)

Sol

Step 1:-

Linear convolution of two sequences $x_1(n)$ of length n_1 samples and $x_2(n)$ of length n_2 samples is given by

$$y(n) = x_1(n) * x_2(n)$$

$$= \sum_{k=0}^{3} x_1(k) x_2(n-k)$$

$$= \{ \underset{\uparrow}{2}, 5, 10, 16, 12, 11, 4 \}$$

Step 2:- compute length L of the linear convolution result is $L = 7$

Step 3:- compute maximum length of $x_1(n)$ and $x_2(n)$ i.e,

$N = \max(n_1, n_2) = 4$.

Step 4:- compute number of zeros to be padded to linear convolution result using

$$P = 2 \times N - L = 8 - 7 = 1$$

Step 5:- Pad P number of zeros to the end of convolution result

$$y(n) = \{ \underset{\uparrow}{2}, 5, 10, 16, 12, 11, 4, 0 \}$$

step 6:- length of convolution result is $N_1 = L+P = 8$

step 7:- To compute circular convolution of two sequences, add 0th sample to $(N_1/2)$ sample of convolution result, next, add first sample with $(N_1/2)+1$ sample, and repeat this procedure for all the sample up to $(N_1/2 - 1)$.

$$x_1(n) * x_2(n) = \{ \underset{\uparrow}{14}, 16, 14, 16 \}$$

each sequence consists of four nonzero points

Now $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with $x_2(n)$ as specified by $x_3(m) = \sum_{n=0}^{N-1} x_1(n) x_2((m-n))_N \quad m = 0, 1, \ldots N-1.$

Beginning with $m=0$ we have

$$x_3(0) = \sum_{n=0}^{3} x_1(n) x_2((-n))_N$$

$x_2((-n))_4$ is simply the sequence $x_2(n)$ folded.

The product sequence is obtained by multiplying $x_1(n)$ with $x_2((-n))_4$, point by point. Finally, we sum the values in the product sequences to obtain.

$$x_3(0) = 14$$

For $m=1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

It is easily verified that $x_2((1-n))_4$ is simply the sequence $x_2((-n))_4$ rotated countorclockwise by one unit in time as illustrated ~~in~~. This rotated sequence multiplies $x_1(n)$ to yield the product sequence, ~~and~~ finally we sum the values in the product sequence to obtain $x_3(1)$.

$$x_3(1) = 16$$

For $m=2$ we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

Now $x_2((2-n))_4$ is the folded sequence By summing the four terms in the product sequence, we obtain

$$x_3(2) = 14$$

For $m=3$ we have

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2((3-n))_4$$

$$x_3(3) = 16$$

we observe that if the computation above is continued beyond $m=3$, we simply repeat the sequence of four values obtained above. There fore, the circular convolution of the two sequences $x_1(n)$ and $x_2(n)$ yields the sequences.