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SECTION: B

DEPT:- BE Civil

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Assignment No: 02.

Ques 1. 01:

$$x^3 y''' + 2x^2 y' + 2y = 10x + \frac{10}{x}$$

Solution:

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{dy}{dx} + 2y = 10x + \frac{10}{x}$$

$$x^3 D^3 y + 2x^2 D^2 y + 2y = 10x + 10x^{-1}$$

$$y (x^3 D^3 + 2x^2 D^2 + 2) = 10x + 10x^{-1} \quad \text{--- (1)}$$

Let: $x = e^t \implies t = \ln x$

$$xD = D$$

$$x^2 D^2 = D(D-1) = D^2 - D$$

$$x^3 D^3 = D(D-1)(D-2)$$

substituting into eq (1)

$$(D - 3D^2 + 2D + 2(D^2 - D) + 2)y = 10x + 10x^{-1}$$

$$(D^3 - D^2 + 2)y = 10x + 10x^{-1}$$

$$(m^3 - m^2 + 2)y = 10e^t + \frac{10}{e^t}$$

using synthetic division.

$$\begin{array}{r|rrrr}
 -1 & 1 & -1 & 0 & 2 \\
 & & -1 & 2 & -2 \\
 \hline
 & 1 & -2 & 2 & 0
 \end{array}$$

$$D^2 - 2D + 2 = 0$$

By using quadratic formula:

$$a = 1, b = -2 \text{ and } c = 2.$$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$D = \frac{2 \pm \sqrt{4 - 8}}{2}$$

$$D = \frac{2 \pm \sqrt{-4}}{2}$$

$$D = \frac{2 \pm \sqrt{-1} \cdot \sqrt{4}}{2}$$

$$D = \frac{2 \pm 2i}{2}$$

$$D = \frac{2(1 \pm i)}{2}$$

$$D = 1 \pm i$$

Since roots are complex.

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$$y_c = e^{-x} (C_1 \cos t + C_2 \sin t).$$

Now particular integration.

$$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^3 - D^2 + 2} \cdot 10/e^t.$$

$$= \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^{-t}}{(1)^3 - (1)^2 + 2}$$

$$= \frac{10e^t}{2} + \frac{10e^{-t}}{2}$$

$$= 5e^t + 5e^{-t}$$

$$y_p = 5e^t + 5e^{-t}$$

⇒ General Solution:

$$y = y_c + y_p.$$

$$y = e^{-x} (C_1 \cos t + C_2 \sin t) + 5e^t + 5e^{-t}$$

Put $e^t = x$ and $t = \ln x$

$$y = e^{-x} (C_1 \ln x + C_2 \sin \ln x) + 5e^x + 5e^{-x} \quad \text{Ans}$$

Ques : No #02:

$$x^3 \frac{d^3 y}{dx^3} + 4x^2 \frac{d^2 y}{dx^2} - 5x \frac{dy}{dx} - 15y = x^4$$

Solution:

$$\text{let } d/dx = D$$

$$x^3 D^3 y + 4x^2 D^2 y - 5x D y - 15y = x^4$$

$$(x^3 D^3 + 4x^2 D^2 - 5x D - 15)y = x^4$$

$$\text{let } ; x = e^t \Rightarrow t = \ln x.$$

$$xD = D.$$

$$x^2 D^2 = D(D-1) = D^2 - D.$$

$$x^3 D^3 = D(D-1)(D-2) = D^3 - 3D^2 + 2D$$

Now substituting :-

$$(x^3 D^3 + 4x^2 D^2 - 5x D - 15)y = x^4$$

$$[D^3 - 3D^2 + 2D + 4(D^2 - D) - 5(D) - 15]y = e^{4t}$$

$$(D^3 + D^2 - 7D - 15)y = e^{4t}$$

By Synthetic division.

	1	+1	-7	-15
5		3	12	15
	1	4	5	0

$$D^2 + 4D + 5 = 0.$$

Quadratic formula:

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here; $a = 1, b = 4, c = 5.$

$$D = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$$

$$D = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$D = \frac{-4 \pm \sqrt{-4}}{2}$$

$$D = \frac{-4 \pm \sqrt{-1} \cdot \sqrt{4}}{2}$$

$$D = \frac{-4 \pm 2i}{2}$$

$$D = -2 \pm i$$

$$y_c = e^{3x} (C_1 \cos t + C_2 \sin t)$$

For $y_p = ?$

$$y_p = \frac{1}{D^3 + D^2 - 7D - 15} \cdot e^{4t}$$

$$y_p = \frac{1}{(4)^3 + (4)^2 - 7(4) - 15} \cdot e^{4t}$$

$$y_p = \frac{1}{64 + 16 - 28 - 15} \cdot e^{4t}$$

$$y_p = \frac{1}{80 - 43} \cdot e^{4t}$$

$$y_p = \frac{1}{37} \cdot e^{4t}$$

Now; $y = y_c + y_p$.

$$y = (C_1 \cos t + C_2 \sin t) + \frac{1}{37} e^{4t}$$

again put $t = \ln x$ & $x = \ln u$.

$$y = e^{3x} (C_1 \cos \ln x + C_2 \sin \ln x) + \frac{1}{37} e^{4x}$$

Ans:

Ques : No: # 03:

$$x^2 y'' + 2xy' - 6y = 10x^2 ; y(1) = 1, y'(1) = 6$$

Solution:-

$$x^2 y'' + 2xy' - 6y = 10x^2$$

$$y(1) = 1 \quad \text{and} \quad y'(1) = 6.$$

$$x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} - 6y = 10x^2$$

$$\Rightarrow (x^2 \frac{d^2}{dx^2} + 2x \frac{d}{dx} - 6)y = 10x^2$$

$$\text{put } xD = D \Rightarrow x^2 D^2 = D(D-1) = D^2 - D$$

$$x = e^t \quad \text{and} \quad \log x = t.$$

$$(D^2 + 2D - D - 6)y = 10e^{2t}$$

$$(D^2 + D - 6)y = 10e^{2t}$$

The characteristic equation.

$$D^2 + D - 6 = 0$$

$$D^2 + 3D - 2D - 6 = 0$$

$$\Rightarrow D(D+3) - 2(D+3) = 0$$

$$(D-2)(D+3) = 0$$

$$D = 2, \quad D = -3.$$

Since root are real and distinct.

For $y_c = 7$

$$y_c = C_1 e^{-3t} + C_2 e^{2t}$$

For $y_p = ?$

$$y_p = \frac{1}{D^2 - D - 6} \cdot 10 e^{2t}$$

$$y_p = \frac{10}{D^2 - D - 6} e^{2t}$$

$$= 10 \frac{1}{0} e^{2t}$$

Now,

$$10 \frac{1}{d/dD(D^2 + D - 6)} e^{2t}$$

$$\Rightarrow 10 \frac{t}{2D + 1} e^{2t}$$

$$= 10 \frac{1 - t}{4 + 1} e^{2t}$$

$$y_p = 2t e^{2t}$$

General Solution

$$y = y_c + y_p$$

$$y = C_1 e^{-3t} + C_2 e^{2t} + 2te^{7t}$$

$$y = C_1 x^{-3} + C_2 x^2 + 2(\log x)x^2 \text{ --- (B)}$$

put $y(1) = 1$ i.e. $x = 1$ & $y = 1$ in (B)

$$1 = C_1 (1)^{-3} + C_2 (1)^2 + 2(\log(1))(1)^2$$

$$1 = C_1 + C_2 \text{ --- (C)}$$

Now differentiate eq (B) w.r.t x .

$$y' = -3C_1 x^{-4} + 2C_2 x + \frac{2(x^2)}{x} + 4x \log x.$$

Now put $y'(1) = -6$ i.e. $y' = -6$ & $x = 1$

$$-6 = -3C_1 + 2C_2 + 2 + 0.$$

$$\Rightarrow -6 = -3C_1 + 2C_2 + 2.$$

$$\Rightarrow -6 - 2 = -3C_1 + 2C_2.$$

$$-8 = -3C_1 + 2C_2 \text{ --- (D)}$$

X-ing eq (C) with (2) and +ing from (D)

$$2C_1 + 2C_2 = 2$$

$$= 8C_1 + 2C_2 = 8$$

$$5C_1 = 10$$

$$5C_1 = 10.$$

$$C_1 = 10/5 = 2.$$

$$-8 = -3(2) + 2(C_2).$$

$$-8 = -6 + 2C_2.$$

$$-8 + 6 = 2C_2$$

$$-2 = 2C_2.$$

$$C_2 = -1.$$

Now put this values of C_1 & C_2 in (B)

$$y = 2x^{-3} - x^2 + 2 \ln x \cdot x (x^2).$$

$$y = \frac{2}{x^3} - x^2 + 2x^2 \log x \quad \text{Ans:-}$$

Ques : No # 04:

$$x^2 y'' + 7xy' + 5y = x^5$$

$$y(0) = 2 \quad \& \quad y'(1) = 2$$

Solution:-

$$x^2 \frac{d^2 y}{dx^2} + 7x \frac{dy}{dx} + 5y = x^5$$

$$\Rightarrow \left(x^2 \frac{d^2}{dx^2} + 7x \frac{d}{dx} + 5 \right) y = x^5 \quad \text{--- (A)}$$

$$\text{put } xD = D \Rightarrow x^2 D^2 = D(D-1) = D^2 - D$$

$$x = e^t \Rightarrow \log x = t \quad \text{in eq (A)}$$

$$\Rightarrow (D^2 - D + 7D + 5) y = e^{5t}$$

$$\Rightarrow (D^2 + 6D + 5) y = e^{5t}$$

By Quadratic formula:-

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

$$D = \frac{-6 \pm \sqrt{36 - 20}}{2}$$

$$D = -6 \pm \sqrt{16}$$

$$D = -6 \pm 4$$

$$D = 2(-3 \pm 2)$$

$$D = -3 \pm 2.$$

Since roots are real & distinct.

$$y_c = C_1 e^{-5t} + C_2 e^{-t}$$

For $y_p = ?$

$$y_p = \frac{1}{D^2 + 6D + 5} e^{5t}$$

$$= \frac{1}{(5)^2 + 6(5) + 5} e^{5t}$$

$$= \frac{1}{60} e^{5t}$$

Now general solution. is

$$y = y_c + y_p.$$

$$y = C_1 e^{-5t} + C_2 e^{-t} + \frac{1}{60} e^{5t}$$

$$y = C_1 x^{-5} + C_2 x^{-1} + \frac{1}{60} x^5 \quad \text{--- (3)}$$

$x=0$ put in this eq B
 $e^0 = 1$.

put $y(0) = 2$ i.e $y = 2$ at $x = 0$.

$$2 = c_1 (2)^{-5} + c_2 (2)^{-1} + \frac{1}{60} (2)^5$$

$$2 = -32c_1 - 2c_2 + \frac{1}{60} (30)$$

$$2 = -32c_1 - 2c_2 + \frac{8}{18}$$

$$2 - \frac{8}{15} = -32c_1 - 2c_2$$

$$\frac{22}{15} = -32c_1 - 2c_2 \quad \text{--- (C)}$$

Now differentiating eq (B) w.r.t x .

$$y' = -5c_1 x^{-6} - c_2 x^{-2} + \frac{1}{12} x^4$$

put $y'(1) = 2$ i.e $y' = 2$ at $x = 2$
 in above eq.

~~$$2 = -5c_1 (2)^{-6} - c_2 (2)^{-2} + \frac{1}{12} (2)^4$$~~

$$2 = -5c_1 (2)^{-6} - c_2 (2)^{-2} + \frac{1}{12} (2)^4$$

$$2 = -5c_1 (64) - c_2 (4) + \frac{1}{12} (16)$$

$$2 = -320c_1 + 4c_2 + \frac{4}{3}$$

$$2 - \frac{4}{3} = 320C_1 + 4C_2$$

$$\frac{2}{3} = 320C_1 + 4C_2 \quad \text{--- (D)}$$

X-ing eq (C) with (2) and
then adding eq (C) from (D).

$$\frac{-44}{15} = 64C_1 + 4C_2$$

$$\frac{-44}{15} = 64C_1 + 4C_2$$

$$+ \frac{2}{3} = + 320C_1 + 4C_2$$

$$\frac{34}{15} = -256C_1$$

$$C_1 = 580$$

put the value of C_1 in eq (C)

$$\frac{22}{15} = -18560 - 2C_2$$

$$\Rightarrow \frac{22}{15} + 18560 = -2C_2$$

$$C_2 = -9280$$

put these values in (c)

$$\frac{22}{15} = -32(580) - 2C_2$$

$$\frac{22}{15} = -18560 - 2C_2$$

$$\frac{22}{15} + 18560 = -2C_2$$

$$+2C_2 = \frac{18561}{-2}$$

$$C_2 = -9280$$

Now put the value of C_1 , E_1 , C_2 in (B)

$$y = 580x^5 - 9230x^{-1} + \frac{1}{60}x^5$$

$$y = \frac{580}{x^5} - \frac{9230}{x} + \frac{1}{60}x^5$$

Ans:-

ques: # 05:

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$$(x+1)^2 y'' - 3(x+1)y' + 4y = x^2$$

Solution-

$$(x+1)^2 y'' - 3(x+1)y' + 4y = x^2$$

$$(x+1)^2 \frac{d^2 y}{dx^2} - 3(x+1) \frac{dy}{dx} + 4y = x^2$$

$$\Rightarrow \left[(x+1)^2 \frac{d^2}{dx^2} - 3(x+1) \frac{d}{dx} + 4 \right] y = x^2$$

$$\Rightarrow \left[(x+1)^2 D^2 - 3(x+1)D + 4 \right] y = x^2 \quad \text{--- (A)}$$

$$\text{put } (x+1)D = D \Rightarrow (x+1)^2 D^2 = D(D-1) \\ = D^2 - D$$

$x = e^t$ in eq (A)

$$\Rightarrow [D^2 - D - 3D + 4] y = e^{2t}$$

$$\Rightarrow (D^2 - 4D + 4) y = e^{2t}$$

for yc we find the roots.

$$D^2 - 4D + 4 = 0$$

$$D^2 - 2D - 2D + 4 = 0$$

$$D(D-2) - 2(D-2) = 0$$

$$D = 2, D = 2.$$

The roots are real & repeat.
The general solution is;

$$y = (C_1 + C_2 x)^{mx}$$

$$y = (C_1 + C_2 x)^{2x}$$

For $y_p = ?$

$$y_p = \frac{1}{D^2 - 4D + 4}$$

$$y_p = \frac{2}{2D - 4} e^{2t}$$

If we put "2"

$$2D - 4 \Rightarrow 2(2) - 4 = 0.$$

We take again derivative.

$$y_p = \frac{2}{2} e^{2t} = e^{2t}$$

$$y = (C_1 + C_2 x)^{2x} + e^{2x}$$

General Solution : Ans.