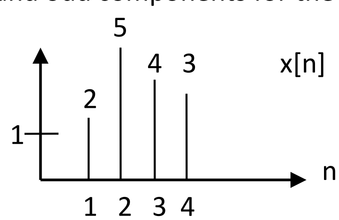
Department of Electrical Engineering
Sessional Assignment
Date: 05/05/2020

Course Details

Course Title: Signals & Systems **Module:** 04
Instructor: Engr Mujtaba Ihsan **Total Marks:** 20

Student Details

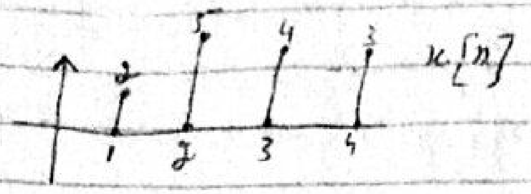
Name: Irshad Khan **Student ID:** 12403

Q1.		**Evaluate** the even and odd components for the given function. 	Marks 05 CLO 1
Q2.		**Calculate** the inverse Laplace transform of the given equation. $$\mathbf{Y(s)} = \frac{\mathbf{s+4}}{\mathbf{s^2 + 4s - 12}}$$	Marks 07 CLO 3
Q3.	i. ii.	**Discuss** the procedure of converting an analog signal into a digital one. Suppose an analog signal has a highest frequency of 60Hz. **Outline** the steps that will ensure that no aliasing occurs.	Marks 02+02 CLO 2
Q4.		**Show** that: $x[n] * [h_1[n] * h_2[n]] = [x[n] * h_1[n]] * h_2[n]$	Marks 04 CLO 2

Assignment: signal and system

Irshad Khan ID # 12403 (1)

Q1: Evaluate the even and odd components for the given function



Solution: As we know that

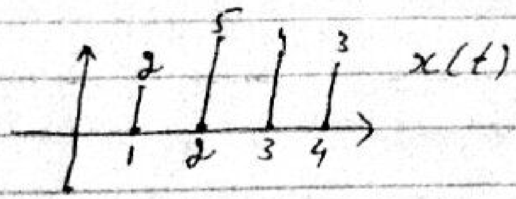
For even component, $x_e(n) = x(n) + x(-n)$

For odd component, $x_o(n) = \dfrac{x(n) - x(-n)}{2}$

$\Rightarrow x_e(t) = \frac{1}{2}[x(n) + x(-n)]$

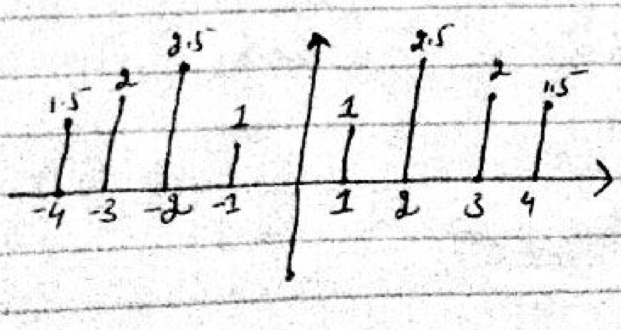
$\Rightarrow x_o(t) = \frac{1}{2}[x(n) - x(-n)]$

First to determine $x(-n)$



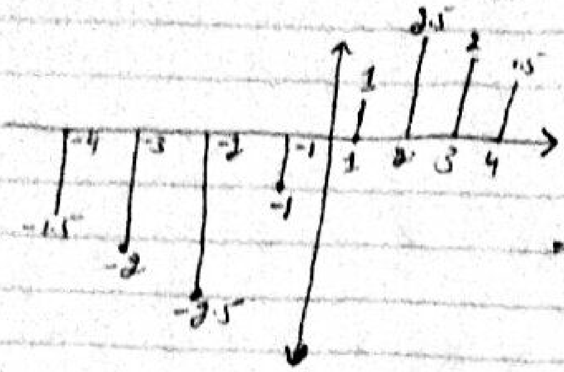
To obtain x_e we need to half the amplitudes as so, $x_e(n)$ can be drawn as

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$



$\Rightarrow$ even component

similarly $x_o[n] = \dfrac{x[n] - x[-n]}{2}$



→ odd component.

So above two graphs are the required even and odd components of the given function.

Q2:

calculate the inverse laplace.

$$Y(s) = \frac{s+4}{s^2+4s-12}$$

Solution: $Y(s) = \dfrac{s+4}{s^2+6s-2s-12}$

$$Y(s) = \frac{s+4}{s(s+6)-2(s+6)}$$

$$Y(s) = \frac{s+4}{(s+6)(s-2)}$$

using partial Functions.

$$\frac{s+4}{(s+6)(s-2)} = \frac{A}{s+6} + \frac{B}{s-2} \longrightarrow ①$$

$$s+4 = \cancel{(s+6)}(s-2)\frac{A}{\cancel{s+6}} + (s+6)\cancel{(s-2)}\frac{B}{\cancel{s-2}}$$

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$s+4 = A(s-2) + B(s+6)$

For $s=2$

$\Rightarrow 2+4 = A(\cancel{2-2}) + B(2+6)$

$6 = 8B \Rightarrow B = 6/8 \Rightarrow \boxed{B = 3/4} \longrightarrow (2)$

For $s = -6$

$\Rightarrow -6+4 = A(-6-2) + B(\cancel{-6+6})$

$+2 = +8A \Rightarrow A = 2/8 \Rightarrow \boxed{A = \frac{1}{4}} \longrightarrow (3)$

put (2) and eq (3) in (1).

$$\frac{s+4}{(s+6)(s-2)} = \frac{1}{4(s+6)} + \frac{3}{4(s-2)}$$

so $Y(s) = \frac{1}{4(s+6)} + \frac{3}{4(s-2)}$

Applying L^{-1} on B.H.S.

$$L^{-1}(Y(s)) = L^{-1}\left(\frac{1}{4} \cdot \frac{1}{s+6}\right) + L^{-1}\left(\frac{3}{4} \cdot \frac{1}{s-2}\right)$$

$$y(t) = \frac{1}{4}\left(L^{-1}\frac{1}{s+6}\right) + \frac{3}{4} L^{-1}\left(\frac{1}{s-2}\right)$$

$$\boxed{y(t) = \frac{1}{4}e^{-6t} + \frac{3}{4}e^{2t}}$$ required laplace inverse

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Q3:- Discuss the procedure of converting ......
(i) into a digital ones.

Ans:- Converting any analog signal into a digital signal requires three processes or we can call them steps.

1. Sampling
2. Quantization
3. Coding.

First an analog signal which is continious time and continious amplitude signal is converted to discrete time continious amplitude. and only time axis is discretized and this process is called sampling.

Then through quantization the obtained discrete time continious amplitude signal is converted into discrete time and discrete valued signal that has finite represented values.

Then finally through coding the obtained finite value are given binary codes as zeros and ones to explicity obtained the final digital output.

Q3: suppose an analog signal ....
(ii) .... no aliasing occurs.?

Ans:- As the highest frequency given is, $F_{max} = 60Hz$ and we require to ensure there is no aliasing, so we need to apply Nyquist rate.

$$F_N \geq 2 \times F_{max}$$
$$F_N \geq 2 \times 60Hz$$
$$F_N \geq 120Hz.$$

so by keeping the sampling rate equal or above 120 Hz we can ensure that there will no aliasing occur

Q 4: Show that

$$x[n] * \{h_1[n] * h_2[n]\} = [x[n] * h_1[n]] * \{h_2[n]\}$$

Solution: Taking R.H.S & let $n = k$

$$[x[k] * h_1[k] * h_2[k]$$

as $[x[k] * h_1[k]]$ can be obtained by convolution sum so we can write (by multiplying $x[k]$ & $h_1[k]$).

as. $[x[k] * h_1[k] = [\sum_{m=-\infty}^{\infty} x[m] h_1[k-m]]$

so R.H.S becomes.

$$[x[k] * h_1[k]] * h_2[k] = [\sum_{m=-\infty}^{\infty} x[m] h_1[k-m]]$$

$* h_2[k]$.

Now again the star sign indicated another convolution sum b/w

$$\sum_{m=-\infty}^{\infty} x[m] h_1[k-m] \;\&\; h_2[k].$$

So, $\sum_{n=-\infty}^{\infty} [\sum_{m=-\infty}^{\infty} x[m] h_1[n-m]] h_2[k-n]$

$$= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x[m] h_1[n-m] h_2[k-n]$$

we can rearrange the summations as

$$\sum_{m=-\infty}^{\infty} x[m] \sum_{n=-\infty}^{\infty} h_1[n-m]\, h_2[k-n]$$

Now let $k-n = r$

$n = k - r$

for $n = \infty$, $r = -\infty$

& $n = \infty$ and $r = -\infty$, $r = \infty$

So now substitute n by $k-r$.

$$= \sum_{m=-\infty}^{\infty} x[m] \sum_{r=+\infty}^{-\infty} h_1[k-r-m]\; h_2[r]$$

$$= \sum_{m=-\infty}^{\infty} x[m] \underbrace{\sum_{r=-\infty}^{\infty} h_2[r]\, h_1[k-m]-r]}$$

As $\sum_{r=-\infty}^{\infty} h_2[r]\, h_1[(k-m)-r] = h_2[m] * h_1[k-m]$

using commutative property and time shift property

let $z[k] = h_1[k] * h_2[k]$

hence

$h_2[m] * h_1[k-m] = h_1[k-m] * h_2[m] = z[k-m]$

Now

$$\sum_{m=-\infty}^{\infty} x[m]\, z[k-m]$$

$= x[k] * z[k]$

put the value of $z[k]$

$: x[k] * h_1[k] * h_2[k]]$ is equal to L.H.S.

Hence proved.