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Semester:- BS (SE) 4th Section:- A

Q1) There are total of 5 machines and five employments are to be relegated and the related cost network is as per the following. Located the best possible task.

Solution:-

J	Machines					
		A	B	C	D	E
O	1	6	12	3	11	15
B	2	4	2	7	1	10
S	3	8	11	10	7	11
	4	16	19	122	23	21
	5	9	5	7	6	10

Balanced assignment problem

Hungarian method

Phase I:- Row and column reduction

Step 0:- subtract the minimum value of each row from the entries of that column

Phase (2) optimization of the problem

a) Row scanning

b) Column scanning

		Machines					
		A	B	C	D	E	
J O B S	1	6	12	3	11	15	3
	2	4	2	7	1	10	1
	3	8	11	10	7	11	7
	4	16	19	122	23	21	16
	5	9	5	7	6	10	5

		Machines				
		A	B	C	D	E
J O B S	1	3	9	0	8	12
	2	3	1	6	0	9
	3	1	4	3	0	4
	4	0	3	106	7	5
	5	4	0	2	1	5
		0	0	0	0	4

Columns minimum:

		Machines					
		A	B	C	D	E	
J O B S	1	3	9	0	8	8	-> Row scanning
	2	3	1	6	0	9	
	3	1	4	3	0	4	-> Column scanning
	4	0	3	106	7	5	
	5	4	0	2	1	5	

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	Machines				
	A	B	C	D	E
1	6	12	13	11	15
2	4	2	7	11	10
3	8	11	10	7	11
4	16	19	22	23	21
5	9	5	7	6	10

Job	Machines	Times
1	C	3
2	D	1
3	E	11
4	A	16
5	B	5

Total hours = 36

Solution is optimal and Job machine operator.



Q2) $\text{min } z = 2x_1 + 3x_2$
 subj = $(\frac{1}{2})x_1 + (\frac{1}{4})x_2 \leq 4$
 $x_1 + 3x_2 \geq 20$
 $x_1 + x_2 = 10$
 $x_1, x_2 \geq 0$

Solution:- $(\frac{1}{2})x_1 + \frac{1}{4}x_2 + s_1 = 4$
 $x_1 + 3x_2 - s_2 + a_1 = 20$
 $x_1 + x_2 + a_2 = 10$

$$z = -2x_1 - 3x_2$$

$$z = -2x_1 - 3x_2 - mA_1 - mA_2$$

$$z + 2x_1 + 3x_2 + mA_1 + mA_2 = 0$$

$$2x_1 + 3x_2 + mA_1 + mA_2 + z = 0$$

	x_1	x_2	s_1	s_2	a_1	a_2	z	
$\rightarrow R_1$	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4
$\rightarrow R_2$	1	3	0	-1	1	0	0	20
$\rightarrow R_3$	1	1	0	0	0	1	0	10
$\rightarrow R_4$	2	3	0	0	m	m	1	0

$$R_4 + (-mR_3)$$

$$\begin{array}{cccccccc} 2 & 3 & 0 & 0 & m & m & 1 & 0 \\ -m & -m & 0 & 0 & 0 & -m & 0 & -10m \\ \hline 2-m & 3-m & 0 & 0 & m & 0 & 1 & -10m \end{array}$$

x_1	x_2	s_1	s_2	a_1	a_2	z	
$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4
1	3	0	-1	1	0	0	20
1	1	0	0	0	1	0	10
$2-m$	$3-m$	0	0	m	0	1	$-10m$

$R_4 + (-mR_2)$

$$\begin{array}{r}
 2-m \quad 3-m \quad 0 \quad 0 \quad m \quad 0 \quad 1 \quad -10m \\
 -m \quad -3m \quad 0 \quad m \quad -m \quad 0 \quad 0 \quad -20m \\
 \hline
 2-2m \quad 3-4m \quad 0 \quad m \quad 0 \quad 0 \quad 1 \quad -30m
 \end{array}$$

x_1	x_2	s_1	s_2	a_1	a_2	z	
$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4 = 16
1	3	0	-1	1	0	0	20
1	1	0	0	0	1	0	10
$2-2m$	$3-4m$	0	m	0	0	1	$-30m$

$\times 4$ to R_1

x_1	x_2	s_1	s_2	a_1	a_2	z	
2	1	4	0	0	0	0	16
1	3	0	-1	1	0	0	20
1	1	0	0	0	1	0	10
2-2m	3-4m	0	m	0	0	1	-30m

$\times \frac{1}{3}$ to R_1

x_1	x_2	s_1	s_2	a_1	a_2	z	
2	1	4	0	0	0	0	16
$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	$20/3$
1	1	0	0	0	1	0	10
2-2m	3-4m	0	m	0	0	1	-30m

$R_1 + (-1)R_2$

2	1	4	0	0	0	0	0	16
$-\frac{1}{3}$	-1	0	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	0	$-20/3$
$5/3$	0	4	$1/3$	$-1/3$	0	0	0	$28/3$

x_1	x_2	s_1	s_2	a_1	a_2	z	
$5/3$	0	4	$1/3$	$-1/3$	0	0	$28/3$
$1/3$	1	0	$-1/3$	$1/3$	0	0	$20/3$
1	1	0	0	0	1	0	10
2-2m	3-4m	0	m	0	0	1	-30m

$R_2 + (-1 R_1)$

$$\begin{array}{cccccccc}
 1 & 1 & 0 & 0 & 0 & 1 & 0 & 10 \\
 -\frac{1}{3} & -1 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & -\frac{20}{3} \\
 \hline
 \frac{2}{3} & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 1 & 0 & \frac{10}{3}
 \end{array}$$

x_1	x_2	s_1	s_2	a_1	a_2	z	
$\frac{5}{3}$	0	4	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	$\frac{28}{3}$
$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{20}{3}$
$\frac{2}{3}$	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	1	0	$\frac{10}{3}$
$2-2m$	$3-4m$	0	m	0	0	1	$-30m$

$R_4 + (- (3-4m R_2))$

$$\begin{array}{cccccccc}
 2-2m & 3-4m & 0 & m & 0 & 0 & 1 & -30m \\
 -1+4m & -3+4m & 0 & 1-4m & -1+4m & 0 & 0 & -10+80m \\
 \hline
 1+2m & 0 & 0 & 1-3m & 1+4m & 0 & 1 & -10+50m
 \end{array}$$

	x_1	x_2	s_1	s_2	a_1	a_2	z	
x_2	$\frac{5}{3}$	0	4	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	$\frac{28}{3}$
s_1	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{20}{3}$
a_1	$\frac{2}{3}$	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	1	0	$\frac{10}{3}$
z	$1+2m$	0	0	$1-3m$	$1+4m$	0	1	$-10+50m$

$x_2 = \boxed{\frac{28}{3}}$

$s_1 = \boxed{\frac{20}{3}}$

$a_2 = \boxed{\frac{10}{3}}$

$z = \boxed{-10 + 50m}$

Solution.

Q3 Use Vogel's Method to obtain the initial feasible solution of:

Origin	Destination				Supply
	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Demand	60	40	30	110	240

240 = 240

Demand = Supply

(Balanced transportation problem)

	A	B	C	D	supply
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Demand	60	40	30	110	240

	A	B	C	D	supply
1	40	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Demand	60	40	30	110	240

	A	B	C	D	supply
1	4	8	8	3	13
2	4	-	8	3	2
3	8	-	11	8	2
4	8	-	-	8	5

Total cost =

$$= 40(22) + 80(4) + 10(24) + 30(9) + 30(7) + 50(32)$$

$$= 880 + 320 + 240 + 270 + 210 + 1600$$

Ans = 3520 =)